Qos- based rate allocation in wireless mesh networks

Airin mardokhpour¹

¹ Young researchers club, Islamic Azad University, buinzahra Branch, buinzahra, Iran

Abstract

In this paper, we present a centralized algorithm for QoS based rate allocation in wireless mesh networks. The main objective is to find approach that also satisfy user-specified QoS constraints, specifically with respect to rate and delay demands. Our approach provides higher priority to real-time flows than elastic flows by reserving the necessary bandwidth for the former and fairly allocating the left-over bandwidth to the latter. We first consider the network with truthful nodes. Then we extend that to cases where nodes are selfish and noncooperative. We propose an efficient and protocol-compliant mechanism to incentivize nodes to be truthful. Although earlier algorithms in this area have demonstrated performance improvements in terms of QoS parameters, the proposed QoS based rate allocation approach provides a framework that guarantees QoS constraints are actually met over the network.

Keywords: Qos, rate allocation, game theory, wireless mesh networks.

1. Introduction

Wireless mesh networking is an emerging technology that uses multi-hop communication to provide cost efficient broadband Internet access for community or enterprise users. A typical wireless mesh network consists of mesh routers and mesh clients [1]. Mesh routers are connected to from a static multi-hop backbone. Mesh routers that are connected to Internet serve as Internet gateways. Mesh clients, such as laptops and PDAs, connect to mesh routers to access the Internet and share network resources among themselves. In wireless mesh networks, inter-router and client-router communications usually use different radio technologies to reduce interference. For example, IEEE 802.16 [2] can be used for inter-router communication while client-router communication uses IEEE 802.11 [3].

One major challenge for the wide deployment of wireless mesh networks is to provide QoS support and fair rate allocation. It is well-known that using TCP and CSMA/CA MAC protocols (e.g., IEEE 802.11 [3]) in multi-hop wireless networks results in severe unfairness [4]. Users that are farther away from the Internet gateway receive less bandwidth and are sometimes starved. In addition, QoS support for real-time applications, such as video and voice over the mesh, is still an open problem. Flows within wireless mesh networks may be classified as real-time and elastic [5]. In such networks, it is important to provide good delay performance to real-time flows, while still maintaining acceptable throughput levels for elastic flows. The random-access nature of uplink in the 802.11 protocol is inherently distributed; that is, users self-classify the priority of their own flows. In a scenario where users are well-behaved, then the network can trust users to correctly classify their flows. But in realistic scenarios where users are strategic, they may have an interest in misrepresenting the priority of their flows even at the expense of overall network performance. For example, a user can improve the throughput of its lowpriority traffic by classifying it as high-priority – an action that adversely impacts the performance of other users. If all users act similarly, then the system no longer supports any OoS differentiation [6]. The main contributions of this paper are two-fold. cooperative game framework and noncooperative game framework. Providing QoS support for real-time flows in wireless networks is an active research area. SWAN [7] is a service differentiation framework for wireless ad hoc networks. The SWAN framework provides service guarantee for real-time flows by controlling the rate of elastic flows. Some type of service differentiation framework focuses on modifying the IEEE 802.11 MAC protocol [8,9,10]. In fact, IEEE 802.11e [11] has been proposed to provide a set of QoS enhancements to the IEEE 802.11 standard. A contention graph and utility maximization based framework has been used for fair rate allocation in several research papers. Some of them [12,13] focus on rate allocation in single hop wireless networks. Others [14,15,16] pay more attention to multi-hop wireless networks. In [17], Higher priority is given to real-time flows by reserving bandwidth for them while utility maximization based rate allocation is used as a tool to allocate the left-over bandwidth among elastic flows fairly. rate allocation is performed at the Internet gateway instead of relying on distributed mesh routers. To support the QoS of real-time flows in mesh networks, a centralized method is more suitable than a distributed one, and employs an adaptive method for accurately estimating the capacity of the network. Our framework in cooperation game theory is based on [17]. Then we consider selfish behavior of users and present non- cooperation game theory frame work based on [18].

The remainder of the paper is organized as follows: Section 2 discusses the capacity of the flow contention graph and introduces the model for the bandwidth allocation problem. Section 3 introduces a cooperative game framework for the system. Section 4 discusses the non-cooperative framework and proves the existence of a unique equilibrium for the game, and gives some simulation results for both the non-cooperative and cooperative game frameworks in their sections. Finally, Section 5 concludes the paper.

2. model and problem formulation

In this paper, we focus on the wireless mesh backbone formed by several mesh routers. We consider the scenario where all the mesh routers are deployed by a single organization, for example, an Internet service provider (ISP). In this case, all the mesh routers are configured and controlled by the organization. Different branches can be configured to use orthogonal channels to eliminate interference between them [19]. we assume that wireless links are bidirectional and the capacities of all links are equal. To illustrate this, consider the example in Fig. 1, and assume node F is the Internet gateway. We denote the set of network flows (end-to-end application flows) as F. Also we denote the set of link flows (flows between directly connected nodes) as L. Every network flow $f \in F$ consists of one or more link flows. Every link flow $l \in L$ carries at least one network flow.



Fig.1 An example of mesh network

2.1 Link contention graph

In an IEEE 802.11 MAC based wireless network, two link flows contend for channel access if the source or destination of one link flow is within the interference range of the source or destination of another one [12,16]. We can define a link contention graph G (V, E) based on the contention relationship between different link flows. The vertex set V contains all the link flows in the network. An edge in set E indicates that two vertices (two link flows) contend with each other. Fig. 2 shows the link-flow contention graph that corresponds to the node graph of Fig. 1. The link contention graph captures the interference among different link flows. An important concept associated with the link contention graph is the maximal clique. A complete sub graph of a graph is called a clique. A maximal clique is defined as the clique that is not contained in any other clique [20]. We denote the set of maximal cliques of a contention graph as C. The maximal

455

using the Bierston algorithm [20]. In fact, a maximal clique represents a contention region in which all link flows interfere with each other. A maximal clique can also be considered as a limited resource shared and contended by different link flows within it. For any maximal clique, at any time, only a single link flow can be active. The above constraint is denoted as the *clique constraint*.



Fig. 2 Link contention graph and its decomposition in maximal cliques

2.2 Problem Formulation

We can formulate the bandwidth allocation problem based on the flow contention graph. For flow contention graph *G*, assume the number of flows in the graph is *N*. The set of flows is denoted as $N = \{1, ..., N\}$. The rate for flow *i* is defined as *xi*, *i* = 1,..., *N*. The set of maximal cliques in *G* is denoted as $M = \{1, ..., M\}$. The capacity of clique *j* is defined as *cj*, *j* \in *M*. One flow may belong to several maximal cliques. These relations of belonging can be described by matrix *A* as follows:

$$a_{j,i} = \begin{cases} 1, \text{ if flow } i \text{ belongs to clique } j, i \in N \\ 0, \text{ if flow } i \text{ does not belong to clique } j, j \in M \end{cases}$$

The capacity constraints of the flows can therefore be defined as:

$$Ax \le C$$
 (1)

where $x = (x_1, ..., x_N)$ is the flow rate column vector and $C = (c_1, ..., c_M)$ is the clique capacity vector. In addition, flow rates must take non-negative values:

$$xi \ge 0, i=1, ..., N$$
 (2)

The set of flow rate vectors that satisfy conditions (1) and (2) is called a feasible set. Scheduling feasibility does not guarantee that the rate vector is throughput feasible, which means the throughput of a flow equals to the allocated rate. This is due to the inefficiency of the underlying scheduling protocols. For example, due to the random nature of the IEEE 802.11 MAC, some idle time is wasted during the back off period. We introduce the effective clique capacity τ_n for every clique n \in C such that



the constraint. Then the throughput feasibility constraint can be written as:

$$4x \le \tau$$
 (3)

The effective capacity of a clique depends on several factors, such as the underlying scheduling protocol, the number of competing link flows in this clique and the location of link flows [13]. It is difficult to determine the effective capacity of a clique in advance. In our framework, the effective capacity of a clique has the maximum value of (2/3)b and is adaptively estimated according to the network conditions [12,17].

3.cooperative game framework

3.1 Admission control

Assume that a network planner has a global view of the contention graph and the traffic pattern, and the network flow set F is divided into real-time flow set F^r and elastic flow set F^e . We give a higher priority to real-time flows over elastic flows by performing admission control and bandwidth reservation first and allocate the left-over bandwidth to elastic flows afterwards. We sequentially process all realtime flows. For a real-time flow $f \in F^r$, it is admitted if

$$P_{nf}r_f < \dot{\tau_n}, \quad \forall n \in C$$
 (4)

which means its requested rate can be supported by all cliques, the rate allocation x_f is equal to r_f . Otherwise, $r_f = x_f \beta$ is suggested to real-time flow sender. It can use compression methods to reduce its requested rate and send with that rate. So, number of accepted flows can be increase. P_{nf} denotes the number of link flows in clique n that carries network flow f; r_f is the rate requested by real-time flow f. So $P_{nf}r_f$ represents the bandwidth used by flow f in clique n. τ_n is the available bandwidth of clique n, which is set to the effective capacity of the clique at the beginning of the admission control process. In addition, the available bandwidth of every clique n is updated as $\tau_n = \tau_n _ P_{nf} x_f$.

3.2 Utility maximization based rate allocation

After the admission control is completed for real-time flows, we may allocate bandwidth to elastic flows. In order to guarantee fairness among different elastic flows, we use a well-developed utility maximization framework. we choose the utility function $U_f(x_f) = \ln(x_f)$ for every elastic flow, to achieve proportional fairness [21,22]. More formally, the objective can be written as

P: max $\sum_{f \in F} U_f(X_f)$ subject to : $Px < \tau$

 $x_f \ge 0$

The above problem is referred to as the system primal problem. It is a typical convex optimization problem [23], which can be solved by using the Lagrange duality [21,24]. The dual problem is defined as

$$D:\min D(\lambda) \quad \lambda \ge 0 \tag{6}$$

In fact, the Lagrange multiplier λn can be interpreted as the shadow price [21] of clique n, which is the cost of a unit flow accessing the channel in clique n [16]. To solve the dual problem, we use an iterative algorithm with the help of the gradient projection method [16]. The algorithm involves all network flows and maximal cliques and is given an initial rate allocation vector x. For real-time flow f, x_f is determined by the admission control process described in the previous subsection and does not change during the iterative process. For elastic flows, initial rate can be randomly chosen. At every iteration, for every clique n, it receives the rate information of all flows f where $P_{nf} \neq 0$ and updates the shadow price λ_n using

$$\lambda_{n} (t+1) = [\lambda_{n} (t) - \gamma (\tau_{n} - \sum_{f \in F} {}^{e} X_{f} P_{nf} - \sum_{f \in F} {}^{r} X_{f} P_{nf})]$$
(7)

Since function $U_f(x_f)$ is strictly concave, the unique solution of the above problem exists and can be expressed as

$$X_{f} = U_{f}^{-1}(\mu_{f})$$
 (8)

$$\mu_{f} = \sum_{n \in c} \lambda_{n} P_{nf}$$
(9)

It has been shown that [14,16], by choosing the appropriate step size c, starting from any initial rate vector x, the above iterative algorithm will converge to the optimal solution (x^*,k^*) , and the solution is primaldual optimal, which means x^* is also the optimal rate vector for the primal problem.

3.3. Adaptive effective clique capacity

The effective clique capacity adjustment is performed before admission control and rate allocation. If the maximal clique set and the traffic pattern are the same as the last round, the initial effective capacity of every maximal clique is set to the value determined in the last round. Then for every real-time flow f, the gateway node checks that if the average measured delay exceeds the delay requirement. If yes, for every clique n, the effective clique capacity τ_n will be reduced by P_{nf} a, where a is a



small constant. This is equivalent to reserving extra bandwidth P_{nf} a for the real-time flow f in clique n. If the maximal clique set or the traffic pattern changes in a new round, the effective capacity of every maximal clique will be set to 0.6b and no further capacity adjustment will be performed in this round [17].

3.4. Simulation result

We use matlab for our simulation. In our framework, the duration of each round is set to be 4 s. In the distributed clique construction phase, 2 s are used for beacon message and link message exchange. Each mesh router periodically sends beacon messages and link messages. The inter-message intervals of beacon and link messages are 0.2 and 0.9 s, respectively. Another one second is used for sending clique messages to the Internet gateway. After admission control and rate allocation, the gateway node sends rate messages to the source of each network flow. The simulated single-branch scenario is shown in Fig. 3. All nodes use two 2.5 Mbps 802.11 radios tuned to orthogonal channels used by data transmission and control message exchange. The transmission range and interference range are set to be 250 m. In this scenario, we have six network flows. The routes for these flows are manually set and are shown in the figure. Flows f_1 , f_2 , f_4 and f₅ are elastic flows with constant bit rate (CBR) traffic source. The packet size is set to be 1000 bytes. Elastic flows last for the whole simulation period, which is 400 s. Real-time flows f_3 and f_6 are from voice applications simulated as CBR traffic sources with 32 kbps rate requirement and 50 ms maximal tolerable oneway delay. The packet size is set to be 80 bytes. Realtime flow f_7 and f_8 is similar to f_3 and f_6 but its rate requirement is 72 kbps. Real-time flows start at 100 s and end at 300 s. we consider this scenario by two value of β .



fig.3 A single-branch scenario. For $\beta = \frac{1}{2}$, Fig. 4 shows the rate allocation at different rounds under the our framework. The rate allocations of flow f₄ and flow f₆ are not shown in the figure since they

are the same as flow f_2 and flow f3, respectively. During [0, 100] seconds, there are no real-time flows in the network and the available bandwidth is allocated to elastic flows. At 100 s, three real-time flows enter the network and the rate allocation is quickly adjusted to meet the QoS requirements of them. When real-time flows leave the network at 300 s, the available bandwidth for elastic flows increases and the new rate allocation is the same as that in the period [0, 100]. Fig. 5 shows the average delay of real-time flows in logarithmic scale at different rounds under our framework. When real-time flows enters the network, the delay requirement is not satisfied. After a few rounds of clique capacity adjustment, the delays of real-time flows are under the maximal tolerable value and are stably maintained during the whole session. The average delay of flow f₆ and flow

accepted are more than QUOTA framework and their delay requirement is satisfied [17]. 0.35 allocated rate of flow 1 allocated rate of flow 2 0.3 allocated rate of flow 3 allocated rate of flow 5 allocated rate of flow 7 0.25 allocated rate(Mbps) 0.2 0.15 0.1 0.05

 f_7 are not shown in the figure since they are the same as

flow f_3 . In our framework, real-time flows that can be

Fig.4. Allocated rate of different flows at different rounds.

50 60

round number

30 40

0

10 20



90

Fig. 5. Average delay of three real-time flows at different rounds.

For $\beta = \frac{1}{4}$, Figures 6 and 7 shows the rate allocation and average delay at different rounds under the our framework. In this scenario, When real-time flow (f₈) enters the network, is accepted but delay requirement for this flow is not satisfied. The rate allocations of flow f₈ is the same as flow f₇.

more realtime flows will be accepted for higher β values and delay requirement for this flow will be satisfied. as the β decreases, the number of realtime flows will incrises but but delay requirement for this flow is not satisfied.



Fig.6. Allocated rate of different flows at different rounds.



Fig. 7. Average delay of three real-time flows at different rounds

4. Non-cooperative game framework

Quality of Service (QoS) provisioning has become an important aspect of MAC layer design in networks where elastic traffic (e.g. data) and real-time traffic (e.g. voice and video) coexist. In such networks, it is important to provide good delay performance to real-time traffic, while still maintaining acceptable throughput levels for elastic traffic. In 802.11e networks, QoS differentiation is accomplished by classifying traffic into priority classes. Users maintain separate queues for each priority class, and packets in each queue contend for the channel with a probability that is dependent on the priority class. In other words, high priority traffic contends for the channel more aggressively than low priority traffic. So we design incentive compatible schemes for MAC-layer QoS.

4.1 System model

Throughout this paper, we use the following model. There are N mesh routers, each with high-priority (HP) and low-priority (LP) traffic. To simplify our analysis, we assume that both sets of queues are saturated. Furthermore, the system guarantees a minimum QoS to mesh routers in the form of minimum throughput guarantees: T^H for HP traffic and T^L for LP traffic. Time is divided into slots. Each slot operates in either the contention phase (CP) or the contention-free phase (CFP). We denote α the probability that a slot operates in CFP, where α is a fixed-value chosen by the gateway. In each slot of the contention-free phase, the gateway polls a single node, where v_i is the probability that node i is polled. The polled node may choose to send either HP or LP traffic. During the contention phase, HP queues attempt to transmit with probability p, while LP queues attempt to transmit with probability q < p. γ_i is abet or punishment multiplier for node i in contention phase. We assume that users exercise internal collision resolution; that is, if a user attempts to transmit from both its HP and LP queues in the same slot, only the HP queue will actually contend for the channel. A transmission attempt is successful if and only if there is a single transmission attempt. With this in mind, we can write the throughput of users as follows:

$$\begin{aligned} T_i^{\,H} &= (1 - \alpha) \, p \, [\, 1 - (p + q(1 - p))]^{N-1} \\ T_i^{\,L} &= (1 - \alpha) \, \gamma_i \, q \, (1 - p) \, [\, 1 - (p + q(1 - p))]^{N-1} \\ T_i^{\,poll} &= \alpha v_i \end{aligned}$$

We formulate a Stackelberg multi-stage game with two types of players: the gateway and the users or mesh



routers. The gateway is a benevolent player whose only goal is ensure that users transmit LP traffic with probability q. The users are selfish players interested in maximizing their own utility. The game is structured such that the values of α and N are fixed and known, and users know apriority that their polling probabilities will be assigned according to some rule based on the actions of the players. In this case, v_i is chosen as 1/number of truthful players, if player i is truthful, and 0 otherwise. In contention phase, if player is truthful, γ_i in next CP is increased, otherwise γ_i is deceased even 0. Since this is formulated as a one-shot game, users are modeled as announcing their choice of transmits probability to the gateway. In reality, the gateway would observe the frequency of transmission attempts over some period of time and assign polling probabilities based on its observations.

4.2 Game Formulation

Formally, we denote by $F = [N, \{\sum_i\}, \{U_i(\cdot)\}]$ the noncooperative game. Let $N = \{0, 1, ..., N\}$ denote the set of players, where player i = 0 is the gateway and players i = 1, ..., N are the users. The strategy space for the gateway is

$$\sum_{0} = \{ v, \gamma: 0 \le v_{i} \le 1, 0 \le \gamma_{i} \le 1, \sum_{i=1}^{N} v_{i} \le 1 \}$$

, and for the users is $\sum_{i} = \{p,q\}$. Notice that the strategy space for users is purposely restrictive; that is, the misbehavior of users is restricted to transmitting LP traffic with probability p. Users are not allowed to modify protocol parameters by using arbitrary transmission probabilities1, nor are users strategic with respect to their HP traffic. We denote by $\sigma_i \in \sum_i$ the action of user i, and by $\sigma_{-i}=(\sigma_1,...,\sigma_{i-1},\sigma_{i+1},\sigma_N)$ the actions of the other users. Since users are not strategic with respect to their HP traffic, we need only model the utility of LP traffic. Since LP traffic is delay tolerant, it is reasonable to model LP utility as a function of throughput. We have

$$U_{i}(\sigma, \sigma, \gamma, v_{i}) = T_{i}^{L} + T_{i}^{poll}$$

= $(1 - \alpha) \gamma \sigma_{i} \sigma_{i} (1 - p) \Pi_{j \neq i} [1 - (p + \sigma_{i} (1 - p))] + \alpha v_{i}$
(11)

Since the gateway is a benevolent player whose only goal is to ensure QoS differentiation, the utility of the gateway is defined as $U_0=(\sigma,v)=0$.

Having constructed our game, we see that, an appropriate choice of v and γ incentivizes users to transmit their LP traffic with the correct probability, and transmitting LP traffic with probability q is a dominant strategy equilibrium of Game F for certain values of α and N [18].

4.3 Simulation result

In this section, we examine the behavior of our proposed scheme through numerical examples. We have 30 mesh routers or users that attempt for transmit their flows in 1000 time slots. Interval rate and service rate for real-time flows in high-priority queue is according to a Bernoulli random variable with parameter p, and for elastic flows in low priority queue is according to a Bernoulli random variable with parameter q. The probability of operating in the contention free phase in any given slot is modeled as a Bernoulli random variable with parameter α . Once in contention-free phase, the gateway polls a single user. Users are equally likely to be polled. If the polled users' high-priority queue is non-empty, it sends a high-priority flow. Otherwise, it sends a low priority flow. We use values p = .05, q = .01, TH = .01, and TL = TH q(1-p)/p. When implementing the contention free phase we use $\alpha =$.0821 and $\gamma = 0.1$. Fig. 8 shows the delay performance of high-priority traffic in two scenarios: strategic users and our incentive compatible mechanism. We see that the delay performance of our proposed scheme is strictly better than the delay performance achieved when users are strategic. In fact, as the number of users grows, the system can no longer support minimum throughput guarantees for HP traffic when users are strategic. This is what causes the delay to grow exponentially in the case when users are strategic. We see that by incentivizing users, the delays remain stable for a larger number of users. Fig. 9 shows the throughput performance of low priority traffic. for large numbers of users our scheme actually gives higher LP throughput.





Fig. 9. Average Throughput of LP Traffic

5. CONCLUSION

In this paper, we proposed a framework for QoS support and fair rate allocation in wireless mesh networks. Our framework uses link contention graph and utility maximization framework to perform admission control and rate allocation. Simulation results show that our framework successfully guarantees the QoS of real-time flows and fairly, efficiently allocates bandwidth for elastic flows in different wireless mesh network scenarios. Then we have presented a scheme that incentivizes users to correctly classify the priority of their traffic. By using contention-free operation and designing appropriate rules for polling probabilities, we have shown it is possible to construct a scheme in which it is a dominant strategy for users to transmit their low-priority traffic with the correct probability. Furthermore, we have shown that this scheme improves the throughput performance of LP users, while maintaining desirable delay performance for high priority users. One promising direction is to combine this framework with a routing protocol.

References

[1] I.F. Akyildiz, X. Wang, W.Wang, "Wireless mesh networks: a survey," computer network journal, March 2005.

- [2] " IEEE 802.16 Standard."
- [3] "IEEE 802.11 Standard."

[4] V. Gambiroza, B. Sadeghi, E. Knightly, "End-to-end performance and fairness in multihop wireless backhaul networks," in: Proceedings of Mobicom, 2004.

[5] S. Shenker, "Fundamental design issues for the future internet," IEEE JSAC 13 (7) (1995) 176–1188.

[6] D. Fudenberg and J. Tirole, "Game Theory." The MIT Press, 1991.

[7] G.S. Ahn, A.T. Campbell, A. Veres, L.H. Sun, "SWAN: service differentiation in stateless wireless ad hoc networks," in: Proceedings of the IEEE INFOCOM'2002, New York, June 2002.

[8] Y. Yang, R. Kravets, "Distributed QoS guarantees for realtime traffic in ad hoc networks," in: IEEE International Conference on Sensor and Ad Hoc Communications and Networks (SECON), 2004.

[9] Y. Yang, R. Kravets, "Achieving delay guarantees in ad hoc networks through dynamic contention window adaptation," in: IEEE INFOCOM,2006.

[10] Y. Yang, R. Kravets, "Throughput guarantees for multipriority traffic in ad hoc networks," Elsevier Ad Hoc Networks Journal (2006).

[11] IEEE 802.11 Standard. Available from: http://standards.ieee.org/getieee802/802.11.html>.

[12] Z.Y. Fang, B. Bensaou, "Fair bandwidth sharing algorithms based on game theory frameworks for wireless ad-hoc networks," in: Proceedings of the IEEE INFOCOM'04, 2004.

[13] V. Gambiroza, E. Knightly, "Congestion control in CSMAbased networks with inconsistent channel state," in: Proceedings of ACM International Wireless Internet Conference (WICON), Boston, MA, August 2006.

[14] L. Chen, S.H. Low, J.C. Doyle, "Joint congestion control and media access control design for wireless ad hoc networks," in: IEEE INFOCOM'05, 2005.

[15] B. Li, "End-to-end fair bandwidth allocation in multi-hop wireless ad hoc networks, in: ICDCS 2005, 2005, pp. 471–480.

[16] Y. Xue, B. Li, K. Nahrstedt, "Optimal resource allocation in wireless ad hoc networks: a price-based approach," IEEE Transactions on Mobile Computing (2005).

[17] Bo Wang, Matt Mutka, "*QoS-aware fair rate allocation in wireless mesh networks*", Computer Communications 31, 2008, pp. 1276–1289.

[18] J. Price, P. Nuggehalli, and T. Javidi, *"Incentive Compatible MAC-Layer QoS Design"*, Consumer Communications and Networking Conference, CCNC 2008. 5th IEEE, 2008, pp. 586 - 590.

[19] V. Gambiroza, B. Sadeghi, E. Knightly, "End-to-end performance and fairness in multihop wireless backhaul networks," in: Proceedings of Mobicom, 2004.

[20] J.G. Augustson, J. Minker, "An analysis of some graph theoretical cluster techniques," Journal of the Association of Computing Machinery 17 (4) (1970) 571–586.

[21] F.P. Kelly, A.K. Maulloo, D.K.H. Tan, "Rate control in communication networks: shadow prices, proportional fairness and stability," Journal of the Operational Research Society 49 (1998) 237–252.

[22] B. Radunovic, J.Y.L. Boudec, "Rate performance objectives of multihop wireless networks," in: Proceedings of the INFOCOM, 2004.

[23] S. Boyd, L. Vandenberghe, "Convex Optimization," Cambridge University Press, 2004.

[24] S.H. Low, D.E. Lapsley, "Optimization flow control – I: basic algorithm and convergence," IEEE/ACM Transactions on Networking 7 (1999) 861–874.