

# Combining Mahalanobis and Jaccard Distance to Overcome Similarity Measurement Constriction on Geometrical Shapes

Siti Salwa Salleh<sup>1</sup>, Noor Aznimah Abdul Aziz<sup>1</sup>, Daud Mohamad<sup>1</sup> and Megawati Omar<sup>2</sup>

<sup>1</sup>Faculty of Computer and Mathematical Sciences  
University Technology MARA  
Shah Alam, Malaysia

<sup>2</sup>Research Management Institute  
University Technology MARA  
Shah Alam, Malaysia

## Abstract

In this study Jaccard Distance was performed by measuring the asymmetric information on binary variable and the comparison between vectors component. It compared two objects and notified the degree of similarity of these objects. After thorough pre-processing tasks; like translation, rotation, invariance scale content and noise resistance done onto the hand sketch object, Jaccard distance still did not show significance improvement. Hence this paper combined Mahalanobis measure with Jaccard distance to improve the similarity performances. It started with the same pre-processing tasks and feature analysis, shape normalization, shape perfection and followed with binary data conversion. Then each edge of the geometric shape was separated and measured using Jaccard distance. The shapes that passed the threshold value were measured by Mahalanobis distance. The results showed that the similarity percentage had increased from 61% to 84%, thus accrued an improved average of 21.6% difference.

**Keywords** - Jaccard Distance, Mahalanobis distance, distance measures, shape recognition, similarity measurement

## 1. Introduction

Similarity measure using distance measure techniques is one of the popular methods that are able to evaluate the similarity between two objects. It works in the manner that small distances correspond to large similarities and large distances do the small. Similarity measure has been used widely in the shape recognition classifier. Recently, recognition, which is based on shape, has been used in image processing, medical diagnosis, trademarks, shape retrieval, image retrieval, and industrial parts. Shape features have been used to measure similarity between objects [1] as it is a dominant feature of an object as it consist lines, contours, curves, and vertices. It is normally presented by discrete sets of points or sets of pixels value which are sampled from the regions or internal and external contours of the object [2].

It is widely used for measuring the similarity between patterns as reported in [1, 3, 4, 5, 6, 7], where patterns that are similar will have a small distance. While uncorrelated pattern in the feature space will have a far apart distance. The best way to select a distance or similarity measure is by identifying the most minimum vector space values by a simple function [7]. Another factor to consider is how close one point in the vector is to another between the two points [8], working on vector model and classification, to measure the similarity between two feature matrices of objects.

This measure is extensively employed in content-based image retrieval, shape-based image retrieval, and planar object recognition. It contributes to an accurate automatic retrieving system for trademark images [4], handwritten digit recognition [3], automatic bank cheque processing application [5], and texture [8].

To date, various distance measures techniques have been proposed and investigated theoretically. Distance measures that provide reasonable results for images comparison includes Euclidean Distance [6], Mahalanobis Distance [5], Chord Distance [1], Cosine Distance [4], Trigonometric Distance [7], Jaccard Distance [3] and others. Among these, the Jaccard Distance is the simplest and effective since it is based on the shape features and comparison [3]. Furthermore, the computation involved in the Jaccard Distance is simple, which leads to low computation time.

The next section of this paper briefly describes the sketch property and issues, Mahalanobis and Jaccard distances measures and their semantics. Section 3 discusses the feature analysis and transformation, and the methodology of combining the Mahalanobis and Jaccard distance measures. Section 4 explains the results and discussions.

Along the way, a few interesting findings were discovered which are presented in Section 4. Finally Section 5 concludes and discusses future work.

## 2. Problem Statement and Objectives

Each distance function has its own strength and weakness; therefore researchers must take extra care to select distance measures to match their needs and applications.

In this case a preliminary experiment [9] was conducted to identify the Jaccard Distance similarity measurement capability and to learn how the preprocessing tasks can improve its recognition performance. Briefly, the Jaccard Distance is performed by measuring the asymmetric information on binary variables and the comparison between vectors components. Further explanation on Jaccard Distance is presented in the later section.

This study evaluated the recognition performance on isolated digits hand writing using a pen-based device. The outcome of the experiment showed that low recognition was achieved by the conventional Jaccard Distance when there was no preprocessing task done on the input. However, after translation, rotation, invariance scale content and noise resistance (ROITRS) were added, the recognition progress showed a little improvement. But, the improvement was not significant. The result is shown in Table 1 and it can be seen that the degree of accuracy only improved on 20% average.

Table 1: Preliminary Experiment on Jaccard Distance Measurement Performance Without and With Pre Processing Tasks

DIGIT	Without ROITRS (50% of Respondents handwritten)			With ROITRS (50% of Respondents handwritten)			Gap (%)
	Mean	Std Dev	Similarity (%)	Mean	Std Dev	Similarity (%)	
1	0.69	0.05	31.0	0.49	0.04	50.6	19.6
2	0.70	0.05	30.1	0.48	0.04	52.0	21.9
3	0.69	0.05	30.9	0.49	0.04	51.0	20.1
4	0.68	0.05	32.5	0.49	0.05	51.5	19
5	0.68	0.05	32.2	0.46	0.05	53.7	21.5
6	0.69	0.06	31.1	0.47	0.03	52.7	21.6
7	0.69	0.05	30.8	0.48	0.04	51.9	21.1
8	0.68	0.05	31.8	0.48	0.04	52.3	20.5
9	0.71	0.06	29.2	0.48	0.05	51.7	22.5

The outcome of this study helped the present researchers in the next step that was to detail the pre processing tasks and obtain certain measures that would improve the similarity measurement. It also improved the researchers' understand

of Jaccard Distance basic strength and weakness in handling pen-based handwritten input.

From here, an automatic preprocessing task was developed for unsupervised input from using a pen. This input was to recognize the similarity between the geometrical images of two-dimensional uncontrolled handwriting sketch.

This paper then combined the Jaccard and Mahalanobis Distances without modifying their formula. The combination of their conventional equations of distance measurement was to improve the recognition accuracy without affecting the computation time. Therefore, the objectives of this study are: (i) to conduct a heuristic approach that combines Jaccard and Mahalanobis distance measurement without modifying their conventional equations; and (ii) to test the algorithm recognition accuracy.

## 3. Preliminaries on Shape and Similarity Measures

This section provides brief discussion on hand sketching using pen-based input.

As known, some distance measures work well in stroke but some do only in strong shape objects. Literature shows there are a few shortcomings in distance measures, which are (i) not all distance functions are able to handle strokes successfully; (ii) most distance functions require thorough pre-processing tasks such as noise removal, object enhancement and this is not applicable to automatic processing/classification, and finally, (iii) most distance functions are sensitive (not invariant) to object transformation.

Another problem is, using pen-based input, the writing will be different as compared to writing on paper using a pen or a pencil. Most users are not familiar with the non-ink pen and surface. While writing, the distance between the pen nozzle must be held at a certain hoover distance (a distance between the pen and the surface). Furthermore, the writing which does not appear on the surface forces the writers to always glance at the screen rather than concentrating on their hand while writing. Thus this might distort the handwriting or hand sketches, reduce smoothness, and affect the shape of the input.

### 3.1 Shape Properties and Issues

In this study, a shape is defined as a spatial information of an object which is determined by its external boundary. It can be categorized as geometrical or organic shapes. Geometric shapes are usually angular and appearing frequently in man-made objects, which also possess orientation. Each shape is precise. Basic geometric shapes in mathematics are square, triangle and circle where others are of these variations or combinations. They are diamond, oval, rectangle, parallelogram, trapezoid, pentagon, pentagram, hexagon and octagon.

In current advancement, mobile devices input surfaces allow users to draw or sketch objects shapes using a stylus of which has become free-hand drawing or sketching. However, the most crucial issue in designing new application falls on the strokes or line variations of the shapes. Even though holding a stylus is similar to holding a pen, the users will still produce variations in stroke size, density, length, width, continuity, smoothness, and pointing connection [12]. Thus this may produce unintended shapes of drawing. Other challenges are on input surface calibration and surface sensitivity. Users do not calibrate their surface coordinate in a regular basis. Likewise, users who are not familiar with technicalities may also do not realize that the calibration does not fit. This will also lead to variations in the sketched objects and shapes. Since this variability stems from several distinct sources, a reliable algorithm is highly required in recognizing sketches [14].

Therefore, in relation to the shortcomings mentioned above, this study analyzed how well Jaccard Distance measures the similarity of pen-based input handwriting shape that contains strokes. The required pre-processing tasks to support Jaccard Distance in improving the similarity measurements was also studied. Secondly, the study observed how far Jaccard Distance could improve the measurement. Finally, since most distance measures are variant to transformation, this study also helped us learn how sensitive the Jaccard Distance is to object transformations.

### 3.2 Jaccard Distance

Jaccard Distance is basically employed to compare two objects in a binary format and it also does not require a large set of data for training and testing purposes. It works by measuring the asymmetric information on binary variables, the comparison between two vector components. The computation of the Jaccard Distance measure determines the correlated or uncorrelated patterns based on pixel-based description where it takes into account the number of pixels in the foreground and the background of both patterns that lead to similar or dissimilar patterns.

The conventional Jaccard Distance similarity measuring between two patterns is calculated [10] as:

$$d(x_i, x) = 1 - \frac{p}{p+q+r} \quad (1)$$

Where:  
 $p$ : Number of pixels, which are in foreground for both patterns.  
 $q$ : Number of pixels, which are in background for  $x_i$  and in foreground for  $x$ .  
 $r$ : Number of pixels, which are foreground for  $x_i$  and in background for  $x$ .

If the two patterns have a maximal number of pixels in foreground,  $p$  for both, while the others,  $q$  and  $r$  will be zero, which leads to  $d(x_i, x) = 0$ , both of the patterns is similar. In contrary, the significant result shows that both patterns is uncorrelated and dissimilar if the value of  $p$  is zero which leads to  $d(x_i, x) = 1$ .

Conventional Jaccard Distance is basically employed to compare two prototypes. The comparison is made between suggested shapes and drawn shapes. The characteristics in computation using conventional Jaccard Distance are:

- i) The measures data should be in the original form (binary or grey level) where the data cannot be used if one employs any data features or transforms.
- ii) The unique condition to compute Jaccard Distance is that the size of both shape supports (images) should be similar and can be converted in the binary employ in vector representation
- iii) The comparison between the vector components, for example, the handwritten digit recognition [10].

Figure 1 shows the sample of calculation using the conventional Jaccard Distance [10].

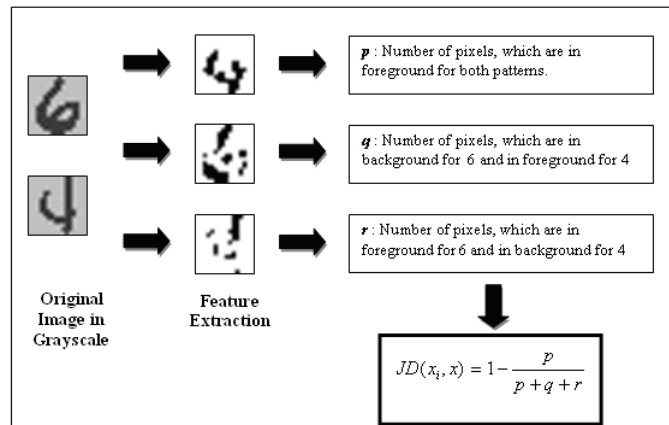


Figure 1: Example of Conventional Jaccard Distance Measurement(Source [1]).

Basically, the Jaccard Distance consists of a simple distance function and requires a minimal computation time. In this sense, Jaccard Distance is better than the other distance measurement in shape recognition. However, despite its strength and wide use, Jaccard Distance's disadvantage is its variant to image transformations. This distance function is commonly applied on binary data where the similarity computation computes the values of 0 and 1. The input object must be in binary form and therefore, Jaccard's method cannot be used if any transformations are employed to the object features. Another requirement is that the size of both binary object and shapes must be of similar size. However, Jaccard's limitation can be improved by adding pre-processing tasks prior to the computation.

### 3.3 Mahalanobis Distance

While Mahalanobis distance is always used for data clustering, calculated by measuring two data points in the space defined by relevant features, clustering technique groups data into clusters so that the data objects within a cluster have high similarity to one another. This process is not easy as the data objects will be dissimilar to those in other clusters. As it describes unequal variances and correlations between features, it will adequately evaluate the distance by giving different weights to the features of data points [14].

Mahalanobis Distance between two points  $x = (x_1, \dots, x_n)^t$  and  $y = (y_1, \dots, y_n)^t$  is defined as:

$$d_{MH}(\bar{x}, \bar{y}) = \sqrt{(\bar{x} - \bar{y})^T \Sigma^{-1}(\bar{x} - \bar{y})} \quad (2)$$

Where,

$\bar{x}$  and  $\bar{y}$  is means of two groups of data

$(\bar{x} - \bar{y})^T$  is the transpose of mean difference

$\Sigma^{-1}$  is the inverse of the covariance matrix of group

The Mahalanobis distance is quadratic multiplication of mean difference and inverse of pooled covariance matrix.

Despite its strength in clustering data, Mahalanobis is always known for its major weakness of computation times and its requirements for large number of samples for training. To overcome Mahalanobis weaknesses, quasi-Mahalanobis distance (QMD), Modified Mahalanobis distance (MMD), Simplified Mahalanobis distance (SMD) and incremental Mahalanobis distance (cited in [11]) have been proposed by previous researches. Those approaches are effective to reduce computation time since they use only parts of the dimensions or incrementally compute the

distance. Hence, in this present study sees that a low computation time pertaining Mahalanobis can be obtained by only taking the most extreme points or vertices of the shapes.

## 4. Experiment

### 4.1 Dataset

The dataset was obtained from the sketches produced by 10 male and 10 female university students ranging from 23 to 26 years of age. The students (respondents) were identified as regular stylus users of mobile phones. They were instructed without guidance to draw basic geometric shapes of triangle, square and diamond (TSD) ten times; therefore their handwriting was considered uncontrolled data. The devices used to obtain data were laptops and tablets and (for stylus). Data collection was conducted in a parallel mode where the respondents were in a sitting position at their convenience time. During the data collection, the respondents were not supervised, nor did they train or have any opportunity to use the pen in prior to the sketching in the data collection session. And the end of the data collection session we managed to collect 200 triangle, 200 square and 200 diamond hand sketching objects. In this experiment, the dataset were divided into training and testing data sets to ensure consistency throughout the experiment.

### 4.2 Feature Analysis and Transformations

All sketches of 350 X 1180 pixel size were cleaned from noises. Next, the shapes size and orientations were normalized and transformed into a 150 X 150 pixel format. The corner points of each shape were located and identified as extreme vertices. Based on the extreme vertices, the staggered edges drawn by the respondents were straightened or perfected. But even though every edge of the shape was straightened, the shape was still neither identical nor perfect [13].

Next, the shapes were divided into two data sets. The first group of data contained strokes or edges of the shape without corner points or edge intersections. The removal of edges was conducted by applying a 6 X 6 pixel mask at the intersection points. The remaining pixels were taken as strokes. In the first dataset, the triangle possessed two diagonal strokes and one horizontal; the square two horizontal and two vertical edges; and diamond four diagonal edges. Another set of data consisted extreme vertices of the shapes. These intersection points of each shape were defined as extreme vertices. The triangle had three extreme vertices, the squares and diamonds four extreme vertices.

### 4.3 The Combined Mahalanobis and Jaccard Distance

In common practice, Mahalanobis distance is only used to cluster or find similarity of a group of data. To date, little effort has been tried to measure shape similarity of strokes or lines using the Mahalanobis Distance. On the other hand, as mentioned, Jaccard is inflexible which works based on overlapping binary. However, despite being time consuming and inflexibility in computing overlapping vertices, both pose a great strength in terms of computation simplicity. This prompted an approach to combine both measures (which is named by this paper JM Approach) without modifying their formula. The sequence of processes involved in JM approach is shown in Figure 2.

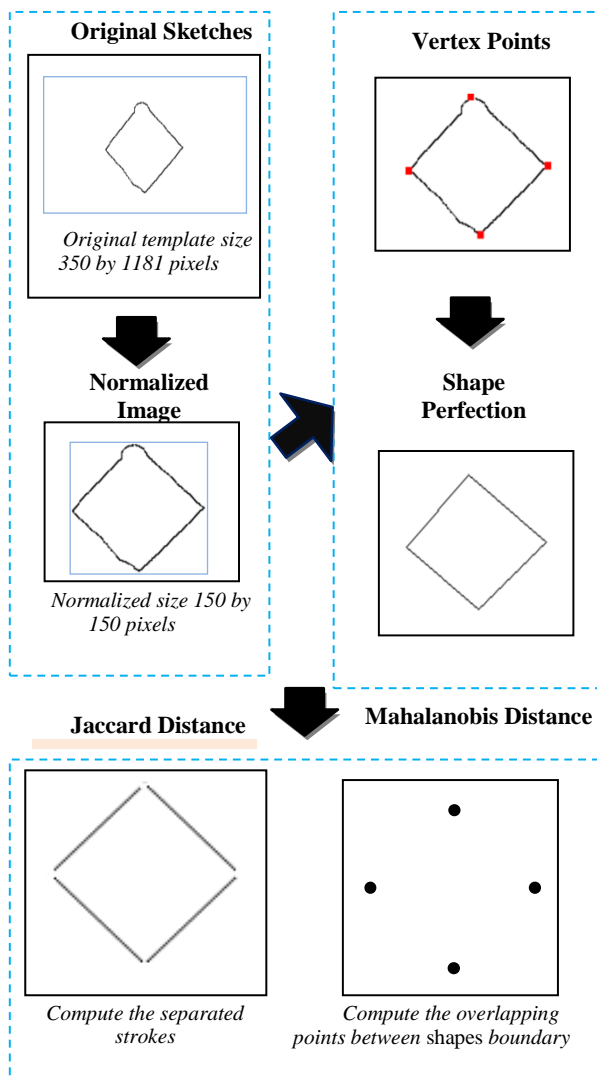


Figure 2: Flow Similarity Measurement Process

The process started with feature analysis and transform as described in Section IV-B. This was followed with similarity computation in three steps. The first step was similarity measurement using Jaccard distance. As the shape passed the threshold value, it was marked as a suspected shape. Next, the suspected shape was measured using Mahalanobis distance. The pseudo code of the JM steps is shown in Figure 3.

Normalized shapes consisted of separated strokes were then measured using the Jaccard distance and as the shape passed the threshold value of 30% similarity they were flowed into the Mahalanobis measurement clustering computation. It is important to note that the threshold value set in Jaccard distance was low as discovered in previous experiments [9]. The similarity percentage did not exceed 30% even though the shape drawn was clearly understood as a triangle, square or diamond in our manual checking. The objects that passed the threshold value were categorized as suspected objects.

```

Step 1: Compute Jaccard Distance ()
Compute ConvertToBinary (img1 && img2)
Given img1.Width == img2.Width && img1.Height
== img2.Height
Given maxW= maximum number of iterations
Given maxH= maximum number of iterations
For i = 0 to maxW
    For i = 0 to maxH
        Compute jd = (q + r) / (p + q + r)
        If 30 < similarity then
Step 2: Compute Mahalanobis Distance ()
Extract each corner using mask
Compute Masking ()
Given Reference [n1, k1]=size(A)
Given User [n2, k2]=size(B)
n = rawScoreA+rawScoreB
if (k1~=k2)
    columns No of A and B must be the same
else
transpose = a[j, i]
meanDifference = mean(A) - mean(B)
covarA = covariance(A)
covarB = covariance(B)
pooledCovar = rawScoreA/n*covarA +
rawScoreB/n*covarB
md = sqrt(((transpose(MeanDifference) *
inverse(pooledCovar)) * meanDifference)
Step 3: Weighted Average
For w = 1 to n
    Sumxw = xiwi
results = (Sumxw)/w
If (results > 70)
    Shape is Similar
else
    Shape is Dissimilar
    
```

Figure 3: Combined Method Pseudo Code

In Mahalanobis, the similarity is measured by clustering the extreme vertices against the referenced shape vertices. The referenced shape extreme vertices are vertices that are formed by 3 X 3 pixel masks. In this study the mask was located at a fixed location which was deliberated from 150 X 150 centroid. Figure 4 shows the location of the masks. The 6X6 mask was used for all shapes.

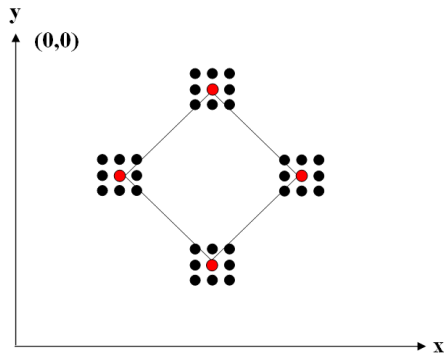


Figure 4. Masking Technique for Mahalanobis Similarity Measurement

In Step Three, the weighted average of both similarity values was calculated. The weighted average applied a concept where instead of each data point contributing equally to the final average, some data points would contribute more than the others.

The weighted average is shown in Equation 3.

$$y = \frac{\sum_{i=1}^n x_i w_i}{\sum_{i=1}^n w_i} \quad (3)$$

in which  $w_i$  are the weights that act upon the attributes  $x_i$ . Normalization is achieved by dividing the weighted numerator sum by the sum of all of the weights [9]. The sum of the normalized weights that act upon each  $x_i$  add to one, it is not a requirement that the sum of the un-normalized weights must add to one.

## 5. Result and Discussion

### 5.1 Results of Using Jaccard Distance

Before the result obtained from combined distance measures is presented, the initial result obtained from experiment showing the improvement made by having thorough pre-processing is shown first. The following table shows the result of measuring the similarity of shape using Jaccard distance without and with ROITS.

Table 2: Jaccard Distance Measurement Performance Without and With Pre Processing Tasks

S H A P E	Without ROITS (50% of training dataset)			With ROITS (50% of training dataset)			GAP (%)
	Mean	Std Dev	Average Similarity (%)	Mean	Std Dev	Average Similarity (%)	
	1	0.86	0.09	14	0.73	0.15	
2	0.75	0.09	25	0.64	0.17	36	11
3	0.84	0.07	16	0.76	0.1	26	10

Note: 1-Triangle, 2 - Square and 3 - Diamond

In Table 2, the average recognition or the similarity of the objects are: 0.86 for triangle, 0.75 square and 0.84 diamond. With standard deviation of the similarity measurement are 0.09, 0.09 and 0.07 for triangle, square and diamond respectively, these results showed that the similarity measurement were likely to be close to each other. In other words, the similarity measure for objects without ROITS were only between 14% and 25%. The recognition showed some improvement between 10% to 13% after implementing ROITS. This again shows that some pre-processing tasks are needed and using Jaccard distance alone is still not effective in recognizing objects.

However separating the strokes provided promising result as the strokes can be adjusted (shrinking and expanding) to match the reference object coordinates without changing the shape of the sketches. The Table 3 shows the result after the intersection points were removed between two strokes and formed the shapes and the separated strokes.

Table 3: Jaccard Distance Measurement Performance for Separated versus connected Strokes

S H A P E	Connected Strokes (50% of training dataset)			Seperated Strokes (50% of training dataset)			GAP (%)
	Mean	Std Dev	Average Similarity (%)	Mean	Std Dev	Average Similarity (%)	
	1	0.73	0.15	27	0.38	0.04	
2	0.64	0.17	36	0.32	0.04	55	19
3	0.76	0.1	26	0.35	0.04	61	35

Note: 1-Triangle, 2 - Square and 3 - Diamond

The result in above table shows that there was an increase in shape similarity measurement consisting of separated strokes of the shape against the object with complete edges (of the shapes). The average increased percentage was 23% for the triangle, 19% the square and 36% diamond. The increased percentage was almost similar for all shapes. Figure 5 shows some of the improvement made in 20 objects of each shapes.

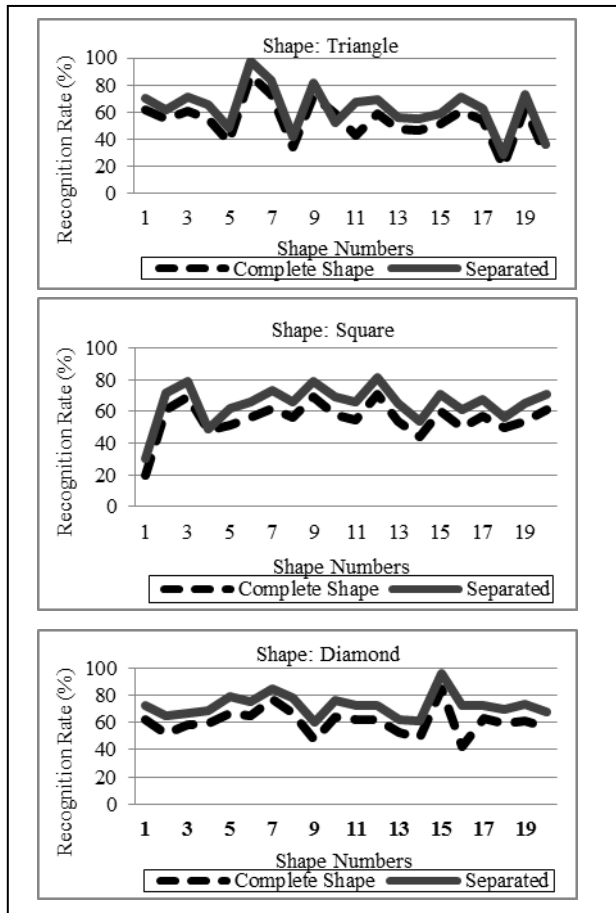


Figure 5. Jaccard Measurement on Separated Versus All Edges

On average, the performance increased by 22% and this shows that by separating all strokes, the similarity measurement using Jaccard would increase. However, the overall recognition is still considered low and that encouraged us to search for alternative by combining it with Mahalanobis Distance.

### 5.2 Results of Using Mahalanobis Distance

In the Mahalanobis similarity measurement, the standard deviation computed using four extreme vertices did not return much difference compared to all vertices taken. Averagely, the differences of standard deviation were 0.006 for triangle, 0.012 for square and 0.023 diamond. This can be concluded that taking four extreme vertices would not disregard the basic shape of a drawn object. This data dimension reduction is acceptable in keeping with the fact that geometric shapes are precise and possesses a precise number of corner points. Figure 6 shows the standard deviation calculated in the Mahalanobis similarity measurement.

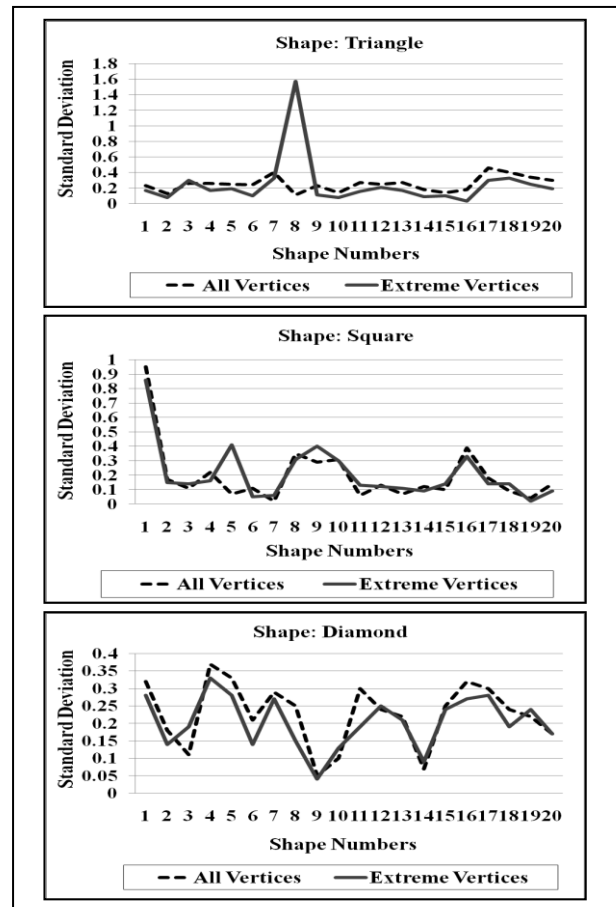


Figure 6. Mahalanobis Measurement for Extreme Vertices Versus All Vertices

### 5.3 Results of Using Combined JM Approach

The recognition averages obtained in the JM approach showed significant changes in recognition after Mahalanobis verified the suspected shape. The changes were from 61% to 84% for diamond, 53% to 77% for triangle and 61% to 77% square. Overall, the diamond shape showed the highest similarity measurement. Table 4 and Figure 7 show the summary of the result of the JM approach.

Table 4: Summary of Similarity Measurement Performance

Shape	Distance Measurement		
	Jaccard Distance	Mahalanobis Distance	Combined Jaccard and Mahalanobis
	Similarity (%)	Similarity (%)	Similarity (%)
Triangle	53	80	77
Square	55	75	73
Diamond	61	80	84

Improvement of each shape was also observed. The following figure (Figure 8) depicts the improvement imposed by each shape. We found that the triangle shape showed the highest improvement of 24%, followed closely by diamond which with 23% improvement and finally the diamond 23%. The square had only 18% improvement.

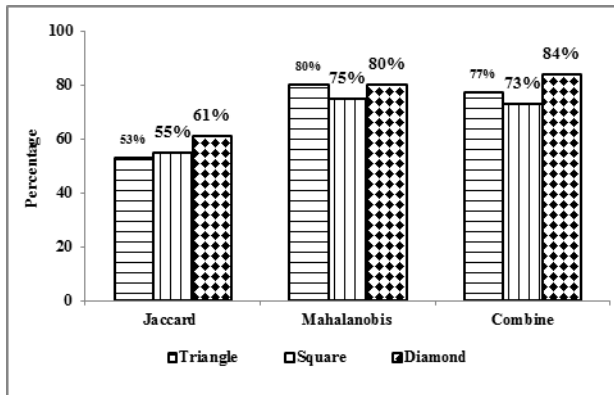


Figure 7: Summary of Results

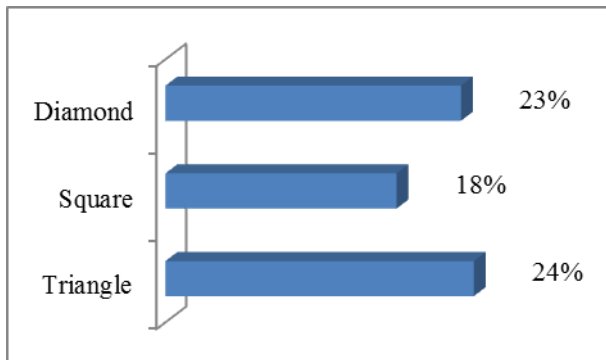


Figure 8. Improvement Made Using Combined Approach

To justify the triangle's highest improvement, the sketches were manually checked. The manual checking showed that only 55% of the sketches were drawn in an acceptable shape and orientation. However, due to the inflexibility of Jaccard, the preliminary measurement of a complete shape with all three intersect edges had gained a low percentage of similarity. This is due to the fact that geometric shapes such as a triangle have perfect and uniform angular measurements and orientation. In addition, when drawing triangles, the respondents had to combine two diagonal lines with 45-degree angle edges and one horizontal line moving from left or right or vice versa. Orientation would change while one was drawing the edge or stroke which made the line distort.

## 5.4 Discussion

Again, the outcome of the experiment shows that low recognition were achieved by the conventional Jaccard Distance when there was no pre processing task done onto the input. However, after translation, rotation, invariance scale content and noise resistance (ROITRS) were applied, little improvement on the recognition progress can be seen with degree of accuracy improve averagely by 11.3%. However, the improvement was also not very significance. To overcome that we separated the shape strokes and some promising improvement made in average by 25%. As mentioned previously, it is almost impossible for users to redraw shapes exactly like their previous sketches.

There have been attempts to combine techniques where [2] recognition modules were designed based on a single stroke method. The like of excluding the intersection points in this study was in most data inspected, they were always not perfectly drawn and intersected. But using separated strokes in Jaccard is considered new as certain edges deployed can be shrunk or stretched to make each shape overlap without changing its original shape. This approach is proposed knowing that it is very difficult for the users to re-point on the same end point of the lines drawn. The users cannot press on the previous point (or dot) as the glass surface of an input device is normally glossy. Furthermore, if the point tips do not perfectly stand between 80 – 90 degree, the neighboring pixel might be activated instead of the intended pixel. In this study, in Mahalanobis, only the extreme points of the object shapes were extracted to reduce its computation time. The most extreme points for common geometrical shapes of four or six or eight, using extreme point masking in Mahalanobis measurement greatly affect the processing time.

The present work may open a window to discoveries such as:

- Sketching by stylus is very different from a pen or a pencil sketching. Therefore geometrical shapes produced in the sketching are rarely connected perfectly at the intersection points. Therefore, the shape should be manipulated and transformed into a proper shape in order to improve the similarity measurement. This may also increase the recognition accuracy.
- Whenever simple similarity measures like Jaccard and Mahalanobis distance are to be used in recognition process, the object should be simplified too.

## 6. Conclusion

This paper studied how to measure the stylus-drawn or stylus-sketched shape efficiently. We found that the combination of Mahalanobis and the Jaccard measures is



comparatively effective to sketch shape similarity measurement. Mahalanobis distance is effective in clustering the extreme points of shapes that produce high probability of similarity for Jaccard to confirm the shapes. We can see that the combination of both measures improved the accuracy. Both similarity calculations are brief and this makes the computations involved low and suitable for mobile application with a low memory and low processing power. Two contributions were made in this study. First, it studied how Jaccard and Mahalanobis distance can be used effectively where strokes separation and masking make the clustering measures useable effectively. Second, an algorithm that combined Mahalanobis with Jaccard distance was designed. The study also showed that Jaccard and Mahalanobis can be used together judiciously, since the manner of combination can greatly affect the performance. For future work, real-time captured sketches will be tested which may tell us its computation time in real practice.

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### References

- [1] Y. Mingqiang, K. Kidiyo, and R. Joseph, 2008. Shape Matching and Object Recognition Using Chord Contexts. *In Proceedings of the 2008 International Conference Visualisation*, 2008. Washington, DC, USA: IEEE Computer Society, pp. 63-69.
- [2] S. Belongie., G. Mori., and J. Malik, 2002. Shape Matching and Object Recognition Using Shape Contexts. *IEEE Transaction on Pattern Analysis and Machine Intelligence*, April 2002. pp. 509-522.
- [3] H. Nemmour and Y. Chibani, 2008. New Jaccard-Distance Based Support Vector Machine Kernel for Handwritten Digit Recognition. *ICTTA 2008. 3rd International Conference*. IEEE Computer Society, pp. 1-4.
- [4] B. J. Zou and M. P. Umugwaneza, 2008. Shape-based Trademark Retrieval using Cosine Distance Method. *Proceedings 2002 IEEE International Conference*, 26-28 Nov. 2008. Washington, DC, USA: IEEE Computer Society, pp. 498- 504.
- [5] G. Chen, H. G. Zhang, and Jun Guo, 2007. Efficient Computation of Mahalanobis Distance in Financial Efficient Hand-Written Chinese Character Recognition. *In Proceedings of the Sixth International Conference on Machine Learning and Cybernetics*, 19-22 August 2007, Hong Kong. IEEE, pp. 2198-2201.
- [6] L. Wang, Y. Zhang, and J. Feng, 2005. On the Euclidean Distance of Images. *IEEE Transactions on Pattern Analysis and Machine Intelligence* Aug 2005. IEEE Computer Society, pp. 1334-1339.
- [7] Z. Li., K. Houl, and H. Li, 2006. Similarity Measurement Based on Trigonometric Function Distance. *Pervasive Computing and Applications, 2006 1st International Symposium*. IEEE Computer Society, pp. 227-231.
- [8] C. C. Chen. And H. T. Chu, 2005. Similarity Measurement between Images. *In Proceedings of the 29th Annual International Computer Software and Applications Conference (COMPSAC'05)*, 2005. Washington, DC, USA: IEEE Computer Society, pp. 41-42.
- [9] N.A. Abdul Aziz, S.S. Salleh, D. Mohamad and M. Omar, 2010. Investigating Jaccard Distance Similarity Measurement Constriction on Handwritten Pen-based Input Digit. *In Proceedings of the International Conference on Science and Social Research. CSSR 2010*. pp. 1-5.
- [10] N.A. Abdul Aziz, S.S. Salleh, D. Mohamad and M. Omar, 2010. A Review on The Use of Similarity Distance Measurement in Shape Recognition. *In Proceedings of the 1st National Postgraduate Seminar 2010. NAPAS 2010*. pp. 1-8.
- [11] B. J. Zou and M. P. Umugwaneza, 2008. Shape-based Trademark Retrieval using Cosine Distance Method. *In Proceedings of the 2002 IEEE International Conference*, 26-28 Nov. 2008. Washington, DC, USA: IEEE Computer Society, pp. 498- 504.
- [12] A. Suebsing and N. Hiransakolwong, 2009. Feature Selection Using Euclidean Distance and Cosine Similarity for Intrusion Detection Model. *In Proceedings of the 2009 1st Asian Conference on Intelligent Information and Database Systems*, 1-3 April 2009, Donghoi. pp. 86-91
- [13] H. Nemmour and Y. Chibani, 2008. New Jaccard-Distance Based Support Vector Machine Kernel for Handwritten Digit Recognition. *3rd International Conference. ICTTA 2008*. IEEE Computer Society, pp. 1-4.
- [14] G. Chen, H. Zhang and J. Guo, 2008. Efficient Computation of Mahalanobis Distance in Financial Hand-Written Chinese Character Recognition. *In Proceedings of the Sixth International Conference On Machine Learning and Cybernetics*, Hong Kong. pp. 317-32.
- [15] D. Wu and J.M. Mendel, 2006. The Linguistic Weighted Average. *In Proceedings of the IEEE International Conference on Fuzzy Systems*, 2006. Vancouver, BC, Canada, pp. 566-573.