Midgets of Transcendental Superior Mandelbar Set

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Abstract

Antipolynomial of a complex polynomial is generated by applying iteration on a function $\bar{z}^{d}+c$ for $d>=2$. This complex function has been intense area for researcher. If we use transcendental function like sine, cosine etc with antipolynomial, i.e. *cos* $(\bar{z}^{d}+c)$, it becomes a more elite area to design beautiful images of fractal. The purpose of this paper is to generate the fractals using function *cos* $(\bar{z}^{d}+c)$, having combined characteristics of conjugacy and transcendental function. One of the most striking features of these fractals is the presence of tricorn and Mandelbrot set on the external rays i.e. midgets of Mandelbrot set for different values of constant variable. After a specific number of iterations, the fractals converge to fixed point.

Keywords: *Superior antiMandelbrot set, Superior antiJulia set, Midgets, Mann Iteration & Fixed Point.*

1. Introduction

Mandelbrot set mathematically defined as a set for which the Hausdroff Besicovich dimension exceeds the topological dimension [7]. The Mandelbrot set and its various extensions and variation have been studied by using Mann, Picard and Ishikawa iteration methods.

In this paper all fractals generated by iterating polynomials of type $\bar{z}^{d}+c$ with transcendental function defined in the complex plan. Here c is a complex parameter, d is an integer, bar sign denotes complex conjugation & transcendental function can be sine, cosine or tangent. Here we will generate fractals for cosine function. For any degree d, there is one holomorphic & one antiholomorphic family of polynomials, labelled by parameter c [2]. A holomorphic function is a complex valued function of one or more complex variables that is complex differentiable in a neighbourhood of every point in its domain. An Antiholomorphic Function of the complex variable z defined on an open set in the complex plane is said to be antiholomorphic if its derivative with respect to z^* exists in the neighbourhood of each & every point in that set, where z^* is the complex conjugate [14]. It has been observed that after applying much iteration to these polynomials, complex number z either move away from the origin and escape towards infinity or bounded to origin. Points with bounded orbit under iteration make up the filled in Julia set, which we call K_c^d for the holomorphic and K_c^{*d} for the antiholomorphic case. The topological boundaries of these bounded sets are Julia sets [2].

Fractal with conjugate of z values referred to as Mandelbar set, name is given by Crown et al. because of its formal analogy with Mandelbrot set and also brought its features bifurcations along axes rather than at points. The critical point of Mandelbar set is zero as in Mandelbrot set [13]. The study of connectedness locus for antiholomorphic polynomials defined as Tricorn, coined by Milnor, plays intermediate role between quadratic and cubic polynomials. Recently Shizuo [12] has quoted the Multicorn as the generalized Tricorn or the Tricorn of higher order and presented beautiful images and their various properties. This set is particularly interesting because its topological properties are different from the ones believed to be true for the Mandelbrot set.

This paper focuses the formation of midgets and the properties of superior Mandelbar set with its usual fractals. The midgets of the superior Mandelbar set are the small mini Mandelbrot set like images found in the scattered surrounding of the superior Mandelbar set [1]. The study of midgets in the Mandelbrot set is given by Philip [8] and Romera [11].

2. Preliminaries & Definitions

This paper focused on the formation of images for an antipolynomial with transcendental function. An iteration method i.e. Mann iteration is applied to function and superior tricorn and multicorn are obtained as a resultant. Some related definitions are as follows:

Mann Iteration: The method is given by [1] [5] [6]

$$z_d = sf(z_{d-1}) + (1-s)z_{d-1},$$

Where z is a complex number and s is convergent to a non-zero number. The value of s exists as $0<s<1$.

Superior Orbit: The sequence $\{z_d\}$ constructed above is called superior sequence of iterates, denoted by, SO(f,z_0,s) [5] [6].

Superior Tricorn and Multicorn: A superior multicorn, for the complex-valued polynomial $Ac=\bar{z}^{\,d}+c$, for $d \geq 2$, is the collection of all $c \varepsilon C$, for which superior orbit $SO(Ac, z_0, s)$, with $z_0 = 0$ is bounded, i.e.

$Ac = \{c \varepsilon C: \{ A_c^k (0), d = 0, 1, 2,\ldots\}$ is bounded with respect to $SO\}$.

A superior multicorn with $d=2$ is known as superior tricorn [4].

A holomorphic function whose domain is the whole complex plane is called an entire function. For an entire function, there are two types of singular values that play an important role in determining the dynamics. These are the critical and asymptotic values [10].

For a transcendental function [3]

(a) a is a critical value of function if there exists a z_0 belongs to c such that $f(z_0)=c1$ & $f'(z_0)=0$. In this case, z_0 is called a critical point of f. (Algebraic Singularity)

(b) a is an asymptotic value of f if there exists a curve “any symbol” tending to infinity such that $f(z) \to a$ as $z \to \infty$ along “any symbol”.

Singular values are important because any attracting fixed point must have a singular value in its immediate basin of attraction.

3. Fractal Generation Criteria

Escape criteria is used to compute transcendental superior AntiMandelbrot set and transcendental superior AntiJulia set. The values of escape criteria are:

For a complex quadratic function of the form $f(z_n)=z^2+c$, the escape criterion is $max\{|c|, (2/s)\}$ and for the cubic polynomial $Q_{a,b}(z) = z^3 + az + b$, the escape criterion is $max\{|b|, (|a| + 2/s)^{1/2}\}$. In the similar way for general polynomials, the escape criterion is $max\{|c|, (2/s)^{1/(d-1)})$.

Note that the initial value z_0 should be infinity, since infinity is the critical point of z for $cos(\bar{z}^{\,d}+c)$. However instead of starting with $z_0 = \infty$, it is simpler to start with $z_1 = c$, which yields the same result. A critical point of $z \rightarrow f(z)+c$ is a point where $f'(z)=0$. The point z in Mandelbrot set for cosine function has an orbit that satisfies $imag(z) > 50$. At this value the orbit of z escapes [9].

4. Geometrical Analysis of Superior AntiMandelbrot Set & Superior AntiJulia Set for Cosine Function

4.1 Description of Superior AntiMandelbrot set for cosine function

This paper shows the fractals obtained by applying conjugacy as well as transcendental cosine function to the Mandelbrot set i.e. $cos(\bar{z}^{\,d}+c)$ .These both terms have their specific geometrical shape images. Another interesting feature of the images is the existence of midgets in terms of tricorn, multicorn and Mandelbrot sets with different number of bulbs on external rays of key image.

The detailed description of images is as follows:

Case 1 (s=1)

- After applying Mann iteration to conjugate cosine function, obtained fractal is having $(2d+1)$ number of arms where d is the power.
- The midgets consist of multicorn with ($2d+1$) number of arms surrounded by Mandelbrot sets with ($2d-1$) number of bulbs.
- Mandelbrot set with the same number of bulbs is enclosed by much multicorn with the previously calculated number of arms.
- In case of quadratic polynomials, on zooming at the backside of image much multicorn with ($2d+1$) arms encircled one Mandelbrot set and in the similar way, for cubic polynomials much multicorn with ($2d+1$) arms encircled one tricorn.

Case 2 (0<s<1)

For $n \geq 2$, we observe that superior antiMandelbrot set of n^{th} order contains its own unique geometrical image symmetrical along real axis. On zooming along the superior antiMandelbrot set for $n \geq 2$, we get the midgets on the antenna or external rays of the fractal. A remarkable feature is observed that all the midgets are either basic Mandelbrot set or tricorn irrespective of the degree of polynomial. However the midgets are dependant on the degree of polynomial at $s=1$.

4.2. Description of superior AntiJulia set for cosine function

Transcendental function $cos(z)$ with superior AntiJulia set follows the law of having $2d$ wings, where d is the power of z. The images for all polynomials possesses symmetry about both x and y axis. In case of cubic polynomial, we have analyzed that the central part of image is in contracted form for $s=0.1$ but expanded for further values of s. As the value of s changes to 1, the central black part of image converted to flower shape with long-drawn-out rays. The fractal formed with large central body having rotational and reflection symmetry along with axes symmetry.

4.3. Generation of superior antiMandelbrot sets for cosine function:

Tricorn:

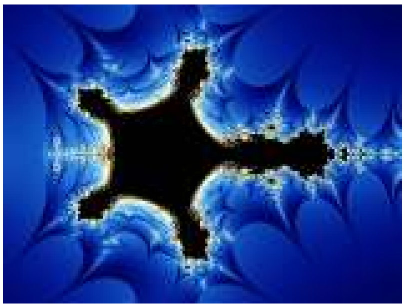

Fig 1: F(z) = $cos\ (\bar{Z}^{2}+c)$ & s=1

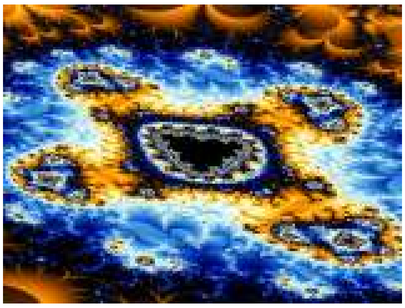

Fig 2: Midgets for f(z) = cos $(\bar{Z}^{2}+c)$ & s=1

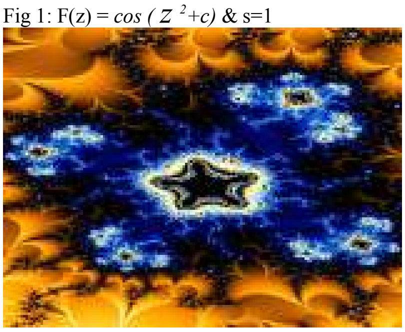

Fig 3: Midgets for f(z) = $cos\ (\bar{Z}^{2}+c)$ & s=1

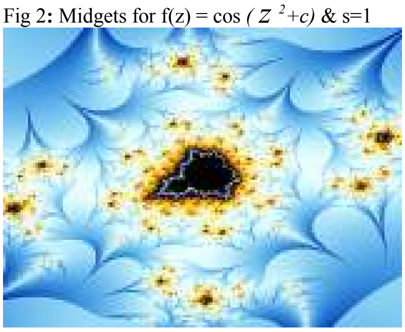

Fig 4: Midgets for f(z) = cos $(\bar{Z}^{2}+c)$ & s=1

Multicorns:

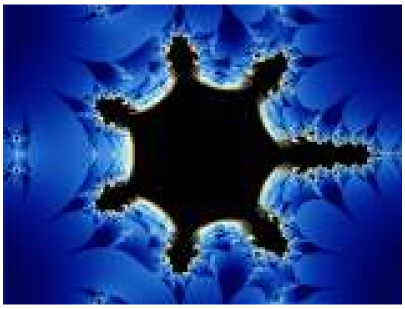

Fig 5: F(z) = $cos\ (\bar{Z}^{3}+c)$ & s=1

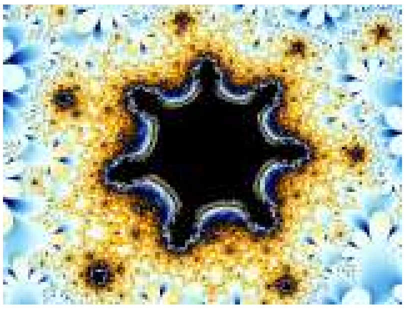

Fig 6: Midgets for f(z) = $cos\ (\bar{Z}^{3}+c)$ & s=1

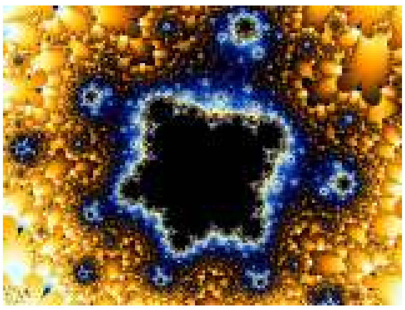

Fig 7: Midgets for f(z) = $cos\ (\bar{Z}^{3}+c)$ & s=1

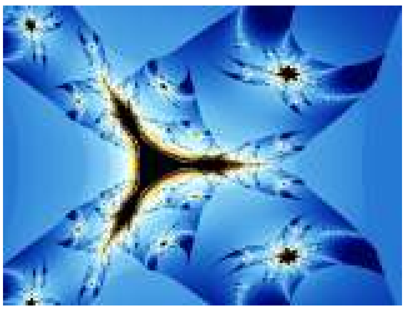

Fig 8: Midgets for f(z) = $cos\ (\bar{Z}^{3}+c)$ & s=1

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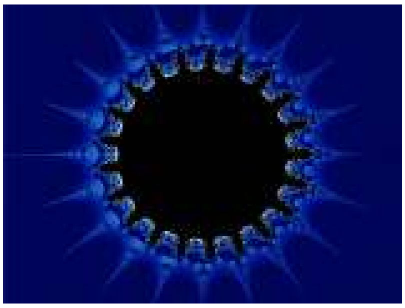

Fig 9: F(z) = *cos* ($\overline{Z}^{10}$+*c*) & s=1

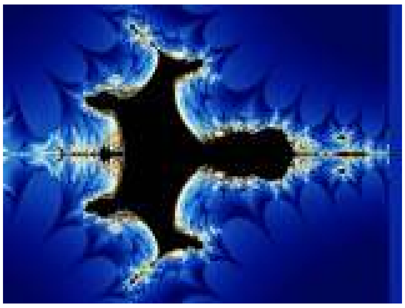

Fig 10: F(z) = *cos* ($\overline{Z}^{2}$+*c*) & s=0.5

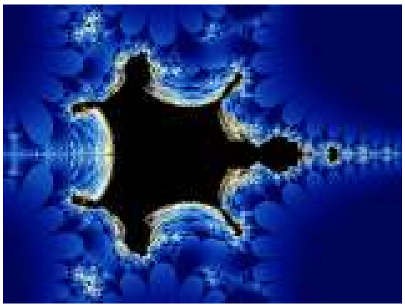

Fig 11: F(z) = *cos* ($\overline{Z}^{3}$+*c*) & s=0.5

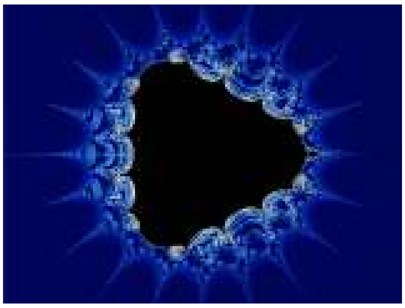

Fig 12: F(z) = *cos* ($\overline{Z}^{10}$+*c*) & s=0.5

4.4. Generation of superior AntiJulia sets for cosine function:

Cubic Function with different value of constant s:

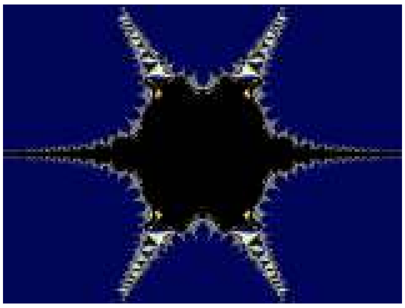

Fig 13: F(z) = *cos* ($\overline{Z}^{3}$+*c*),
s=0.1 and C==-8.785714286+0i

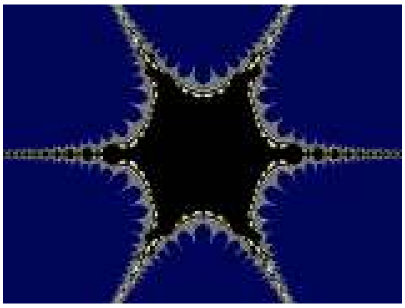

Fig 14: F(z) = *cos* ($\overline{Z}^{3}$+*c*),
s=0.5 and C=-1.4875+0.0625i

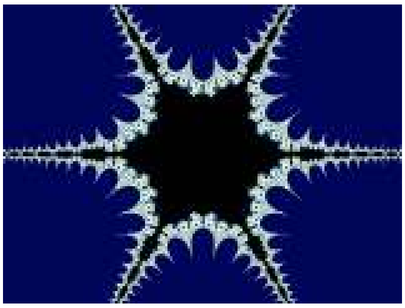

Fig 15: F(z) = $\cos(\bar{Z}^3+c)$,
s=1 and C=-0.3392857143+0.01785714286i

4.5. Fixed Point

4.5.1. Fixed point of cubic Function at s=0.1

Table 1: Orbit of F(z) at s=0.1, (C=-8.785714286+0i)

Number of Iteration i	***F(z)***	***Number of Iteration i***	***F(z)***
1	0.1214	5	0.1154
2	0.1148	6	0.1154
3	0.1155	7	0.1154
4	0.1154	8	0.1154

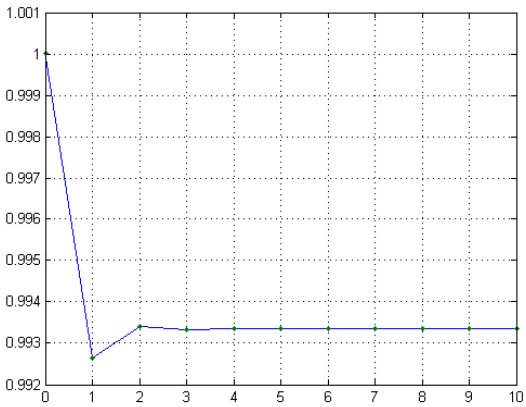

Fig 1: Orbit of F(z) at s=0.1 for (C=-8.785714286+0i

4.5.2. Fixed Point for Cubic Function at s=0.5

Table 2: Orbit of F(z) at s=0.5, (C=-1.4875, 0.0625i)

Number of Iteration i	***F(z)***	***Number of Iteration i***	***F(z)***
1	0.2562	5	0.2420
2	0.2401	6	0.2420
3	0.2422	7	0.2420
4	0.2419	8	0.2420

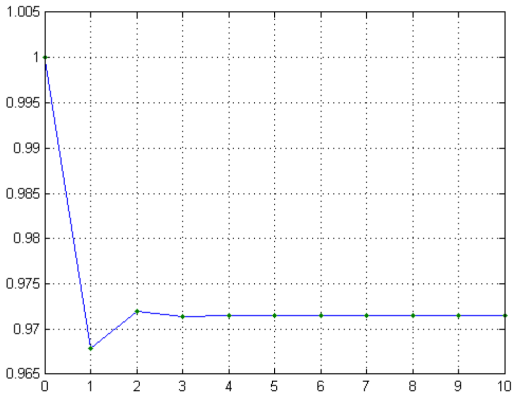

Fig 2: Orbit of F(z) at s=0.5 for (C=-1.4875, 0.0625i)

4.5.3. Fixed Point for Cubic Function at s=1

Table 3: Orbit of F(z) at s=1, (C=-0.3392857143+ 0.01785714286i

Number of Iteration i	*F(z)*	*Number of Iteration i*	*F(z)*
1	0.6607	6	0.6302
2	0.6198	7	0.6303
3	0.6331	8	0.6303
4	0.6254	9	0.6303
5	0.6305	10	0.6303

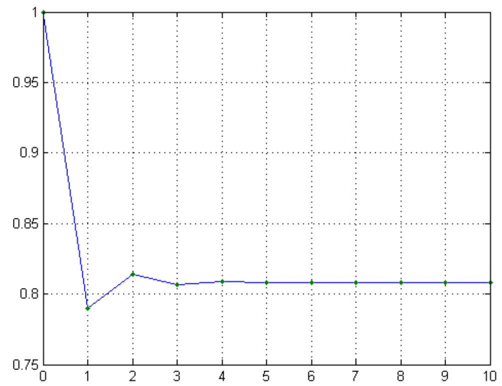

Fig 3: Orbit of F(z) at s=1 for (C=-0.3392857143+ 0.01785714286i)

5. Conclusion

This paper represents the fractals for cosine function with conjugacy of Mandelbrot equation using Mann iteration. Our analysis emphasize to the beauty of midgets appeared in superior antiMandelbrot set for the function $cos\ (\bar{z}^{d}+c)$, where d=2, 3 & 10. We find that the number of arms of the fractal follows the rule of (2d+1) branches for a particular value of constant s. One fascinating feature of these fractals is the appearance of midgets in such a stunning pattern.

Superior AntiJulia set possess 2n wings with central black region. As the value of constant s move from 0 to 1, the central black part expanded to external division. All the images are symmetrical about x and y axis as well as having reflection and rotational symmetry. Another remarkable observable fact is that the number of iterations required to achieve convergence for a particular choice of the parameter c, is much less in the superior orbit.

References

[1] Ashish Negi & Mamta Rani,"Midgets of Superior Mandelbrot set", Chaos, Solitons and Fractals 36 (2008) 237–245.

[2] Eike, Lau and Dierk Schleicher, " Symmetries of fractals revisited", The Mathematical Intelligencer, Vol 18, No. 1, 45-51, DOI: 10.1007/BF03024816.

[3] Kin-Keung Poon, "Fatuo-Julia Theory on Transcendental Semigroups", Bull Austrad. Math Soc. Vol. 58 (1998) [403-410].

[4] M Rani,,"Superior Antifractals",volume 1, IEEE, 978-1-4244-5586-7/10/2010.

[5] M Rani and V Kumar. Superior Mandelbrot sets, J. Korea Soc. Math. Educ. Ser. D; Res. Math. Educ. (8)(4)(2004), 279-291.

[6] M Rani, V. Kumar. Superior Julia set. J Korea Soc Math Educ Ser D Res Math Educ 2004;8(4):261–77.

[7] N. Shizuo and Dierk Schleicher, "On multicorns and unicorns: I. Antiholomorphic dynamics. Hyperbolic components and real cubic polynomials", Internat. J. Bifur. Chaos Appl. Sci. Engrg, (13)(10)(2003), 2825- 2844.

[8] Philip AGD. Wrapped midgets in the Mandelbrot set. Comput Graphics 1994;18(2):239–48.

[9] R. L. Devaney, A First Course in Chaotic Dynamical Systems: Theory and Experiment, Addison-Wesley, 1992. MR1202237 Zbl 0768.58001

[10] R. L. Devaney, "Complex Exponential Dynamics", Boston University, August 6, 2000. {5}

[11] Romera M, Pastor G, Montoya F. On the cusp and the tip of a midget in the Mandelbrot set antenna. Phys Lett A 1996; 221(3–4): 158–62. MR1409563 (97d:58073).

[12] Shizuo Nakane, and Dierk Schleicher, Non-local connectivity of the tricorn and multicorns, Dynamical systems and chaos (1) (Hachioji, 1994), 200-203, World Sci. Publ., River Edge, NJ, 1995. MR1479931.

[13]. W. D. Crowe, R. Hasson, P. J. Rippon, and P. E. D. Strain-Clark, On the structure of the Mandelbar set. Nonlinearity (2)(4)(1989), 541-553. MR1020441 Zbl 0701. 58028.
[14]. http://en.wikipedia.org/wiki/

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