Combined coding and companding to reduce both PAPR and BER in OFDM systems

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Abstract

A major problem in telecommunications is to adapt the information to transmit to the propagation channel. For frequency selective channels, a technique is to use multi-carrier modulation such as OFDM (Orthogonal Frequency Division Multiplexing) in which an information block is modulated by a Fourier transform. But one of the main disadvantages of OFDM is its high PAPR (Peak to Average Power Ratio).

In this paper we propose a method for reducing PAPR using encoding and companding techniques. Companding is used to reduce PAPR while coding is used to reduce the Bit Error Rate (BER). BCH codes and μ-law compander have been used for this purpose. The effects of modulation have been studied too.

Keywords: *block coding, BCH, OFDM, PAPR, Companding*

1. Introduction

OFDM is used to achieve high transmission rates in a mobile environment. It transforms a multipath channel broadband to a set of single path sub-channels. Furthermore, the use of cyclic redundancy at the transmit side reduces the complexity thanks to the algorithms based on Fast Fourier Transform (FFT) [1]. Because of its multiple frequencies, OFDM has a very high PAPR. The simplest technique and the most widely used to reduce the PAPR is clipping. Unfortunately clipping creates distortion signal which degrades performance. Coding [2] remains the technique of creating less distortion which not only reduces the PAPR but corrects errors. We can find in [3] an overview of different PAPR reduction techniques.

In most cases, the authors propose techniques to reduce the PAPR without worrying about performance at the BER. On the other hand, these techniques used must be independent of the modulation, the OFDM symbol length and the coding rate with low complexity. To this end, we propose a new technique of PAPR reduction which is independent of the above points.

In this paper, we propose a PAPR reduction technique which combines μ-law companding technique and encoding technique. The companding is also a technique used to reduce the PAPR [4]. We studied the effect of μ parameter and the modulation type on the PAPR reduction.

The rest of the paper is organized as follows. Section 2 describes system based on OFDM and defines PAPR. Section 3 presents an overview of PAPR reduction techniques. In section 4 we describe our proposal method to reduce both PAPR and BER in an OFDM transmission. Finally, we present our simulation results and a conclusion.

2. PAPR in OFDM

2.1 PAPR analysis

For an OFDM symbol with N subcarriers, a standard signal can be written as follows:

$$y_s(k) = \frac{1}{\sqrt{\mathrm{N}}}\sum_{\mathrm{n=0}}^{\mathrm{N-1}} \mathrm{s_n\,W^{-nk}}, \mathrm{k} = 0, \dots, \mathrm{N}-1 \qquad (1)$$

where s_n is a complex symbol sequence $S = [s_0, s_1, \dots, s_{N-1}]$ and $W = e^{\frac{-j2\pi}{N}}$. The symbols s_n are taken in an alphabet of size $l = 2^p$ where p is the number of bits per symbol giving l^N separate sequences. The instantaneous power of the signal envelope is given by the function $P_s(k) = |y_s(k)|^2$. The PAPR corresponding to the transmitted signal is given by:

$$papr(s) = \frac{P_s(k)}{mean(P_s(k))} = \frac{P_s(k)}{N} \quad (2)$$

and:

$$P_s(k) = \sum_{m,n} s_m\, s_n^* W^{-mk}(W^{-nk})^* \quad (3)$$

From (3), we obtain:

$$P_s(k) = N + 2\phi_s(k) \quad (4)$$

where

$$\phi_s(k) = \Re\left(\sum_{u=1}^{N-1} C_s(u)W^{-uk}\right) \quad (5)$$

$C_s(u)$ is the aperiodic autocorrelation of the sequence S given by the displacement u

$$C_s(u) = \sum_{m=0}^{N-u-1} s_m\, s_{m+u}^* \quad (6)$$

where m and m+u are between 0 and N-1. ϕ_s can be interpreted physically as the sum of harmonics cosine weighted by the auto-correlation aperiodic function $C_s(u)$. Sequences with low Cs(u) will produce low PAPR. The binary complementary Golay sequences are one of sequences that provide a PAPR which does not exceed 2 (3dB) because of the envelope of the average power of any sequence is N [5]. If S = [1111,…, 1], then the peak envelope power (PEP) is $N + 2\sum_{u=1}^{N-1}(N-u) = N^2$. Therefore, the PAPR of an uncoded message is at most N.

2.2 Golay binary complementary sequences

A pair of complementary sequences has the property that the sum of their aperiodic autocorrelation is zero everywhere except zero. So if S_1 and S_2 are two sequences of length N, they form a complementary pair if:

$$C_{s_1}(u) + C_{s_2}(u) = \sum_{m=0}^{N-u-1}\left(s_{1_m}s_{1_{m+u}}^* + s_{2_m}s_{2_{m+u}}^*\right) \quad (7)$$

with u ≠ 0.
In addition, a new pair of complementary sequences (S_1, S_2) is deduced from the pair (S_1', S_2') by:

$$S_{l,n}' = S_{l,n}e^{\frac{j2\pi n}{m}}, l = 1, 2 \quad (8)$$

This idea can be extended from pairs to identify complementary sequences. Thus, for a pair of sequences with length N the PAPR is at most 2.

$$papr(s_1) + papr(s_2) = \frac{1}{N}(2N + \sum_{i=1}^{N-1}(C_{S_1 S_1}(u) + C_{S_2}(u))e^{i2\pi it}) = 2, \quad u \neq 0 \quad (9)$$

Since the PAPR is a positive quantity, the PAPR of each sequence should not exceed 2. In [6], the authors have made a major theoretical discovery by finding that large sets of binary complementary pairs of length 2^m can be obtained from 2^{nd} order cosets of the Reed-Muller codes.

3. Overview of PAPR reduction techniques

In this section, we present the PAPR reduction techniques commonly used.

3.1 Amplitude Clipping and Filtering

This technique is most used. It limits the signal to a fixed value when the amplitude exceeds a certain value.

$$A(x) = \begin{cases} x, & |x| < B \\ B, & else \end{cases} \quad (10)$$

But this technique creates distortion signal which degrades performance.

3.2 The partial transmit sequence

In this technique, an input data block of N symbols is partitioned into disjoint sub blocks. The subcarriers in each sub block are weighted by a phase factor for that sub block. The phase factors are selected such that the PAPR of the combined signal is minimized. The disadvantage of this technique is that the search complexity increases exponentially with the number of sub blocks.

3.3 Selected Mapping (SLM) technique

The SLM technique [7] involves generating a set of independent data blocks U candidate representing the original data block X ={X_1, X_2… X_N} to transmit over N subcarriers. The data block candidate giving the lowest value of PAPR is selected for transmission. The U candidates data blocks are generated using U distinct vectors defined as follows:

$$P^{(u)} = [p_1^{(u)}, p_2^{(u)}, \dots, p_N^{(u)}], \qquad u = 1,2,\dots,U \quad (11)$$

where $P^{(1)}$ is a vector containing only 1 . These $P^{(u)}$ vectors are each multiplied by the original data block to generate U independent candidates obtained as follow:

$$P^{(u)} = \left[X_1 p_1^{(u)}, X_2 p_2^{(u)}, \dots, X_N p_N^{(u)}\right] \quad (12)$$

The PAPR of each signal is calculated, and the one with the lowest value of PAPR is selected for transmission. To correctly recover the transmitted signal, the channel must be known to the receiver for all the above techniques.

3.4 The interleaving technique

This technique consists to permute the position of the input signal. Thus, the signal $X = [X_0, X_1, \dots, X_{N-1}]$ becomes $X' = [X_{\pi(0)}, X_{\pi(1)}, \dots, X_{\pi(N-1)}]$ where π is any permutation of the numbers 0,1,…,N-1. The complexity of this technique increases exponentially with the size of X.

3.5 Coding technique

Coding technique is used to reduce both PAPR and Bit Error Rate (BER). In general the main idea is to send only the sequences with low PAPR [8]. But this requires an exhaustive search and such search becomes impractical when the length of the codewords increases.
Second order cosets of first order Reed-Muller code based on Golay complementary pairs (PAPR at most 3dB) introduced by Davis and Jewab [5] is the most important work in this domain.
Reed-Muller codes are a family of linear error-correcting codes used in communications. They are named after their discoverers, Irving S. Reed and D. E. Muller. Muller discovered the codes, and Reed proposed the majority logic decoding for the first time [9]. A first order Reed-Muller code is equivalent to a Hadamard code. Reed-Muller codes are listed as RM(*r*,*m*), where *r* is the order of the code, and *m* is a parameter related to the length of code.
Authors in [5] showed in Corollary 2.4
For any permutation π of the symbols 1, 2…, m and for any c, $c_k \in \mathbb{Z}_{2^h}$,

$$a(x_1, x_2, \dots, x_m) = \varphi(x_\pi) + \sum_{k=1}^{m} c_k\, x_k \quad (13)$$

is a Golay sequence over $\mathbb{Z}_{2^h}$ of length 2^m where

$$\varphi(x_\pi) = 2^{h-1} \sum_{k=1}^{m-1} x_{\pi(k)} x_{\pi(k+1)} + c \quad (14)$$

The first order binary Reed-Muller code $RM_{2^h}(1,m)$ of length 2^m is generated by the monomial in the boolean functions x_i of degree 1[5]. So we use the binary case $h = 1$.
In (13), $\varphi(x_\pi)$ represents the coset and $\sum_{k=1}^{m} c_k\, x_k$ is the Reed-Muller codeword. The codes obtained from Reed-Muller codes have a very low PAPR of 3dB. The main drawback of this technique is that it is limited only to M-PSK modulation and has low code rate. The rate of this technique is defined by

$$\tau = \frac{log_2\left(\frac{m!}{2}\right) + h(m+1)}{h2^m} \quad (15)$$

So for RM_2 (1, 8), τ = 0,09.
Since OFDM can use M-QAM modulation, it is therefore important to find a technique based on coding that does not depend on the modulation and having a good coding rate with low complexity.

4. Our proposal

Our proposed method is to combine coding and companding techniques which will be studied in the next sections.

4.1 Companding technique

Companding is simply a system in which information is first compressed, transmitted through a bandwidthlimited channel, and expanded at the receiving end. It is frequently used to reduce the bandwidth requirements for transmitting telephone quality speech, by reducing the 13-bit codewords to 8-bit codewords. Two international standards for encoding signal data to 8-bit codes are A-law and µ-law. A-law is the accepted European standard, while µ-law is the accepted standard in the United States and Japan.
Limiting the linear sample values to 12 magnitude bits, the A-law compression is defined by

$$f(x) = \begin{cases} \dfrac{A|x|}{1+\ln(A)} & 0 \le |x| \le \dfrac{1}{A} \\ \dfrac{sign(x)\left(1+ln(A|x|)\right)}{1+ln(A)} & \dfrac{1}{A} \le x \le 1 \end{cases} \quad (16)$$

Limiting the linear sample values to 13 magnitude bits, the µ-law compression is defined by,

$$f(x) = \frac{sign(x)\ln(1+\mu|x|)}{ln(1+\ |x|)} \quad (17)$$

where µ is the compression parameter (µ =255 in the U.S. and Japan) and x is the normalized integer to be compressed.

4.2 Coding Technique (BCH)

BCH codes (Bose, Chaudhuri, Hocquenghem) are correcting codes used to correct random errors. They were introduced in 1959 by A. Hocquenghem and in 1960 by R. C. Bose and D. K. Ray Chaudhuri. These codes are a class of linear cyclic codes correcting several errors and based on the Hamming metric. For any integer n of the form 2m – 1 there is a *t*-correcting binary BCH code built on GF(q) where q = 2m [10].
The BCH codes check:
Block lenght $n = 2^m - 1$, number of redundant bits $r = n - k$, minimum distance $d_{min} \geq 2t + 1$.
Let C(n, k, t) be a BCH code with generator matrix G, length n, dimension k and error correcting capability t. A coded message is defined by

$$codeword = msg.G \qquad (18)$$

4.3 Transmission chain

Figure 1 describes the general scheme of our proposal. A message from a data source is first encoded. Then it will be modulated by a M-QAM modulation and it will be processed by the IFFT block to form an OFMD symbol. The last one is then compressed by a companding type. The inverse operations are performed at the receiver side. The PAPR is calculated at the transmitter while the BER at the receiver side.

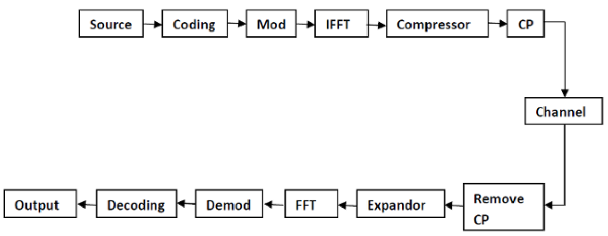

Fig. 1 Transmission chain

5. Results

5.1 Our results

Our PAPR results are presented as CCDF curve. The CCDF shows the probability of an OFDM frame exceeding a given PAPR (dB). In general, the closer the CCDF curve to the vertical axis, the better its PAPR reduction performance.
In our work, we decided to use µ-law compander. So, the first stage is to find the value of µ-law to use. For that, we use different values of µ -law and 64 sub-carriers, Figure 2. We can see that µ = 255 gives better performances. This is due to compression rate. More it compresses and more the peaks are reduced. µ = 255 is the value used in practice in USA and Japan. There is a gain of about 7dB at 10^{-3} compared to a simple coding without companding.

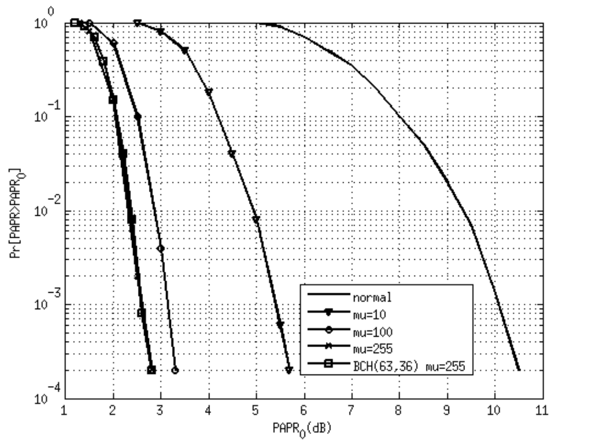

Fig. 2 CCDF for different values of µ-law combined with BCH(15,11)

We can see that the coding has no effect on PAPR results because coding transmission gives the same result that an uncoded transmission.
In Figure 3, we compare the CCDF curves for the proposed method to the SLM method and technique of adding dynamic offsets [11]. The variable U of the SLM technical represents the number of times the OFDM signal is presented for comparison, which is equivalent to calculate U times the IFFT.
The results show that a great value for U are needed for SLM technique to achieve the same performance in terms of PAPR reduction of the algorithm for adding offset, and the proposed method.
In terms of complexity, our method is much less complex because it calculates one time the IFFT, while the SLM technique executes U times the IFFT and the adding dynamic offset method computes in the worst case ω times the IFFT block to transmit one OFDM symbol, where

$\omega = N + (N-1) + (N-2) + \ldots + (N-w)$ and w is the weight of the dynamic offset able to reduce the PAPR for an OFDM signal and N is the number of subcarriers. In this case w = 10 and N = 64.

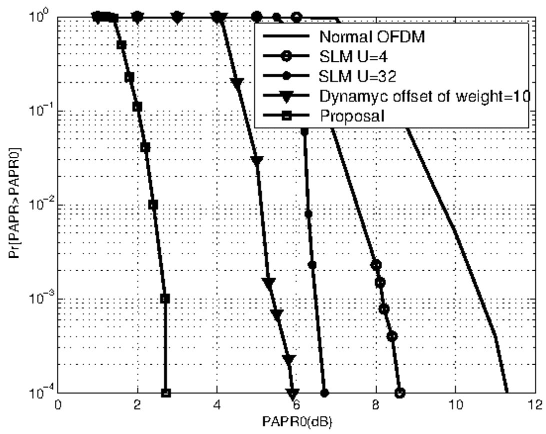

Fig. 3 CCDF comparison of our proposal with BCH(63,51), SLM, and the technique of adding dynamic offset of weight = 10 , N = 64, and 16-QAM

But reducing the PAPR has effects on system performance. The use of coding can thus improve performance. Figure 4 shows the effects of μ parameter on the BER.

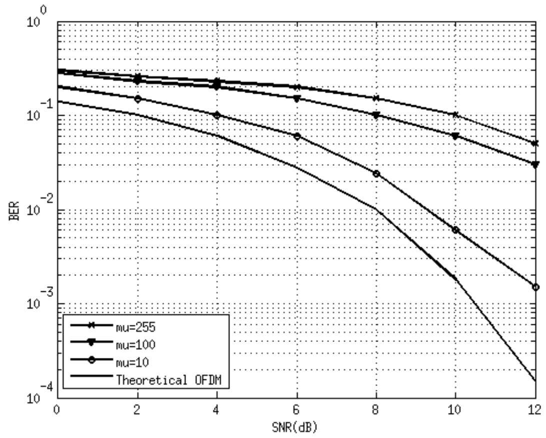

Fig. 4 Effect of μ-law on BER performance

As can be seen in Figure 4, performances degrade when μ increases. This result is expected because more you compress and more you lose information.
So there is a tradeoff between reducing the PAPR and the BER. We will use μ = 100 for a good PAPR reduction with performance less degraded.
Thus, we have a gain of 4.5dB at 10^{-3} for PAPR. This value of μ will be used in the rest of our work.

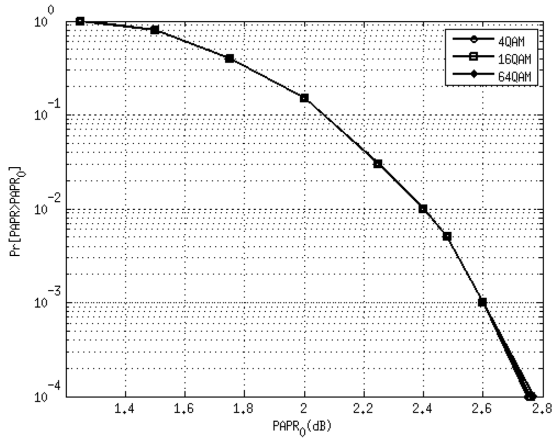

Fig. 5 Modulation effect

In Figure 5, we study the modulation effect. We use BCH (63, 36) and μ=100 with different M-QAM modulations and 64 sub-carriers. We can see that, modulation has not real effect on PAPR reduction.

5.2 Comparison of our proposal with some existing methods

In this paragraph, we compare our results in terms of BER and PAPR with those in [11] where authors joint error correction code and PAPR reduction, this technique is called adding dynamic offset technique on the one hand.
On the other hand, we compare it with the famous SLM technique.
Figures 6 and 7 compare the PAPR reduction performance of our proposed technique with one in [11].

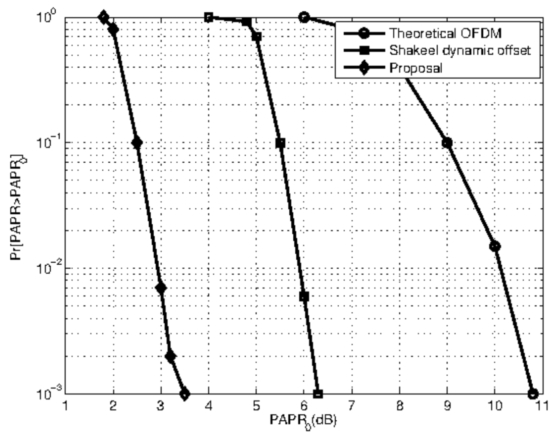

Fig. 6 CCDF comparison of our proposed technique with Shakeel dynamic offset N=128, QPSK, BCH(255, 115, 43)

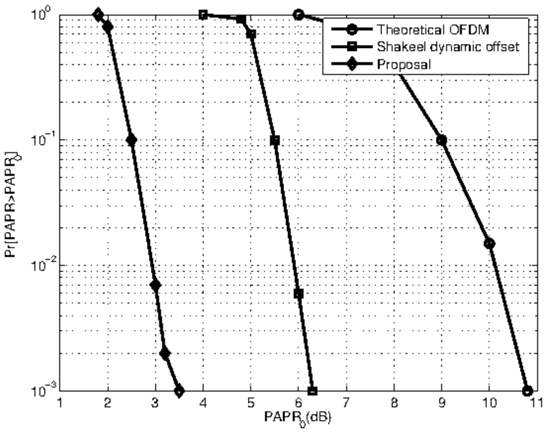

Fig. 7 CCDF comparison of our proposed technique with Shakeel dynamic offset N=64, 16QAM, BCH(255, 123, 39)

In Figure 6 we use 128 sub-carriers, QPSK modulation and BCH (255, 115, and 43), while in Figure 7 we use 64 sub-carriers, 16QAM modulation and BCH (255, 123, 39). As we can see, our technique achieves the best PAPR reductions with a gain of 3dB and 2dB at 10^{-3} respectively in Figure 6 and 7. Whatever the modulation or coding rate, our technique allows to have good performance in terms of PAPR reduction.

Figures 8 and 9 present the performances in terms of BER for OFDM system using our method. Also in both figures, we introduce a comparison with the system Coded OFDM (COFDM) and the technique of adding dynamic offset [11]. The COFDM system is encoded with standard block code C(n, k, d) which has a higher error correcting capacity than that used by Shakeel and Grant C(n, k, d') (d' = d - w) where they used a dynamic offset of weight w=10 for RS codes (63,27,18) and BCH (255,131,18) which become respectively according to this technique RS (63,27 , 8) and BCH (255,131,8). The results in terms of BER performance shows that the C(n, k, d') code ensures the correction better than the standard block code C(n, k, d) even if (d'< d).

We also find that our method achieves performance comparable with those achieved by the method of adding dynamic offset. However our method is better in terms of complexity because it computes one time the IFFT. We also note that the OFDM curves without coding and coded OFDM (COFDM) exhibit the phenomenon of "Error Floor", which is removed using our technique of PAPR reduction, and also by the method of adding dynamic offset. According to the simulation result, we observe that for SNR lower than 12dB, the proposed method and also the method of adding dynamic offset are slightly less efficient than the case of coded OFDM (COFDM). This is because the channel is dominated by AWGN noise rather than the distortions caused by the non-linearity of the amplifier.

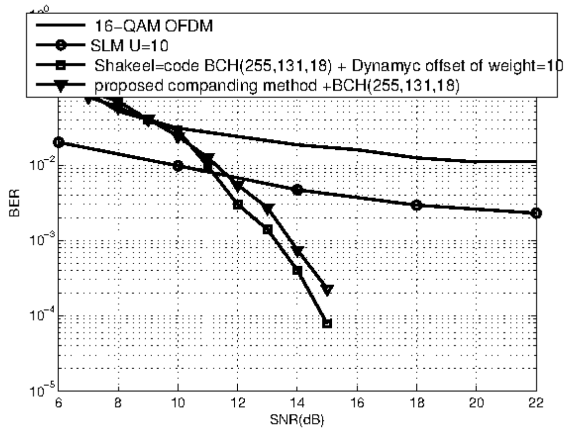

Fig. 8 BER performances of our proposal, dynamic offset technique w =10, SLM U=10, normal OFDM on 16-QAM modulation

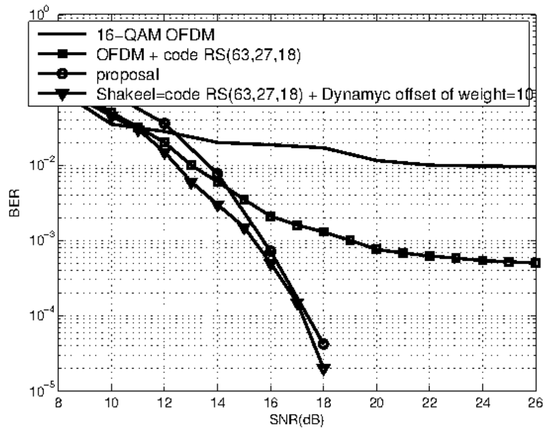

Fig. 9 BER performances of our proposal, dynamic offset technique w = 10 , the Coded OFDM and normal OFDM on 16-QAM modulation

5. Conclusion

In this paper we have considered the problem of the PAPR reduction in OFDM system. The proposed PAPR reduction method is based on combining encoding technique and the companding method. The performances of the proposed method are evaluated using M-QAM modulation. The simulation show that the best value of parameter μ is 255 but this value gives a bad BER. So μ = 100 has been used. With this value, our technique gives better results than [11]. It should be noted that there is a tradeoff between PAPR and BER reduction. We studied also the impact of this technique on BER, the result shows that our method allows to have the performances in terms of BER better than the COFDM and the SLM technique.

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