

Decision Feedback Blind Equalization Based on Recurrent Least Squares Algorithm for Underwater Acoustic Channels

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Abstract

The cost function of constant modulus algorithm (CMA) is simplified to meet second norm form, and the blind equalizer can use recurrent least squares (RLS) algorithm to update the weights. Considering the underwater acoustic channel is usually nonlinear, decision feedback equalizer is used as the blind equalizer. According to the simplified cost function of CMA, the weights of forward part and feedback part of blind equalizer update by RLS algorithm and gradient descent algorithm respectively. Simulation results demonstrate that under conditions of underwater acoustic channel, compared with decision feedback blind equalization based on gradient descent algorithm and transversal equalizer blind equalization based on RLS algorithm, decision feedback blind equalization based on RLS algorithm has faster convergence rate and lowest convergence steady-state error, so it's more suitable for blind equalization for nonlinear time-varying underwater acoustic channel and has better real-time tracking ability.

Keywords: *Blind Equalization; RLS; Underwater Acoustic Channel; Decision Feedback Equalizer*

1. Introduction

In underwater acoustic communication systems, acoustic wave propagation by underwater acoustic channel would bear energy attenuation, Doppler frequency shift, multipath transmission and noise interference, which will cause inter-symbol interference (ISI) at the receiver that seriously affects the quality of communication [1]. Due to underwater acoustic propagation characteristics, the single carrier adaptive equalization is still an effective technology to eliminate ISI so far. Compared with traditional adaptive equalization [2], blind equalization can realize the communication channel compensation and tracking without any training sequence [3], it can save the bandwidth and prevent equalizer unlock, so blind equalization has important significance to improve the efficiency and quality for underwater acoustic communication under the limited channel bandwidth conditions. For the nonlinear distortion underwater acoustic channel, blind equalization with linear equalizer cannot obtain better performance [4]; however, decision feedback equalizer can use feedback weights to

compensate the nonlinear characteristic of the channel [5] to eliminate the residual ISI of the forward equalizer. As Underwater acoustic channel is usually has nonlinear characteristics, so the decision feedback equalizer more suitable for it. CMA is one of the most widely used blind equalization algorithm for its robust with low computational complexity [6], but CMA based on gradient descent algorithm convergence slowly and has big steady residual error after convergence [7], in the complex underwater environment, CMA need thousands of iterations can achieve convergence that means CMA needs to sacrifice a lot of transmitted symbols under the high rate underwater acoustic communication, furthermore, the underwater acoustic channel is fast time-varying, CMA is difficult to realize the real-time tracking for the channel, resulting in equalization failure [8]. Compared with gradient descent algorithm, the RLS algorithm has faster convergence rate and higher convergence precision [9], so it's more suitable for high rate underwater acoustic communication channel equalization and keep better real-time tracking of time-varying channel. While the cost function of CMA does not satisfy second norm form [10], then RLS algorithm cannot used for equalizer weights updating directly.

This paper provides a new algorithm based on the analysis of CMA cost function, approximate transformation method is adopted and CMA cost function is simplified to meet second norm form, RLS algorithm is used for blind equalizer weights updating. For decision feedback blind equalization algorithm proposed in this paper, the weights of forward part and feedback part of blind equalizer update by RLS algorithm and gradient descent algorithm respectively, Simulation results demonstrate that, decision feedback blind equalization based on RLS algorithm has fastest convergence rate and lowest convergence steady-state error, thus more suitable for blind equalization for nonlinear time-varying underwater acoustic channel and has better real-time tracking ability compared with decision feedback blind equalization based on gradient descent algorithm, and transversal equalizer blind equalization based on RLS algorithm under conditions of underwater acoustic channel.

2. Basic principle of decision feedback equalizer

Decision feedback equalizer (DFE) is better than the linear equalizer for it can compensate the amplitude distortion of transmit signal with minimum noise gain [11], meanwhile DFE can obtain ideal equalization when the length of channel does not exceed the equalizer. For decision feedback equalizer, the impulse response of the forward filter need not force to the channel inversion, so as to avoid the noise enhancement, furthermore, feedback filter can remove the ISI in the previously detected symbols [12]. Fig.1 [13] shows the block diagram of decision feedback blind equalization.

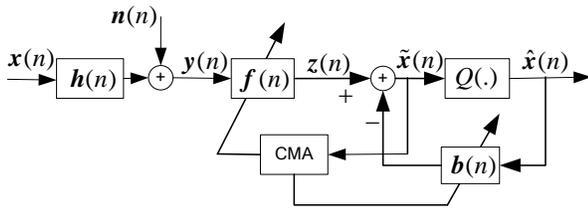


Fig. 1 Block diagram of decision feedback blind equalization.

The basic idea of DFE is that once detected an information symbol, the symbol of ISI is estimated by subtracting in advance. Decision feedback blind equalizer includes three parts: forward filter $f(n)$, feedback filter $b(n)$ and a quantified decision device $Q(\cdot)$. The feedback filter is easy to construct the corresponding zero offset for the minimum phase zero near the unit circle of the channel, therefore does not amplify the noise, of course, it easy to see for the maximum phase zero near the unit circle, due to the feedback filter cannot construct the zero offset outside the unit circle, therefore decision feedback equalizer can only rely on the forward filter to obtain better equalization performance [14]. The input signal $x(n)$ transmits through the unknown channel $h(n)$ adding Gauss white noise $n(n)$, and then we can get the observation signal $y(n)$ before the blind equalizer. $\tilde{x}(n)$ is the output signal of the blind equalizer, and $\hat{x}(n)$ is the decision symbol of $\tilde{x}(n)$. Blind equalization can recover the input signal $x(n)$ only rely on the observation signal $y(n)$, without the information of $x(n)$ and $h(n)$. If let the length of $f(n)$ is N_f and the length of $b(n)$ is N_b , then the symbol representation can be given by

$$y(n) = [y(n), y(n-1), \dots, y(n-N_f+1)]^T \quad (1)$$

$$f(n) = [f(0), f(1), \dots, f(N_f+1)]^T \quad (2)$$

$$b(n) = [b(1), b(1), \dots, b(N_b)]^T \quad (3)$$

$$\hat{x}(n) = [\hat{x}(k-1), \hat{x}(k-2), \dots, \hat{x}(k-N_b)]^T \quad (4)$$

where symbol "T" denotes transpose operation. The output signal $\tilde{x}(n)$ can be given by

$$\tilde{x}(n) = f^H(n)y(n) - b^H(n)\hat{x}(n) \quad (5)$$

Traditional decision feedback blind equalization algorithm updates the equalizer weights based on CMA algorithm, CMA algorithm is one of the Godard algorithms with the parameter $p = 2$, which is the special case of Bussgang blind equalization with the most widely used. The cost function of CMA is [15]

$$J = \frac{1}{2} [R_{CM} - |\tilde{x}(n)|^2]^2 \quad (6)$$

where R_{CM} is the constant modulus can compute by

$$R_{CM} = \mathbf{E}\{|x(n)|^4\} / \mathbf{E}\{|x(n)|^2\} \quad (7)$$

where symbol $\mathbf{E}\{\cdot\}$ denotes mathematical expectation operation. According to the stochastic gradient descent algorithm, decision feedback blind equalization updates weights based on CMA is given by [16]

$$f(n) = f(n-1) + \mu \tilde{x}(n) [R_{CM} - |\tilde{x}(n)|^2] y^*(n) \quad (8)$$

$$b(n) = b(n-1) - \mu \tilde{x}(n) [R_{CM} - |\tilde{x}(n)|^2] \hat{x}^*(n) \quad (9)$$

where μ is the step size and the symbol "*" denotes conjugate operation.

3. Simplified CMA cost function

CMA blind equalization updates the equalizer weights based on stochastic gradient descent algorithm, and the convergence rate is slow and has big steady state error after convergence, furthermore, because the CMA cost function is non-convex, CMA blind equalization easily fall into the local minimum value [17]. Compared with stochastic gradient descent algorithm, RLS algorithm has faster convergence rate and better tracking performance, for the complex underwater acoustic channel, RLS algorithm has more advantages. From Eq.7 can see that the CMA cost function does not meet second norm form and then RLS algorithm cannot applied to directly. According to Eq.5 we can get

$$|\tilde{x}(n)|^2 = [f^H(n)y(n) - b^H(n)\hat{x}(n)] \times [f^H(n)y(n) - b^H(n)\hat{x}(n)]^* \quad (10)$$

after expansion of (10) we can get

$$|\tilde{x}(n)|^2 = \mathbf{f}^H(n)\mathbf{y}(n)\left[\mathbf{f}^H(n)\mathbf{y}(n) - \mathbf{b}^H(n)\hat{x}(n)\right]^* - \mathbf{b}^H(n)\hat{x}(n)\left[\mathbf{f}^H(n)\mathbf{y}(n) - \mathbf{b}^H(n)\hat{x}(n)\right]^* \quad (11)$$

Obviously, if let $\mathbf{u}_f(n)$ and $\mathbf{u}_b(n)$ as follows

$$\mathbf{u}_f(n) = \mathbf{y}(n)\left[\mathbf{f}^H(n)\mathbf{y}(n) - \mathbf{b}^H(n)\hat{x}(n)\right]^* \quad (12)$$

$$\mathbf{u}_b(n) = \hat{x}(n)\left[\mathbf{f}^H(n)\mathbf{y}(n) - \mathbf{b}^H(n)\hat{x}(n)\right]^* \quad (13)$$

then the CMA cost function can be rewritten as

$$J = \frac{1}{2}\left[R_{CM} - \left[\mathbf{f}^H(n)\mathbf{u}_f(n) - \mathbf{b}^H(n)\mathbf{u}_b(n)\right]\right]^2 \quad (14)$$

The cost function with the form of (14) meets the second norm form, but $\mathbf{u}_f(n)$ and $\mathbf{u}_b(n)$ include the equalization weights $\mathbf{f}(n)$ and $\mathbf{b}(n)$ explicit, then RLS algorithm still cannot be applied to update the weights of the equalizer. Here we assume that the equalizer convergence, then the weights vector would has relations as follows

$$\lim_{n \rightarrow \infty} \|\mathbf{f}(n) - \mathbf{f}(n-1)\| \rightarrow 0 \quad (15)$$

$$\lim_{n \rightarrow \infty} \|\mathbf{b}(n) - \mathbf{b}(n-1)\| \rightarrow 0 \quad (16)$$

According to (15) and (16) defined $\tilde{\mathbf{u}}_f(n)$ and $\tilde{\mathbf{u}}_b(n)$ as follows

$$\tilde{\mathbf{u}}_f(n) = \mathbf{y}(n)\left[\mathbf{f}^H(n-1)\mathbf{y}(n) - \mathbf{b}^H(n-1)\hat{x}(n)\right] \quad (17)$$

$$\tilde{\mathbf{u}}_b(n) = \hat{x}(n)\left[\mathbf{f}^H(n-1)\mathbf{y}(n) - \mathbf{b}^H(n-1)\hat{x}(n)\right]^* \quad (18)$$

If use $\tilde{\mathbf{u}}_f(n)$ and $\tilde{\mathbf{u}}_b(n)$ instead of $\mathbf{u}_f(n)$ and $\mathbf{u}_b(n)$ in the cost function expressed by (14), the cost function can be written as

$$J = \frac{1}{2}\left[R_{CM} - \left[\mathbf{f}^H(n)\tilde{\mathbf{u}}_f(n) - \mathbf{b}^H(n)\tilde{\mathbf{u}}_b(n)\right]\right]^2 \quad (19)$$

If let $C^H(n) = [\mathbf{f}^H(n); \mathbf{b}^H(n)]$, $U(n) = [\tilde{\mathbf{u}}_f(n); \tilde{\mathbf{u}}_b(n)]$,

then the CMA cost function can be simplified as

$$J = \frac{1}{2}\left[R_{CM} - C^H(n)U(n)\right]^2 \quad (20)$$

Then the CMA cost function is simplified to meet second norm form as (20) and RLS algorithm can be used directly.

4. Decision feedback blind equalization based on RLS algorithm

On the basis of the analysis in above section, simplified CMA cost function meet second norm form and the blind equalizer weights can be updated by RLS algorithm directly. Considering the stability of the algorithm, the forward filter and the feedback filter using independent update mode, that is the forward filter updates by RLS algorithm and the feedback filter updates by gradient descent algorithm based on the simplified CMA cost function. The block diagram of the RLS decision feedback blind equalization is illustrated in Fig.2.

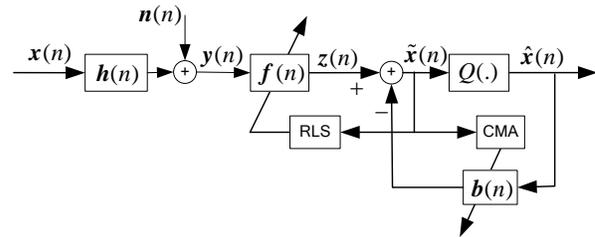


Fig. 2 RLS decision feedback blind equalization principle diagram.

According to the principle of RLS and CMA algorithm, the decision feedback blind equalization algorithm can be summarized as follows.

Parameters set: λ =forgetting factor

δ =value to initialize $\mathbf{P}(0)$

Initialization: $\mathbf{f}(0) = \mathbf{0}$ in addition to the central tap is 1

$\mathbf{b}(0) = \mathbf{0}$

$\mathbf{P}(0) = \delta^{-1}\mathbf{I}$, where is the identity matrix

of rank N_f

Computation: for $n=1,2,\dots$

calculate $\tilde{\mathbf{u}}_f(n)$ and $\tilde{\mathbf{u}}_b(n)$ according to

Eq.17 and Eq.18;

$$\mathbf{e}_f(n) = R_{CM} - \left[\mathbf{f}^H(n)\tilde{\mathbf{u}}_f(n) - \mathbf{b}^H(n)\tilde{\mathbf{u}}_b(n)\right]$$

$$\mathbf{e}_b(n) = R_{CM} - \left|\mathbf{b}^H(n)\hat{x}(n)\right|^2$$

$$\mathbf{k}(n) = \frac{\mathbf{P}(n-1)\tilde{\mathbf{u}}_f(n)}{\lambda + \tilde{\mathbf{u}}_f^H(n)\mathbf{P}(n-1)\tilde{\mathbf{u}}_f(n)}$$

$$\mathbf{P}(n) = \frac{1}{\lambda} \left[\mathbf{P}(n-1) - \mathbf{k}(n)\tilde{\mathbf{u}}_f(n)\mathbf{P}(n-1) \right]$$

$$\mathbf{f}(n) = \mathbf{f}(n-1) + \mathbf{k}(n)\mathbf{e}_f^*(n)$$

$$\mathbf{b}(n) = \mathbf{b}(n-1) - \mu \mathbf{e}_b(n) (\mathbf{b}^H(n-1) \hat{\mathbf{x}}^*(n))^T \hat{\mathbf{x}}(n)$$

end

where $\mathbf{P}(n)$ is the inverse matrix of the autocorrelation matrix $\mathbf{R}(n)$ of the forward filter input signal $\tilde{\mathbf{u}}_f(n)$, $\mathbf{R}(n)$ can calculate according to time recursive that given by [18]

$$\mathbf{R}(n) = \lambda \mathbf{R}(n-1) + \tilde{\mathbf{u}}_f(n) \tilde{\mathbf{u}}_f^H(n) \quad (21)$$

According to the inverse matrix theorem it is easy to get the recursion formula to calculate $\mathbf{P}(n)$.

5. Simulations and discussion

The simulation is using QPSK signal as input signal. The channel noise is band-limited gauss white noise with mean zero. Typical shallow sea and deep sea underwater acoustic channel [19] are used in our simulations, the channel models have verified by the sea experiment. For the shallow sea channel model, the parameters set as follow: carrier frequency is 10kHz, channel bandwidth is 2kHz, transmit baud rate is 1000 symbol/s, wind speed is 20kn, the sender and receiver locate in underwater 10 meters, and the distance is 5000 meters. For the deep sea channel model, the parameters set as follow: the depth of sea is 5000 meters; sound source is located in 1000 meters underwater, receiver is located in the 900 meters underwater, the distance between sound source and receiver is 56 kilometers, carrier frequency is 1kHz, transmit baud rate is 100symbol/s, the parameters of eight rays of the channel model can be shown as Tab.1.

Table 1: the parameters of eight rays of the channel model

ray num	shallow sea channel		deep sea channel	
	t/ms	A	t/ms	A
1	0.000	1.0000	0.0000000	0.4954
2	0.026	-1.0000	0.0265385	-0.1464
3	0.026	-0.3286	0.0319367	0.5079
4	0.100	0.3286	0.0647739	-0.1555
5	0.100	0.3286	0.2056037	0.8399
6	0.240	-0.3286	0.2320864	1.0000
7	0.420	-0.1080	0.2359591	0.6914
8	0.420	0.1080	0.3671784	0.2187

The decision feedback blind equalizer $N_f = 18$ forward taps and $N_b = 10$ feedback taps, forgetting factor $\lambda = 0.98$, study step size $\mu = 0.0015$. In order to verify the performance of decision feedback blind equalization based on RLS algorithm (RLS-DFE-CMA)

proposed in this paper, CMA based on stochastic gradient descent algorithm with linear equalizer (SGD-LE-CMA), linear blind equalization based on RLS algorithm (RLS-LE-CMA) and decision feedback blind equalization based on stochastic gradient descent algorithm (SGD-DFE-CMA) are done in the simulations for comparison. The comparison is in terms of mean square error (MSE), which is defined as

$$MSE(n) = \frac{1}{n} \sum_{k=1}^n |\tilde{x}(k) - \hat{x}(k)|^2 \quad (22)$$

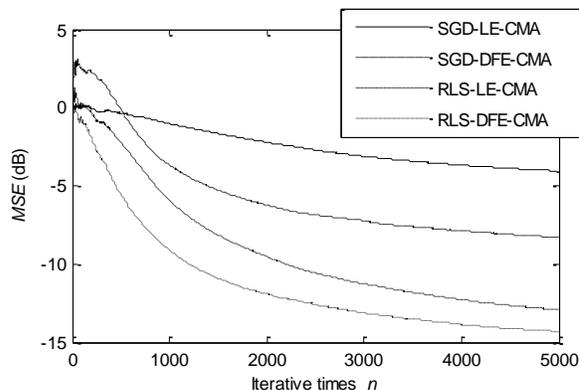


Fig. 3 MSE in shallow sea channel.

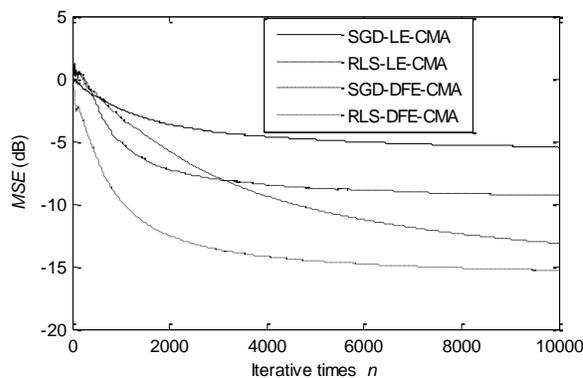


Fig. 4 MSE in deep sea channel.

6. Conclusions

In this study, the cost function of CMA is simplified to meet second norm form, and the RLS algorithm is used to update the blind equalizer directly. Considering the underwater acoustic channel usually has nonlinear distortion, decision feedback equalizer is adopted to compensate the nonlinear characteristic of the channel, the forward filter and the feedback filter using independent update mode, that is the forward filter updates by RLS

algorithm and the feedback filter updates by gradient descent algorithm based on the simplified CMA cost function. The simulations of the typical shallow sea channel and deep sea channel proved the advantage of this method called RLS-DFE-CMA.

Acknowledgments

This work was supported in part by The National Natural Science Foundation of China (61201418), Fundamental Research Funds for the Central Universities (DC12010218) and Scientific and Technological Research Project for Education Department of Liaoning Province (2010046).

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