

Mean Square Error Performance of OFDM Systems against Signal to Noise Ratio under different Fading Channels

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Abstract

The well known problem in an orthogonal frequency division multiplexing (OFDM) system is its sensitivity to frequency offset. Most of the coherent OFDM systems transmit pilot symbols on some of the subcarriers to estimate channel attenuation and also add a cyclic prefix (CP) to avoid inter carrier interference and inter symbol interference. An estimation algorithm based on the redundancy of both cyclic prefix and pilot subcarriers is proposed for the correction of frequency offset. If the frame timing is synchronized in advance, by considering the two kinds of redundancy simultaneously, the performance of the proposed hybrid algorithm achieves significant improvement under low SNR and short CP. As to high SNR and long CP, the performance of the hybrid algorithm is almost identical to that of CP-based algorithm. Some comparative simulations are given to illustrate the advantages of the proposed hybrid estimation scheme

Keywords: OFDM, Cyclic Prefix, Subcarriers, Frequency offset

1. Introduction

The OFDM system is capable of coping with the frequency selective fading and narrowband noise because each subscriber in OFDM system is narrow band with respect to coherent bandwidth. Sensitivity to synchronization errors, including both frequency and time is one of the main problems in OFDM systems. Compared to single carrier modulation schemes, OFDM is very sensitive to frequency offset. Minor carrier frequency offset makes the subcarrier of OFDM system to lose its orthogonality because all the subcarriers in OFDM systems are overlapped and orthogonal to each other. Detection of carrier frequency and time offset is a difficult task in OFDM systems. Using of analogy of sample time offset in time domain causing Inter Symbol Interference (ISI),

frequency offset significantly degrades the system performance and cause Inter Symbol Interference (ISI). Frequency and Time correction algorithms are classified into two types based on whether the additional data is required or not, if required what kind of additional data is to be used. For the applications that require fast and reliable synchronization, data aided schemes are suitable because of the redundancy of the OFDM data frame. Two kinds of redundancy data which are usually used are training symbols (or pilot symbols) and pilot subcarriers. Training symbols are two or more consecutive and identical symbols, used to estimate frequency or time offset [1-3]. Pilot subcarriers are used to estimate channel attenuation in a single carrier system, as pilot-symbol assisted modulation (PSAM).[4] In case of multicarrier systems pilot subcarriers are used, to estimate frequency offset.[5,6] Depending on nature structure of OFDM frames, non-data-aided schemes are used to estimate frequency offset. To obtain better performance null subcarriers are selected.[7] Based on the utilization of the correlation of received data samples, other blind methods are proposed. [8,9] which uses an adaptive algorithm to reduce mean square error of frequency offset. One of the most popular non-data-aided synchronization schemes is based on using the cyclostationarity properties of the OFDM signals because of the insertion of cyclic prefix. The cyclic prefix is a part of a transmitted symbol frames and pilots are always inserted in some specific subcarriers in the modern specification of OFDM systems, such as 802.11a and HIPERLAN/2. An algorithm combining the two features, CP and pilot carriers, in one OFDM frame has good potential to provide better estimation performance. The presence of the cyclic prefix and the redundancy of the pilot sub carriers are considered in this paper. The hybrid maximum likelihood estimator for carrier frequency offset only is derived. As the frequency

synchronization is usually done after the correction of symbol time offset, the time offset is discarded.

2. OFDM System model

In the design of frequency offset estimation, an OFDM system model with CP and pilot insertion is considered. In this system, there are N_p pilot symbols inserted into total N subcarriers, and the length of the CP is assumed to be L . In case of multicarrier systems, the pilot symbols are scattered in time and frequency grids, the idea of PSAM is extended to 2D-PSAM. Therefore, the set of subcarriers indexing carrying pilots is denoted as γ and the set of OFDM frames including pilots is denoted as Γ . By definition, the modulated symbol X_n in frame i , where $i \in \Gamma$ and $n \notin \gamma$, is the data symbol transmitted on the n th subcarrier, and P_n in frame 'i' is the intentionally inserted pilot symbol. The signal is separated into two parts; the one containing the $N-N_p$ subcarriers is denoted as $s(k)$ and the other containing the N_p subcarriers is represented by $m(k)$, after the symbol frame 'i' passes through IDFT. For $k \in [0, N-1]$, $s(k)$ and $m(k)$ are defined as:

$$S(k) = \frac{1}{\sqrt{N}} \sum_{n \notin \gamma} X_n e^{j2\pi kn} \quad (1)$$

$$m(k) = \frac{1}{\sqrt{N}} \sum_{n \in \gamma} P_n e^{j2\pi kn} \quad (2)$$

The data symbol X_n is random with average energy $E\{|X_n|^2\} = \sigma_x^2$. The transmitted signal $m(k)$ is treated as a time-variant, because the pilot symbol P_n is known with the assumption of $E\{|P_n|^2\} = \sigma_p^2$. The statistical properties of $s(k)$ are simplified as a discrete time Gaussian random process with variance $\alpha\sigma_x^2$ (where $\alpha = N - N_p/N$), since the amount of the data-carrying subcarriers is reasonably large ($N \gg N_p$). The CP is added after IDFT modulation in OFDM systems. Hence, once the system uses the cyclic prefix, the tail L samples of the N -sample ($N > L$) transmitted signal $s(k) + m(k)$ are copied and inserted in front of the original signal, that is $s(k) = s(k+N)$ and $m(k) = m(k+N)$ for $k \in [0, L-1]$. Finally, the transmitted signal $s(k) + m(k)$, becomes $N+L$ samples and is not a white process anymore. Considering that additive white Gaussian noise (AWGN) channel is used, the time dispersion and time offset are not introduced. The received signal $r(k)$ is modeled as

$$r(k) = (s(k) + m(k)) e^{j2\pi k \epsilon} + n(k) \quad (3)$$

where $n(k)$ is a complex white zero-mean Gaussian noise with variance σ_n^2 . The exponential term in (3) is used to model the presence of frequency offset, which is caused by the instability of local oscillators and Doppler effects. In a discrete time system, frequency offset is introduced as a fraction of frequency spacing (the distance between each subcarrier) denoted as ϵ , where $|\epsilon| < 0.5$, so it is modeled as $e^{j2\pi k \epsilon}$ in equation (3) mathematically, assuming that all the

subcarriers between transmitter and receiver experience the same frequency offset mismatch.

3. Estimation Algorithm

The ML function of frequency offset is derived step by step with the help of the received signal model in (3). First a simplified assumption about the statistical properties of the correlation of $r(k)$ is considered. As the noise is zero mean Gaussian and $m(k)$ is a deterministic signal known at the receiver, the modeled received signal $r(k)$ is also a Gaussian process with the time varying mean $m(k) e^{j2\pi k \epsilon}$. As the numbers of pilots assumed are much smaller than that of data-carrying sub carriers, the correlation between successive frames is ignored. The autocorrelation of $r(k)$ is calculated as

$$E[r(k)r^*(l)] = \begin{cases} \alpha\sigma_x^2 + \sigma_n^2, & k = l \\ \alpha\sigma_x^2 e^{-j2\pi \epsilon}, & k - l = -N \\ \alpha\sigma_x^2 e^{-j2\pi \epsilon}, & k - l = N \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

Therefore the PDF of $r(k)$ & the joint PDF of $r(k)$ & $r(k+N)$ is analyzed using the above correlation properties

$$\text{as } f(r(k)) = \frac{1}{\pi(\sigma_s^2 + \sigma_n^2)} \exp\left(-\frac{|r(k) - m(k) e^{j2\pi \epsilon}|^2}{\sigma_s^2 + \sigma_n^2}\right) \quad (5)$$

and $f(r(k), r(k+N))$

$$= \frac{1}{A} \exp\left(-\frac{|\Omega(k)|^2 - 2\rho \text{Re}\{\Omega(k)\Omega^*(k+N)e^{j2\pi \epsilon}\} + |\Omega(k+N)|^2}{(\sigma_s^2 + \sigma_n^2)(1 - \rho^2)}\right) \quad (6)$$

To estimate the existing frequency offset ϵ the ML estimation scheme is used. The ML estimator of the frequency offset ϵ is derived by investigating the log-likelihood function of ϵ , which is written as

$$\Lambda(\epsilon) = \log f(r/\epsilon) \\ = \sum_{k=0}^{L-1} \log\left(\frac{f(r(k), r(k+N))}{f(r(k))f(r(k+N))}\right) + \sum_{k=0}^{N+L-1} \log f(r(k)) \quad (7)$$

where $f(\cdot)$ represents for the corresponding probability density function. Therefore, the ML frequency offset estimate can be obtained by maximising the log-likelihood function in (7) over all possible values of ϵ , that is

$$\epsilon_{ML}^* = \arg \max_{\epsilon} \{\Lambda(\epsilon)\} \quad (8)$$

Maximizing the log-likelihood function $A(\epsilon)$ in (7) is equivalent to maximizing the following function.

$$\rho \Lambda_{CP}(\epsilon) + (1 - \rho) \Lambda_p(\epsilon) \quad (9)$$

The log likelihood function in (9) is solved by substituting (5) & (6) into (9). Due to the cyclic prefix insertion with $m(k) = m(k+N)$, $k \in [0, L-1]$. The term $\sum_{k=0}^{L-1} m(k)m^*(k+N)$

becomes a constant, $\sum_{k=0}^{L-1} |m(k)|^2$ and is not relevant to the maximizing argument of the log-likelihood function. Like another constant term $\sum_{k=0}^{L-1} \log(1 - \rho)^2$, both of them are dropped during derivation. Finally the first term of (7) is proportional to

$$\frac{\sum_{k=0}^{L-1} \text{Re}\{r(k)r^*(k+N)e^{j2\pi\epsilon}\}}{\frac{\rho}{2} \sum_{k=0}^{L-1} [|r(k)|^2 + |r(k+N)|^2]} - (1-\rho) \sum_{k=0}^{L-1} \text{Re}\{[r^*(k) + r^*(k+N)e^{j2\pi\epsilon}]m(k)e^{j2\pi\epsilon k}\} \quad (10)$$

and the second term of (5) is proportional to

$$\frac{1-\rho^2}{\rho} \text{Re} \left\{ \sum_{k=0}^{N+L-1} r^*(k)m(k)e^{\frac{j2\pi\epsilon k}{N}} \right\} \quad (11)$$

Where

$$\rho = \left| \frac{\alpha\sigma_s^2 e^{-j2\pi\epsilon}}{\alpha\sigma_s^2 + \sigma_n^2} \right| = \frac{\alpha \text{SNR}}{\alpha \text{SNR} + 1} \quad (12)$$

$$\Lambda_{\text{CP}}(\epsilon) = \sum_{k=0}^{L-1} \text{Re}\{r(k)r^*(k+N)e^{j2\pi\epsilon}\} - \frac{\rho}{2} \sum_{k=0}^{L-1} [|r(k)|^2 + |r(k+N)|^2] \quad (13)$$

$$\Lambda_p(\epsilon) = (1 + \rho)\Lambda_{p1}(\epsilon) - \rho\Lambda_{p2}(\epsilon) \quad (14)$$

$$\Lambda_{p1}(\epsilon) = \sum_{k=0}^{N+L-1} \text{Re} \left\{ r^*(k)m(k)e^{\frac{j2\pi\epsilon k}{N}} \right\} \quad (15)$$

$$\Lambda_{p2}(\epsilon) = \sum_{k=0}^{L-1} \text{Re} \left\{ [r^*(k) + r^*(k+N)e^{j2\pi\epsilon}]m(k)e^{\frac{j2\pi\epsilon k}{N}} \right\} \quad (16)$$

The signal to noise ratio in (12) is $\text{SNR} = \frac{\sigma_s^2}{\sigma_n^2}$. The function in (9) uses the linear combinations of the information from the cyclic prefix and from pilot insertion to estimate the frequency offset. The important weighting factor ρ defined in (12), is determined by the SNR of the signals and the coefficient α , which is the ratio of the number of data carrying sub carriers ($N-N_p$) to the total number of sub carriers (N). ρ will approach 1 if the SNR is high and the proposed hybrid estimator will be dominated by the first term Λ_{CP} in (9). The estimator depends more on the last term $\Lambda_p(\epsilon)$ of (9) for a low SNR. However, more the pilot subcarriers, an OFDM frame has, more the term $\Lambda_p(\epsilon)$ contributes. In special cases, if the transmitted signal does not contain any pilot sub carrier, then $N_p=0, \rho=\text{SNR}/\text{SNR}+1$ and the function in (9) only exploits the cyclic prefix redundancy. Then the proposed hybrid estimator in (9) is similar in form to the estimator to the symbol timing offset in [9]. Furthermore it is important to investigate the two function terms $\Lambda_{\text{CP}}(\epsilon)$ and $\Lambda_p(\epsilon)$ in (9). As the summation range of $\Lambda_{\text{CP}}(\epsilon)$ is from 0 to $L-1$ only, the information of frequency offset in $\Lambda_{\text{CP}}(\epsilon)$ is contributed by the redundancy in the received signal due to cyclic prefix. The likelihood function proposed in [12] depends on the frequency offset information provided by the CP but it discussed the frequency and time offset together in one function Hence if time offset is set to 0 in

[12], its maximum likelihood function and (13) will become same. The function $\Lambda_p(\epsilon)$ in (14) contains the frequency offset information given by the redundancy in the received signal due to the pilot carriers. As defined in (15) and (16) this term can be separated into two parts $\Lambda_{p1}(\epsilon)$ and $\Lambda_{p2}(\epsilon)$ the information of pilot subcarriers contained in $\Lambda_{p1}(\epsilon)$ is provided by whole symbol and the information in $\Lambda_{p2}(\epsilon)$ is provided by CP. $\Lambda_{p1}(\epsilon)$ is same as the corresponding ML function proposed by [5], as these two estimators use ML criteria and the pilot subcarriers. Moreover [5] only contains general OFDM symbol frame without involving CP redundancy so the term $\Lambda_{p2}(\epsilon)$ never appears in its ML function. The proposed estimation algorithm uses the pilot subcarriers and cyclic prefix together to estimate the frequency offset. The proposed method uses special features in one OFDM symbol hence named as hybrid synchronization scheme. The proposed hybrid estimator is the weighting combination of estimator proposed in [12] and that in [5], which in turn depends on value of ρ . Hence the performance comparison of the simulation results for the three different estimators (the proposed hybrid estimator, the CP based estimator and the pilot based estimator) is reported in the following section under different fading channels.

4. Comparisons & Results

Effect of SNR

To observe the performance of the proposed hybrid estimation algorithm, 4000 symbol frames are used for the simulation and the mean square error of estimated frequency offset is calculated. The timing offset is assumed to be zero in comparative simulations, frequency offset is assumed to be 0.18 and the channel is one tap with an additive white Gaussian noise. The 16-QAM mapping scheme is used and no coding technique is applied. The pilot symbols are needed to be inserted to the subcarriers, they are inserted as per the specification of IEEE i.e. 802.11a, where the number of subcarriers is $N=64$. By using the MSE of estimated frequency offset, the performance of three different algorithms are evaluated and normalized with the range 0-0.5. The three maximum likelihood functions are written below

1. Proposed: $\rho\Lambda_{\text{CP}}(\epsilon) + (1-\rho)\Lambda_p(\epsilon)$

2. CP-based:

$$\Lambda_{\text{CP}}(\epsilon) = \sum_{k=0}^{L-1} \text{Re}\{r(k)r^*(k+N)e^{j2\pi\epsilon}\} - \frac{\rho}{2} \sum_{k=0}^{L-1} [|r(k)|^2 + |r(k+N)|^2]$$

3. pilot-based:

$$\Lambda_{\text{APB}}(\epsilon) = \sum_{k=0}^{N-1} \text{Re} \left\{ r^*(k)m(k)e^{\frac{j2\pi\epsilon k}{N}} \right\}$$

The MSE performance of normalized frequency offset estimation against different SNR values with CP length $L = N/32$, $N = 64$ is shown in figure 1. In this figure, the results from the pilot-based algorithm are exactly the same because this algorithm does not take the CP into consideration. In figure 1, where $L = N/32$, it can be observed the MSE of the CP-based algorithm becomes smaller than that of the pilot-based scheme. The proposed algorithm plotted by a solid line with star-can achieve lower mean square error than the other two schemes. With the growth of SNR, as mentioned before, the estimated results depend more on the cyclic prefix. Hence the proposed and CP-based techniques have similar performance. The MSE performance against different SNR values for the length of CP $L = N/32$, are plotted under Rician & Rayleigh channels. Fig 2 & 3 show that the proposed algorithm outperforms the other two schemes. Results of Fig 4 shows that as the length of the cp becomes longer ($L = N/16$), under Rician channel the MSE difference between CP-based and proposed algorithm would become smaller and the MSE performance is far better compared to low CP length ($L = N/32$). Therefore, when the length of CP is longer, the CP-based algorithm is enough to obtain a good performance and the complexity of computation is reduced.

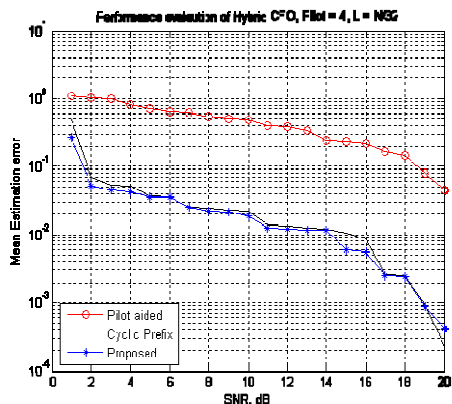


Fig1: Varying SNR under AWGN channel with Pilot=4 and $L = N/32$

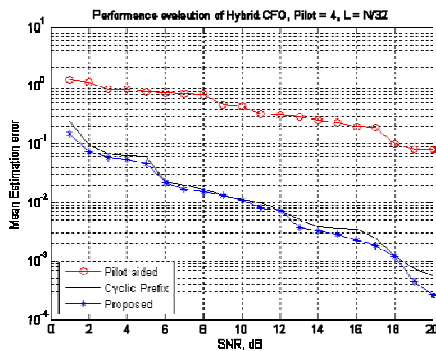


Fig 2: Varying SNR under Rician Channel with Pilot=4 and $L = N/32$

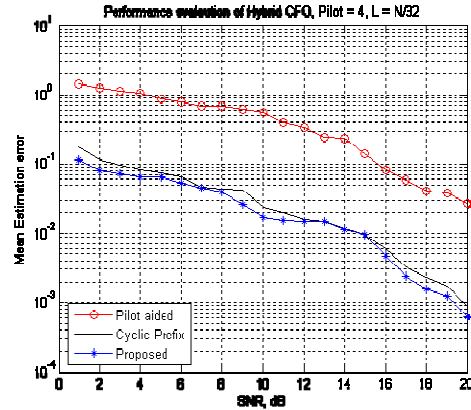


Fig 3: Varying SNR under Rayleigh Channel with Pilot=4 and $L = N/32$

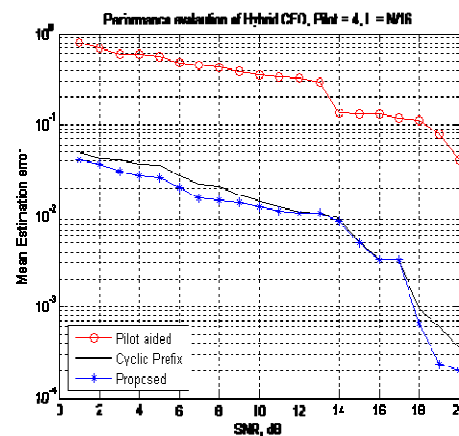


Fig 4: Varying SNR under Rayleigh Channel with Pilot=4 and $L = N/16$

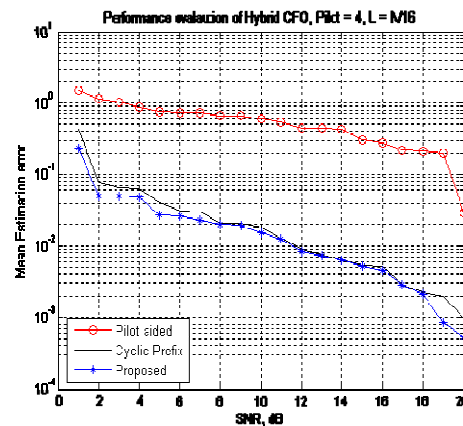


Fig 5: Varying SNR under Rician Channel with Pilot=4 and $L = N/16$

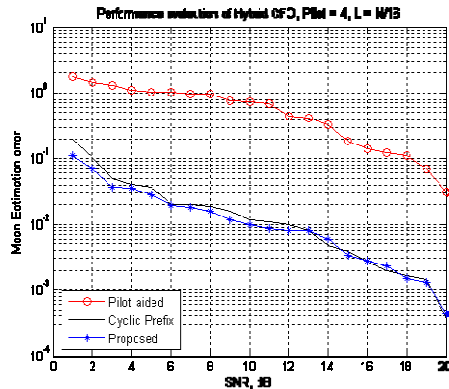


Fig6: Varying SNR under AWGN channel with Pilot=4 and L=N/16

5. Conclusion:

The performance of all the three estimated schemes is discussed. The performance in terms of MSE improves considerably (comparing fig 3 &4) if a Rayleigh fading channel is considered for longer CP length. The performance in terms of MSE degrades (comparing fig 2&5) if a Rician fading channel is considered for longer CP length. The MSE difference between CP-based and proposed algorithm would become smaller when the length of CP is longer. Therefore the CP-based algorithm is enough to obtain a good performance and the complexity of computation is reduced in the proposed model

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