Comparison of the Fuzzy-based Wavelet Shrinkage Image Denoising Techniques

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Abstract

In this paper, a comparative study on the different membership functions which are used for fuzzy-based noise reduction methods is done. This study focuses on the three different membership functions such as Gaussian, Sigmaf and Trapezoidal. The fuzzy wavelet shrinkage method is tested with different membership functions in order to reduce different types of noise such as Gaussian, Salt & Pepper, Poisson and Speckle. The measure of comparison between different membership function is based on PSNR (Peak Signal to Noise Ratio). Experimental results show that on the some well-known images, such as "Lena", "Barbara" and "Baboon", the Gaussian membership function can efficiently remove the additive Gaussian and the Poisson noises from the grey level images. Furthermore, on the Speckle and Salt & Pepper noises, the Sigmaf membership function outperforms the Trapezoidal one to remove noise.

Keywords:*Fuzzy set, Membership function, Noise detection, Noise reduction, Wavelet shrinkage.*

1. Introduction

The goal of image denoising is to reduce noise while preserving important details of image [1]. In order to do, a lot of approaches are proposed. To achieve a good performance, a noise reduction algorithm should adapt itself to the spatial context [1]. For this purpose, the wavelet transform [2] is the impressive approach. The transform construct a matrix coefficients which represent the important details of image. Noise reduction using wavelet transform includes three main steps: i) computes the wavelet coefficients ii) apply the noise reduction algorithm on the coefficients iii) use inverse wavelet transformation The additive noise of the image domain is transformed directly to the transform domain, because the wavelet transform has linear property [1]. One of the most applied methods for image denoising is shrinking which effects on all wavelet coefficients and reduces their values. The best performance of the image denoising is achieved if the shrinkage method has the low and high influences on the free noise component and the noisy pixel, respectively. Furthermore, after shrinking process, the noise component of image should be reduced more. One of the most common methods used in the shrinkage domain is thresholding [3] which if the value of coefficient is lower than the threshold, the coefficient is noise, and otherwise it is a signal component. Neighboring is another solution to wavelet shrinkage that will be described in subsection 3.1. The value of threshold is so critical. Therefore, the fuzzy set theorem [4] has been used to select best value for threshold that the most of the coefficients under the threshold are noise and most of the coefficients above the threshold are signal [1].

This paper is organized as follows: in section 2, related works in this domain are reviewed. The fuzzy shrinkage method with different membership function is described in section 3. In section 4, the dataset, the experimental results and discussion are stated. Section 5 deals with conclusion.

2. Related work

There is some research covering the problem of image denoising. This paper is based on [1] which proposed the fuzzy wavelet shrinkage for image denoising. In [1], Schulte et al. studied on the usage of fuzzy set theory in the domain of image enhancement using wavelet thresholding. Schulte et al. used just the Trapezoidal MF and the Gaussian noise distribution to evaluate the performance of fuzzy image denoising. Tavassoli et al. [6] studied on a new method for impulse noise reduction. In their method, the main algorithm includes 2 steps that in the first step, an Adaptive Neuro-Fuzzy Inference System (ANFIS) model has been used to

detect noisy pixels. In the second step, the fuzzy shrinkage wavelet is used to make some changes on the detected pixels. Tolt and Kalaykov [7] proposed the fuzzy similarity and homogeneity approach for noise cancellation. The proposed method has two advantages: i) simple tuning of fuzzy filter parameters ii) it is very convenient for high-speed real-time image processing. Sendur and Selesnick [10] worked on the dependencies between the wavelet coefficients and their parents for wavelet-based denoising. They proposed a new non-Gaussian bivariate distributions, and also corresponding nonlinear shrinkage functions which are derived from the models using Bayesian estimation theory.
Schulte et al. [11] introduced an innovation based on to reduce the additive noise from digital color images. For this propose, two fuzzy subfilter is proposed. In the first subfilter, s fuzzy distances between the color components of the central pixel and its neighborhood. These distances determine in what degree each component should be corrected. All performed corrections preserve the color component distances. The goal of the second subfilter is to correct the pixels where the color components differences are corrupted so much that they appear as outliers in comparison to their environment.

3. Fuzzy Shrinkage Method

Inspired by human's remarkable capability to perform a wide variety of physical and mental tasks without any measurement and computations, and dissatisfied with classical logic as a tool for modeling human reasoning in an imprecise environment, L. A. Zadeh developed the theory and foundation of fuzzy logic with his 1965 paper, "Fuzzy Sets" [4]. Design of membership function and fuzzy rule base will be described in next subsections.

3.1 Membership Function (MF)

As the mentioned in [3, 8, 9], neighboring is a solution to wavelet shrinkage. In this method, a window is considered: if the main (central) coefficient of window ($w_{s,d}(i, j)$) and the average value of neighbor's coefficients ($x_{s,d}(i, j)$) are both enough large then the central coefficient is a signal component. If the average value of neighbor's coefficients is high then the central coefficient is a signal component. If the central coefficient and the average value of neighbor's coefficients are both enough small then the central coefficient is noisy. Equation (1) shows the formula to compute the $x_{s,d}(i, j)$ [1]. Note that the parameters *s* and *d* refers to scale and orientation of coefficient, respectively.

$$x_{s,d} = \frac{\left(\sum_{k=-K}^{K}\sum_{l=-K}^{K}|w_{s,d}(i+k,i+l)|\right)-|w_{s,d}(i,j)|}{(2k+1)^2-1} \qquad (1)$$

The MF is used to determine that the variable is small, large or degree of being large. For the both wavelet coefficient $|w_{s,d}(i, j)|$ and average value of neighborhood $|x_{s,d}(i, j)|$, two MFs should be defined which are noted as μw and μx, respectively. In [1], the Triangular MF had been used for the two mentioned variables (fuzzy sets), but in our study, three different MFs, such as Triangular, Gaussian and Sigmaf, are used for the both above variables to analyze which MF can help the noise reduction algorithm to reach the best denoised image. Note that in the testing phase, the type of the MFs for both variables is the same. For instance, in the testing phase, the type of the both μ_w and μ_x is Gaussian.
To construct the all above MF, three thresholds are needed: two of them for $\mu_w$and one of them for μx. In this study, the values of the thresholds are chosen from [1]. Therefore, the values of the three thresholds are: T1 = σ, T2 = 2σ and T3 = 2.9σ−2.625. Note that in the testing time, the σ takes different values: 5, 20, 30 and 40 [1]. The Gaussian MF requires two parameters, standard deviation (σ) and median (c), to compute the degree of MF for each input. In this type of MF, the μ_w and μ_x are defined as (2) and (3).

$$\mu_w = \begin{cases} 0 & w < T_1 \\ \frac{-(w-T_1)^2}{2{T_2}^2} & T_1 \le w \le T_2 \\ 1 & w > T_2 \end{cases} \qquad (2)$$

$$\mu_x = \begin{cases} 0 & x = t \\ e^{\frac{-(T_3)^2}{2t^2}} & t \le x \le T_3 \\ 1 & x > T_3 \end{cases} \qquad (3)$$

Note that in (2), the parameter *t* for Gaussian should be set as a negligible value (near zero). In the testing part, the value of *t* is set to 0.01. In general, the Trapezoidal MF has four threshold while, in this work, just two of them are important. Because, the coefficients below the T1 should be 0, the coefficients between 0 and 1 are probability and the value above the 1 should be 1. So, it is required just two of them. For the Trapezoidal MF, the μ_w and μ_x are defined as (4) and (5).

$$\mu_w = \begin{cases} 0 & w < T_1 \\ \frac{w-T_1}{T_2-T_1} & T_1 \le w \le T_2 \\ 1 & w > T_2 \end{cases} \qquad (4)$$

$$\mu_x = \begin{cases} 0 & x = t \\ \frac{x}{T_3} & t \le x \le T_3 \\ 1 & x > T_3 \end{cases} \qquad (5)$$

The Sigmaf MF is a Sigmoidally-shaped function. Equations (6) and (7) demonstrate μ_x and μ_w in the type of Sigmaf.

$$\mu_w = \begin{cases} 0 & w < T_1 \\ \frac{1}{1+e^{-T_1(w-T_2)}} & T_1 \le w \le T_2 \\ 1 & w > T_2 \end{cases} \qquad (6)$$

$$\mu_x = \begin{Bmatrix} 0 & x < t \\ \frac{1}{1+e^{-t(x-T_3)}} & t \le x \le T_3 \\ 1 & x > T_3 \end{Bmatrix} (7)$$

3.2 Fuzzy Rule Base

Rule base is the main part of fuzzy system and the quality of results in the system depends on the fuzzy rules [4]. A reasoning procedure known as the compositional rule of inference enables conclusions to be drawn by generalization from the qualitative information stored in the knowledge base [4]. The fuzzy rules can express him with the natural language in the following way: if x is small and y is middle, then z is great. The variables *x*, *y* and *z* are type linguistic [4]. The fuzzy rules in the domain of wavelet shrinkage can be defined as follow,

IF($|x_{s,d}(i, j)|$ *is a large variable***OR**
($|w_{s,d}(i, j)|$ *is a large coefficient***AND**$|x_{s,d}(i, j)|$ *is a largevariable***)**)
THEN$w_{s,d}(i, j)$ *is a signal of interest*

In the fuzzy model, the AND (intersection) and OR (union) operations are roughly equivalent to the norm and co-norm, respectively [1, 4]. The intersection and union of the two fuzzy sets can be presented by the norm and co-norm. In the above fuzzy rule, math form of the antecedent part $|w_{s,d}(i, j)|$ is a large coefficient AND $|x_{s,d}(i, j)|$is a large variable" is $\mu_x(|x_{s,d}(i, j)|) \cdot \mu_w(|w_{s,d}(i, j)|)$ which is based on (10). Note that the $\mu_w(|w_{s,d}(i, j)|$ shows the MF degree of main coefficient to the fuzzy set "LARGE COEEFICIENT". Furthermore, to prove the math form of the OR operation in the antecedent part of the rule is shown as follow,

$$(\mu x(|xs,d(i, j)|) \cdot \mu w(|ws,d(i, j)|)) + (\mu x(|xs,d(i, j)|)) =$$
$$\mu_x(|x_{s,d}(i,j)|) \cdot \mu_w(|w_{s,d}(i,j)|)) + (\mu_x(|x_{s,d}(i,j)|) -$$
$$\mu_x(|x_{s,d}(i,j)|) . \mu_w(|w_{s,d}(i,j)|) . (\mu_x(|x_{s,d}(i,j)|) \qquad (8)$$

which is based on (9).
Co-norm (probabilistic sum): (x+y-x.y) (9)
Norm (algebraicproduct) : (x.y) (10)

3.3 Fuzzy-based Wavelet Shrinkage

The output of the fuzzy process is a matrix with values in the range of [0--1]: value 0 shows that the corresponding coefficient is not signal of interest, while the value 1 means that the corresponding coefficient is a signal of interest. The value between 0 and 1 indicates the degree of certainty which the coefficient is a signal of interest.

4. Excremental Results

For this work, the three well known images have been tested such as "Lena", "Barbara" and "Baboon". The size of all gray scale images is 512×512. The size of window is set to 3. All images are tested for different values of σ, i.e. 5, 20, 30 and 40. The framework for implementation and testing the algorithm is Matlab7.

1 Additive Noise
In this experiment, all the MFs are tested on the four different additive noises such as Gaussian, Salt &Pepper, Speckle and Poisson that are described as follows:
Gaussian noise: Gaussian noise is a Gaussian white noise which requires two parameters: mean and variance. In the testing phase, the values of mean and variance are set to 0 and 0.1, respectively.
Salt & Pepper: Salt & Pepper noise is a commonly used additive noise. The noise has just one variable, noise distribution. The value of the noise distribution is set to 0.1.
Speckle: Speckle adds noise according to (11),
$J=I+n\times I$ (11)
which I is a noise free image and J is a noisy image. The parameter n is uniformly distributed random noise with mean 0 and variance 0.15.
Poisson: Poisson noise is based on Poisson distribution that is generated from data.

4.2 PSNR Measure

To evaluate the performance of the noted MFs, we use the Peak Signal to Noise Ratio (PSNR) measure. The PSNR is used to evaluate the difference between the original noise free image and the denoised image. Notation "Signal" in the PSNR measure refers to noise free image. The PSNR measure is calculated using (12). Note that the high value of the PSNR shows the high quality of the reconstructed image.

$$MSE = \frac{1}{m\times n}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}[\mathrm{I}(i,j) - K(i,j)]^2 \qquad (12)$$

$$PSNR = 10 \times log_{10}\left(\frac{MAX_I^2}{MSE}\right)(13)$$

Equation (13) calculates the differences between the original noiseless image and the filtered image. In (13), notations *I* and *K* refer to the input noise free image and the filtered image, respectively, and size of the image (original/filtered) is $m\times n$. In (12), MAX_I shows the maximum pixel value in the image. For instance, in the gray level image, MAX_I is equal to 255.

4.3 Discussion

Table 1 shows the PSNR values for the mentioned MFs which were tested on the different additive noises and different values of σ too. As the mentioned, the Gaussian and the Sigmaf MFs which are proposed in this study are compared with the Trapezoidal MF which is proposed in [1]. margins and set them in 9-point type (Fig. 1 shows an example). The distance between text and figure should be

Table 1: PSNR values of the different MFs that are tested on the different additive noise source and different values of σ. Note that the Notations "G", "S" and "T" refer to (G)aussian, (S)igmaf and (T)rapezoidal, respectively.

Noise	MF	Lena				Baboon				Barbara			
		Sigma(σ)											
		5	*20*	*30*	*40*	*5*	*20*	*30*	*40*	*5*	*20*	*30*	*40*
Gaussian	G	17.08	18.19	**19.69**	**19.72**	**17.05**	17.80	**19.27**	**19.19**	**17.06**	17.89	**19.38**	**19.35**
	S	17.10	**19.76**	19.61	19.48	17.01	**19.21**	18.69	18.30	17.02	**19.30**	18.75	18.32
	T	**17.09**	19.59	19.23	19.68	17.04	19.14	19.23	18.91	17.05	18.21	19.32	19.03
Poisson	G	27.23	**34.09**	**32.70**	**31.45**	27.07	**31.12**	**28.78**	**26.90**	27.74	**32.03**	**29.90**	**28.20**
	S	29.62	31.46	29.66	28.45	**28.51**	26.95	24.42	23.09	**29.64**	28.21	24.61	23.18
	T	**27.67**	33.26	31.52	30.24	27.28	29.70	27.00	25.17	28.16	30.86	28.14	25.89
Salt & Pepper	G	**15.46**	15.88	17.93	22.02	15.60	15.93	17.83	21.31	**15.38**	15.70	17.40	20.69
	S	15.41	**21.87**	**27.14**	**26.73**	15.59	**21.50**	**23.70**	22.66	**15.38**	**20.83**	**23.67**	22.56
	T	15.43	17.45	22.51	26.32	**15.62**	17.45	21.73	**23.86**	**15.38**	16.97	21.04	**23.76**
Speckle	G	14.36	14.57	15.47	17.82	14.09	14.22	15.07	17.32	15.02	15.24	16.30	18.55
	S	**14.39**	**16.10**	**24.26**	**24.76**	14.10	**15.81**	**22.37**	**22.98**	**15.02**	**17.04**	**22.89**	**22.97**
	T	19.36	14.86	17.08	21.71	**14.11**	14.44	16.75	20.88	15.01	15.60	17.92	21.65

Fig. 1 Experimental results on the different types of additive noise. Each row refers to a type of the additive noise. First column (left most side), second column, third column, fourth column and fifth column shows the noise free image, different types of additive noise, the filtered image by Gaussian MF, the denoised image by Sigma MF and the filtered image by Trapezoidal MF, respectively.

about 8 mm, the distance between figure and caption about 6 mm.Fig. 1 demonstrates the denoised version of Barbara's image corrupted with σ =40 different additive noises, where different MFs are used for noise reduction. In Table 1, the values of the PSNR prove that the Gaussian and the Sigmaf MFs outperform the Trapezoidal one. The details of comparison are mentioned as follows:

Additive Gaussian Noise: The Gaussian MF achieves the best result and reduces the noise well for the Baboon and Barbara images damaged with σ =5, 30 and 40 and for the all tested images damaged with σ = 30 and 40 too. For this additive noise with σ = 20, the Sigmaf MF reveals the best PSNR values on the all tested image. The Trapezoidal MF function result in the best PSNR value just on the noisy Lena image with σ = 5.

Additive Poisson Noise: For the all noisy image with σ=20, 30 and 40, the Gaussian MF shows the highest PSNR value. The best PSNR value, on the noisy Baboon and the noisy Barbara images with the same σ = 5, belongs to Sigmaf membership function. Again, the best score for the Trapezoidal membership function is on the noisy Lena image with σ = 5.

Additive Salt & Pepper Noise:For the all noisy image with σ = 20 and 30, the Sigmaf membership function gets the elevated PSNR value that is visualized in Fig. 1.On the noisy Lena with σ = 40 and also on the noisy Barbara image with σ = 5, the Sigmaf membership function receives the best PSNR values. The Trapezoidal MF illustrates the best results on the noisy Baboon and the noisy Barbara images with the same σ =5 and 40. The Gaussian MF shows the poor performance except on the noisy Lena and the noisy Barbara with the same σ = 5.

Additive Speckle Noise: In this additive noise, the best PCNR value refers to the Sigmaf MF. The Fig. 1 proves our claim. This MF achieves the best PCNR values on the all noisy images except on the Baboon image which is corrupted with σ = 5 additive noise. The Trapezoidal MF is the best just on the noisy Baboon image with σ=5. The Gaussian MF shows the worst performance.

In general, on the all mentioned additive noise sources, the Gaussian and Sigmaf MF outperform the Trapezoidal one. The Gaussian MF is the best choice for the fuzzy-based wavelet shrinkage in order to denoise the image which is corrupted with the additive Gaussian (Poisson) noise. Furthermore, in the fuzzy-based denoising technique, the best usage MF to reduce the additive Salt & Pepper (Speckle) noise is Sigmaf. As the shown in the Fig. 1, the Gaussian MF is able to preserve the most of the details. The Sigmaf MF reduces themost of the noisy pixels while it removes the details. In other words, the Sigmaf MF generates a smooth image.

5. Conclusion

The main goal of this study is to analyze the effect of different MFs on the well-known images which are corrupted with the some additive noise such as Gaussian, Salt & Pepper, Speckle and Poisson. In this study, two MFs are proposed, Gaussian and Sigmaf, for [1]. In [1], the Trapezoidal MF is just used. Experimental results reveal that the Gaussian and Sigmaf MFs outperforms the Trapezoidal MF. To reduce the additive Gaussian (Poisson) noise from the image, the Gaussian MF is the best choice for the fuzzy-based wavelet shrinkage technique. Furthermore, the Sigmaf MF is the best selection for the fuzzy-based denoising method to remove noise from the image which is damaged by the additive Salt & Pepper (Speckle) noise.

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