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Precise BER Analysis of ½ and ⅔ rated RS-CC Concatenated Coded for Digital Image Transmission in OFDM System

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Abstract

We introduce an analytical approach to evaluate bit error rate (BER) expression is derived for an OFDM system with PSK and QAM encoded image signal are transmitted over AWGN and fading channel models. Different system configurations including time-domain differential modulation, frequency domain differential modulations, delay spread of channels are considered in the exact BER analysis. Both analysis and simulation results are presented for 2PSK, 4QAM and 16QAM. We also investigate the performance of concatenated RS and convolutional code (CC) over AWGN and frequency selective fast fading channels. By concatenating two different codes, we can get the effect of improving the total BER due to benefits of RS codes correcting burst errors while convolutional codes are good for correcting random errors that are caused due to a noisy channel. The resulting scheme is tested for the transmission of images over noisy channels.

Keywords: *OFDM, Bit Error Rate, Color Image, RS-CC Concatenated Code, FEC, Modulation Technique.*

1. Introduction

The increasing demand of higher capacity network with high data rate for broadband communication system with a greater range of services as like as internet services, video conferencing and digital multimedia applications has assisted the development of orthogonal frequency division multiplexing (OFDM) [1].

OFDM is special case of multicarrier modulation technique [2] which splits the whole bandwidth into many narrow band sub channels and a single data stream is transmitted over a number of lower rate subcarriers (SC_s) [3] in OFDM. The (Inverse) Discrete Fourier transform ((I)DFT), particularly fast implementation algorithm of Fast Fourier transform (FFT) is employing to do the frequency division multiplexing very efficiently[4].

The flexibility of OFDM physical layer technologies provides opportunities to use advanced techniques in several LAN, MAN, as IEE802-11a, HIPERLAN/2 and IEEE802.16e [6].

1.1 Basic OFDM

Let $\{d_{n,k}\}_{K=0}^{N-1}$ with E $[d_{n,k}]^2=\sigma_s^2$ be the complex symbols to be transmitted at the n^{th} OFDM subcarrier index, then the OFDM modulated signal can be represented by : [2][5]

$$S(t) = \sum_{K=0}^{N-1} d_{(n,k)} e^{j2HK\Delta ft} \quad 0 \le t \le T_s \qquad (1)$$

Where $d_{(n,k)}$ is the data for stream n^{th} OFDM block, T_s, Δf and N are the symbol duration , the sub channel space and the number of sub channels of OFDM signals, respectively j is the square root of -1.For the receiver to demodulate the OFDM signal, the symbol duration should be long enough such that $T_s\ \Delta f = 1$, which is also called the orthogonal condition since it makes $e^{-j2\pi K\Delta ft}$ orthogonal to each other for different K. At the receiver the transmitted signal by the receiver is detected by

$$S_{(n,k)} = \frac{1}{Ts\int_0^{Ts} d_{(n)}\,(t)e^{-j2\pi K\Delta ftdt}} \qquad (2)$$

If there is no channel distortion.

Now, we rewrite the Eq(1) for discrete time at t=nT to get:

$$\begin{aligned} S_{(n)} &= \sum_{K=0}^{N-1} d_{(n,k)}.\, e^{j2\pi K\Delta fmTs/N} \\ &= \sum_{K=0}^{N-1} d_{(n,k)} e^{j2\pi mK/N} \end{aligned} \qquad (3)$$

Considering the summation of the right hand of the eq(3) which is actually the Inverse Discrete Fourier Transform (IDFT) with $\{d_{n,k}\}_{K=0}^{N-1}$ the data at subcarrier K at time sample n and can efficiently be calculated by fast Fourier Transform (FFT).At the receiver it can be easily seen that demodulation at the receiver can be performed by using Discrete Fourier Transform (DFT) instead of the integral Eq(2).

In this figure, the cyclic prefix or guard interval of signal S(t) can be expressed as

$$S(t) = 1/\sqrt{N} \sum_{t=-\infty}^{t=\propto} \sum_{K=0}^{N-1} \frac{d_{(n,k)} \cdot}{e^{j2\pi n(t- \; T_g \; -iT_s)4(t-iT_s)//T_u}} \quad -T_g \le t \le T_u \tag{4}$$

Fig. 1 shows the function of the CP. If the CP is absent, the length of the OFDM symbol is T_s as shown in Fig. 1, But with the CP, the transmitted signal is extended to $T=T_u+T_g$, the duration of OFDM symbol and u(t) is defined by [6],

$$u(t) = \begin{cases} 1 & 0 \le t \le T_s \\ 0 & Otherwise \end{cases} \tag{5}$$

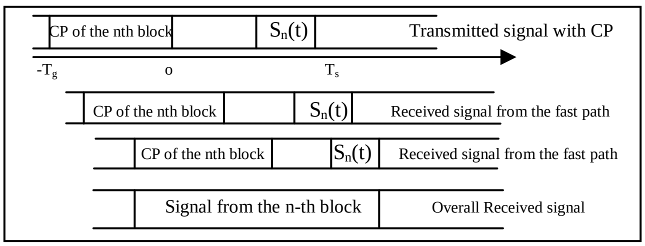

Fig. 1 Function of the CP

Where g_{BPSK} is the complex envelope of the signal of Eq(4) can be written as,

$$S(t)= Re\{\sqrt{(2E_b/T_b)}\, d_{(k,n)}\, e^{j(\varphi_{n-1}[i]+\Delta\varphi_n[i])} \tag{6}$$

Where E_b is the arrange energy per bit, p_{n-1} is the phase of the symbol modulated on the (n-1)th sub carrier of the OFDM symbol.

Accordingly the phase of the carrier of Quadrature phase shift keying (QPSK) takes on one of four equally spaced values, such as 0,π/2,π and 3π/2,then the complex symbol $d_{(n,k)}$ can be written as

$$S(t)=\{\sqrt{E_s}\cos[\frac{(i-1)\pi}{2}\varphi_1(t) - \sqrt{E_s}\sin[(i-1)\pi/2\ \varphi_2(t)\} \quad i=1, 2\ldots \tag{7}$$

Where E_s is the average energy per symbol, each symbol corresponds to two bits, then $E_s=2E_b$.

And in the similar way, the quadrature amplitude modulation (QAM) are applying of the complex data of $D_{(n,k)}$ can be written as

$$S(t)= \sqrt{(2E_{min}/T_s)}\, D_{(n,k)}\, e^{j2\pi f_c\, t} \qquad 0\le t \le T, \quad i=1,2,\ldots..M \tag{8}$$

The channel impulse response of a wireless channel can be expressed by [8]

$$h(\tau,t) = \sum_{i=0}^{l=1} h_l(t)\delta(\tau - \tau_l) \tag{9}$$

Where $h_l(t)$ and τ_l is the complex amplitude and propagation delay of the lth path respectively. Then the received signal can be expressed as

$$X_n(t) = \sum_{i=0}^{l-1} h_l(t)\delta(t - \tau_l) + n(t) \tag{10}$$

Where n(t) represents the additive White Guassian Noise (AWGN) at the receiver.

The signal at sub carrier K (X (k)) after the FFT process the received signal $X_n(t)$ after demodulating the carrier frequency (f_c) given by [6]

$$\begin{aligned} Y(t) &= e^{j2\pi\Delta ft}\sum_{l=0}^{l-1} h_l(t)S(t-\tau_l) + n(t) \\ &= \sum_{n=0}^{N-}\sum_{l=0}^{l-1} S(t-\tau_l)h_l(t)e^{j2\pi\Delta ft} + \sum_{K=0}^{N-1} n(t).\, e^{j2\pi\Delta ft} \\ &= 1/N \sum_{n=0}^{N-1}\sum_{l=0}^{L-1}(h_l(t)e^{j2\pi\Delta ft} \sum_{K=0}^{N-1} d_{(n,k)}\, e^{j2\pi mk/N}) + \sum_{n=0}^{N-1} n(t)e^{j2\pi\Delta ft} \end{aligned} \tag{11}$$

Where n(t) is a complex Gaussians noise process with zero mean and variance σ^2 per dimension.

Let, the N point FFT of the channel response $h_l(t)$ be :

$$\sum_{l=0}^{N-1} h_l(t)e^{j2\pi\Delta ft} = h_{(n,k)} \tag{12}$$

Then from equation (11) can be written as,

$$Y(t)= d_{(n,k)}h_{(n,k)} + n(t)$$

Ignoring the noise, dividing y(t) by the channel response $h_{(n,k)}$ yields the transmitted data .

The block diagram of the system model used in this paper is shown in Fig. 1.

1.2 Concatenated Coding Schemes

Consider an R-S code of (n, k) = (255,239), where each symbol is made up of m=8 bits, since n-k=16 which indicates that t= $|d_{min} - 1/2| = |n - k/2|$ = 16/2=8 , this code correct any eight symbol errors in a block of 255.In the presence of burst error noise ,say 25 contiguous bits must disturb exactly eight symbols. The R-S decoder for the (255,239) code will correct any eight symbol without regard to the type of damage suffered by the symbol [9].

Convolutional codes can be specified as CC (n, k, m), n≥ m, where n is the number of output bits, k is the number of input bits and m is the constraint length of the encoder [10].The codes used in frequency paper are CC (2,2,1) (3,3,2). Decoding is performed using the viterbi algorithm [8].

2. BER Analysis of Coded OFDM System

In this system we will investigate the BER performance of OFDM under RS-CC coded method. Non binary RS code based on symbols from Galois field GF(2^m) , where each symbol consists of m bits , For a t-error correcting RS code , its generator polynomial is ,

$$g(x) = \prod_{i=1}^{2t}(x + \alpha i)$$

Where n-k=2t and α is a primitive element in GF (2^8).Following this strategy, systematic shortened (255,239) Reed Solomon encoding is performed on each received 239 byte packet, with T=8, this mean that 8 erroneous bytes per transport packet can be corrected [11].

The convolutional interleaving process is composed of overlapping error protected packet and can be delimited by synch bytes. The interleaver is connected to the input byte stream cyclically by the input switch. For synchronization purposes, the synch bytes and the inverted sync bytes shall be always routed into the (FIFO) shift register with depth (M) cells (where M=N/I, N= 204= error protected frame length , I= 12= maximum interleaving depth , J= Branch Index) of "O" of the interleaver. The de-interleaver principle is similar to the interleaver where the branch indexes and reversed [12] [18].

The convolutional code is chosen from the following table of code rates which are used in this paper by puncturing ½ and ⅔ rated with constraint length k=7 with Generator Vector G and puncturing patterns P.

Table 1: Convolutional Coding Schemes of ½ and ⅔ rated

Original Code			***Code Rates***			
			1/2		2/3	
K	G_1	G_2	P	d_{free}	P	d_{free}
7	171_{oct}	133_{oct}	X=1	10	X=10	6
			Y=1		Y=11	
			$I=X_1$		$I=X_1Y_2Y_3$	
			$Q=Y_1$		$Q=Y_1X_3Y_4$	

The results presented here are based on concatenated RS and CC code. The performance of the convolutional code can be expressed by the following:

$$P_{cb} \approx \frac{i}{K}\sum_{d=d_{free}}^{d_{free}+N} C_d\, P_d \qquad (13)$$

Where k is the number of bits input into the encoder, d_{free} is the free distance of the convolutional code, C_d is the total number of bit errors, P_d is the probability of choosing an incorrect path and N is the number of significant terms used in the calculation.
By exploiting the Gaussian distribution, the BER of BPSK can be shortly expressed as [13], follows the exact of UB in case of binary signaling. Now the exact error probability of BPSK in AWGN can be expressed as,

$$P_b^{NB} = \lim_{N_{ADC}\to\infty} P_b^{BPSK} = Q\sqrt{\delta} = Q\sqrt{(2E_b}/N_0) \qquad (14)$$

Where the Q function is defined as

$$Q(x) = 1/\sqrt{2\pi}\int_x^{\infty} e^{-x^2/2}\, dx$$

In similar way, the BER for QPSK and 4QAM is expressed by [14].

$$\text{BER}= \frac{\int_0^{t=\infty} Q(\sqrt{|\lambda_1|^2}|m_1|^2}{|\lambda_1|^2\sigma^2{}_{ICI}+\sigma^2{}_{ICI,\beta}+\sigma^2{}_{AWGN}} \times \frac{2|\lambda_1|}{c\lambda_1\,\lambda_1}\exp\,(-|\lambda_1|^2/ c\lambda_1\,\lambda_1)d|\lambda_1| \qquad (15)$$

Which can be solved by numerical techniques such as the Laguerre-Gauss quadrature [15] or calculated by Monte Carlo method.

On the other hand, in the Rician fading channels the integral form of equation for QPSK and QAM is following [14].

$$\text{BER}=\int_{\lambda_{1,NLOS}} Q(\sqrt{\Upsilon}(\lambda_{LOS,}\,\lambda_{1,NLOS}) \times f_{\lambda_{1\,,NLOS}(\lambda_1\,,NLOS)}\, d_{\lambda_{1,}\,NLOS} \qquad (16)$$

Where $f_{\lambda_{1,NLOS}}(\lambda_{1,NLOS})$ is complex Gaussian with zero mean.And obviously it can be said that the Rician factor K determines the LOS termλ_{LOS}.
And another case of Rayleigh fading, the conditional random variable t1 = z1 1 λ_1 becomes,

$$t_1 = A_0\,(m_1\lambda_1 S_1 + \alpha 1\lambda 1 + \beta_1) + \widetilde{\alpha_1}\,\lambda_1 + \widetilde{\beta_1} + V_1 \qquad (17)$$

Where α_1 and β_1 are expressed by [16] and [17] respectively,

$$\widetilde{\alpha_1} = C_{\lambda_1\lambda_1}^{-1}\sum_{n=1}^{N} m_n\, C_{\lambda n}\lambda_1 V_{NL,n}$$
$$\widetilde{\beta_1} = \sum_{n=2}^{N} m_n\, K_n\, V_{NL,n}$$

Such each element of the nonlinear distortion noise vector $V_{NL}[l] = \mathrm{F}W_{NL}[l]$ is obtained by linear combination of N elements, where N is large.

The total system is simulated using parameters which are shown in Table 2.

Table 2: Simulated coding, modulation schemes and noisy channels.

Parameters	***Value***
Number of used subcarriers	200
Cyclic prefix	0.25
N_{FFT}	256
RS code	(255,239,8)
CC code	(1/2 and 2/3)
Modulation	QPSK, 4QAM,2PSK
Noise channels	AWNG,Rayleign,Rician

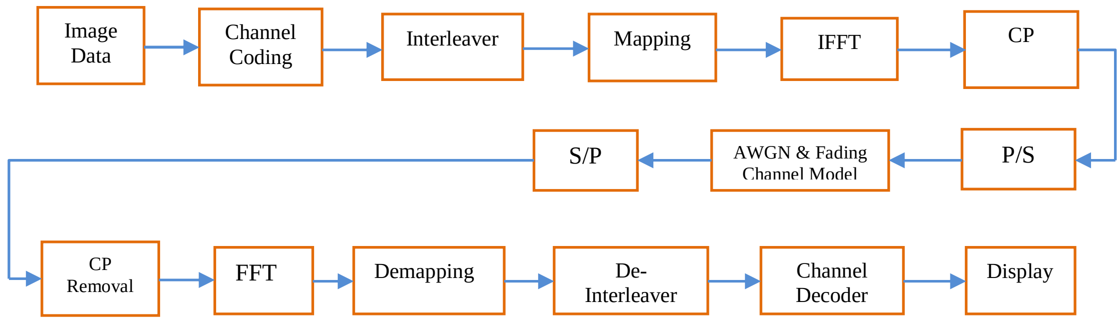

Fig. 2 OFDM with concatenated coding block diagram for image transmission.

3. Simulation Results

The derivation of the BER of OFDM systems with carrier frequency offset and channel estimation error in AWGN, Rayleigh and Rician fading channels model will be given. We consider an OFDM system with N=256 subcarriers with a subcarrier separation of Δf= 1/t=312.5 KHz and with cyclic prefix of length L=16. Fig. 3 shows the BER performance of 2PSK in the ½ and ⅔ rated coding schemes of AWGN and different fading channel models. It is evident that the performance of ½ rated RS-CC coding schemes with different noise channel models give better performance result than ⅔ rated code. Table 3 shows the precise BER performance of fig 3.

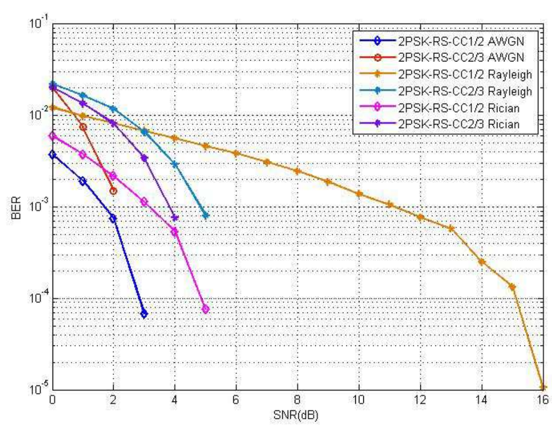

Fig. 3 BER analysis for 2PSK in AWGN and Rayleigh, Rician fading channels for ½ and ⅔ rated RS-CC coded.

Table 3: BER vs SNR for 2PSK modulation.

BER 10^{-235} for 2PSK modulation	***SNR Results***					
	AWGN		*Rayleign*		*Rician*	
	1/2	2/3	1/2	2/3	1/2	2/3
	0	1.8dB	6.5dB	4dB	2dB	2.5dB

From the table 3 it is seen that the performance analysis in the presence of nonlinear fading channel models that both ½ and ⅔ rated RS-CC coding schemes of Rician channels (Rician factor K=8) than Rayleign channels.

Fig. 4 illustrates the BER performance in the 4-QAM modulation technique where AWGN and fading channel models are present. Table 4 represents the BER vs SNR.

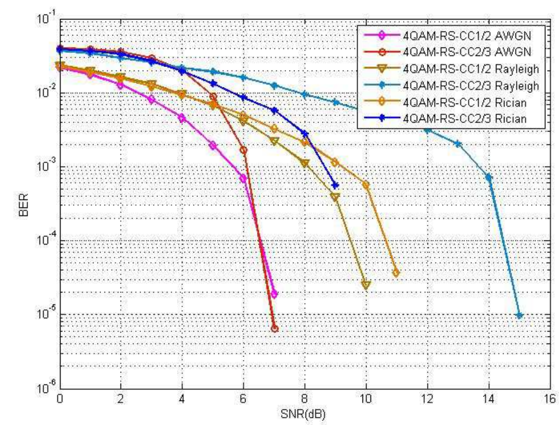

Fig. 4 BER analysis for 4QAM in AWGN and Rayleigh, Rician fading channels for ½ and ⅔ rated RS-CC coded.

Table 4: BER vs SNR for 4QAM modulation.

BER 10^{-3} for 4QAM modulation	***SNR Results***					
	AWGN		Rayleign		Rician	
	1/2	2/3	1/2	2/3	1/2	2/3
	5.8dB	6.0dB	7.1dB	12.1dB	8.5dB	8.5dB

Fig 5 illustrates the BER performance of 16QAM modulation technique where AWGN and fading channels are present .Table 5 presents the BER decreases on the value of SNR increases. So a good agreement is clearly indicates that in the AWGN case, the Gaussian approximation of the ICI leads to the accurate result. However in fading channels, the BER is obtained by averaging P_{BE} (λ_1) over the P_{Df} of λ_1 and hence it is practically imposed by the values of P_{BE} (λ_1) that correspondent to the small values of (λ_1) [19]. For these values, the Gaussian approximation is very good, because P_{BE} (λ_1) is high and therefore the obtained BER is small of any other fading channel models.

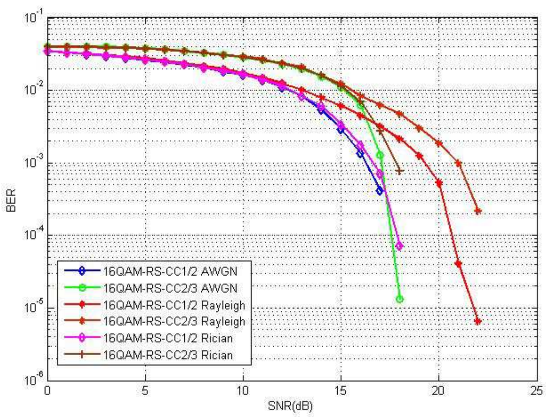

Fig. 5 BER analysis for 16QAM in AWGN and Rayleigh, Rician fading channels for ½ and ⅔ rated RS-CC coded.

Table 5: BER vs SNR for 2PSK modulation.

	SNR Results					
BER 10^{-3} for	*AWGN*		*Rayleign*		*Rician*	
16QAM	1/2	2/3	1/2	2/3	1/2	2/3
modulation	16dB	17.5dB	18dB	22dB	15dB	17dB

Fig. 6-11 shows the color images of 84 X 30 pixels is transmitted through the AWGN and Rayleigh and Rician fading channel models with using various modulation techniques. The figure also highlights the performance of ½ and ⅔ rated Reed Solomon and Convolutional Coded which is to apply the transmitted color images. It is seen to observe all the images that the performance of ½ rated RS-CC coded gives the better performance than the ⅔ rated RS-CC coded techniques.

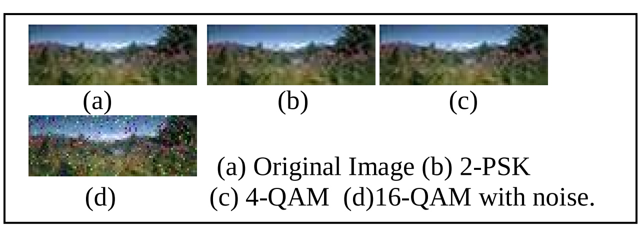

Fig. 6 Transmitted and Received images for ½ rated RS-CC Coded over AWGN Channel

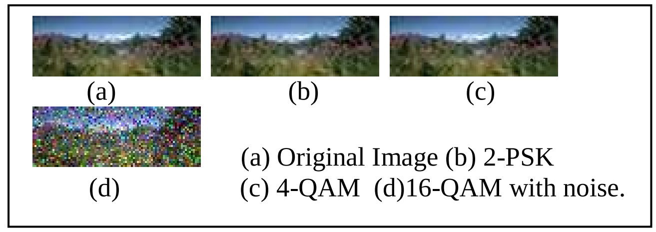

Fig. 7 Transmitted and Received images for ⅔ rated RS-CC Coded over AWGN Channel

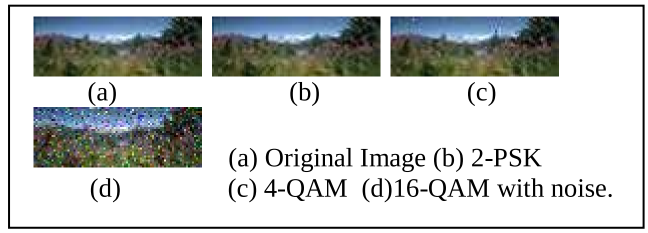

Fig. 8 Transmitted and Received images for ½ rated RS-CC Coded over Rayleigh Channel

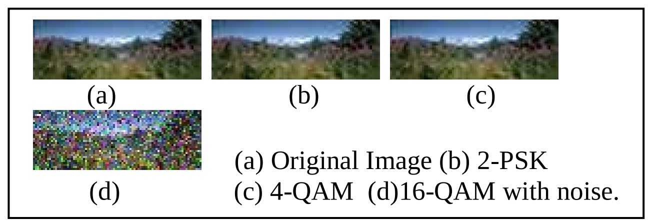

Fig. 9 Transmitted and Received images for ⅔ rated RS-CC Coded over Rayleigh Channel

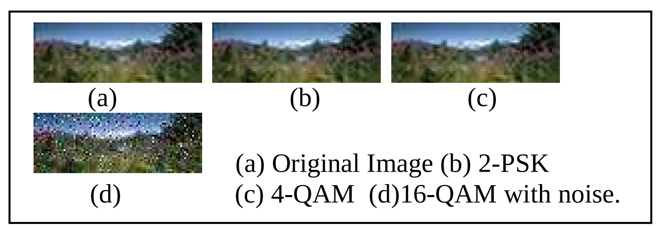

Fig. 10 Transmitted and Received images for ½ rated RS-CC Coded over Rician Channel

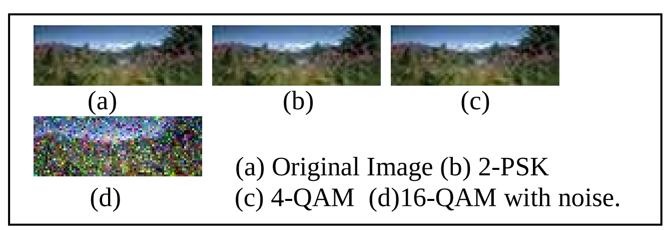

Fig. 11 Transmitted and Received images for ⅔ rated RS-CC Coded over Rician Channel

4. Conclusions

In this paper, we developed an exact method for calculating the BER of ½ and ⅔ rated RS-CC coding schemes OFDM schemes in the presence of frequency offset over frequency selective fast Rayleigh fading channels. An analytical BER expression is obtained. The closed formed BER expression can be calculated directly for N=256 number of subcarriers. In the case of ½ rated RS-CC codes, the BER expression was used to analyze the system performance under several wireless channel configurations. Delay spread and Doppler spread will cause significant performance degradation. In these cases, we simulate different combination of modulating technique with the code rate. We compared the performance of RS-CC ½ and ⅔ code rated for color image transmission, confirms the outperformance of the ½ rated RS-CC coding schemes for AWGN channel.

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