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# Numerical simulation of groundwater level in a fractured porous medium and sensitivity analysis of the hydrodynamic parameters using grid computing: application of the plain of Gondo (Burkina Faso)

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## Abstract

The use of mathematical modeling as a tool for decision support is not common in Africa in solving development problems. In this article we talk about the numerical simulation of groundwater level of the plain of Gondo (Burkina Faso) and the sensitivity analysis of the hydrodynamic parameters. The domain has fractures which have hydraulic coefficients lower than those of the rock. Our contribution is to bring brief replies to the real problem posed in the thesis of Mr. KOUSSOUBE [1]. Namely that what causes the appearance of the piezometric level observed and impact of surface water on the piezometry. The mathematical model of the flow was solved by programming the finite element method on FreeFem++[2]. A local refinement of the mesh at fracture was used. We then conduct a sensitivity analysis to see which hydrodynamic parameters influences much of the solution. The method used for the sensitivity analysis is based on the calculation of the gradient by the adjoint equation and requires great computational power. To remedy this, we used a technique of distributed computing and we launched our application to the Moroccan grid (magrid). This allowed us to reduce the computation time. The results allowed to highlight the role of fractures and contributions of surface water on the evolution of the piezometric level of the plain of Gondo and identified the parameters that greatly influence the piezometric level.

**Keywords:** *fractured porous media, finite elements, hydrodynamic parameters, sensitivity analysis, distributed computing, grid computing*.

## 1 Introduction

The goal of this work is the mathematical modeling and numerical simulation of groundwater level of the plain of Gondo. The mathematical model was solved by the Galerkin finite element method. The programming was done in FreeFem++ [2].

We worked on a geological section of reference of the plain of Gondo. The geological section shows a fractured porous media [1] (see Figure 1). The usual models programmed with standard softwares asume that the medium is homogeneous and continuous [3]. Fractured aquifers cannot be modeled in a simple way. The flow in the fractures does not meet the laws that govern a continuous medium. The presence of fractures greatly influences the hydraulic conductivity. In most cases, the fractures have a hydraulic conductivity greater than that of the rock. In this case they constitute privileged channels for the water circulation. In our case, the fractures have hydraulic conductivities lower than the rock. There are several methods and models for treating fractures [4] [5]. To treat the presence of fractures, we assume that the fractures are sub-domains of a global domain as in [6]. We therefore assimilate them to porous media and we use finer meshes in fractures. Each domain was meshed separately and we have imposed on the nodes common border of two neighboring domains.

Our paper is organized as follows. In the first part, we present the problem to be solved. The second part is devoted to mathematical modeling and to the numerical solution of the model used. A third part is devoted to presenting the results of numerical simulations, followed finally by a fourth section on sensitivity analysis of hydrodynamic parameters, in wich we present the analysis method used and this execution on the grid computing.

## 2 Problem to be solved

### 2.1 Site description: geology and geometry of the site [1]

The plain of Gondo is vast; by lack of data on the whole plain, the domain used for the modeling is a geological section located on the edge of the basin of Gondo (Figure 1). On this zone, many investigations were realized within the framework of the thesis [1]: geophysics, piezometry, correlation of logs of drillings, chemistry and isotopic geochemistry, observations of ground. This vertical section of selected reference extends from the village of Nomou located at the East in the crystalline base to the village of Yensélocated at the West in the sedimentary basin, that is to say a length of 25 km approximately. It has a height of 300 m. On the section of reference, five subverticale fractures of 500 m thickness affect the base and the sedimentary formations as well.

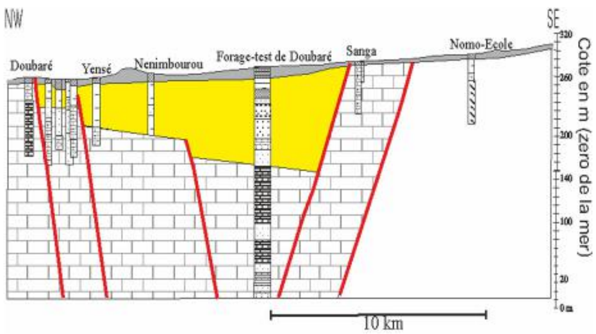


Figure 1: Geology and geometry of the site

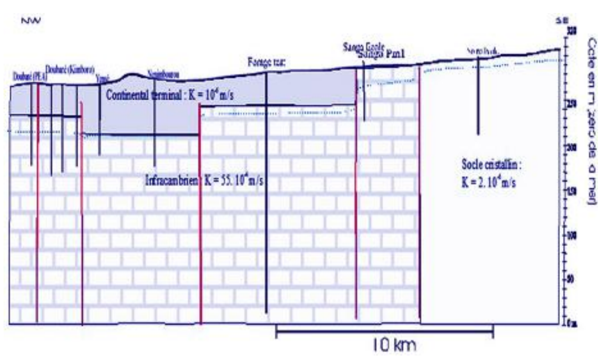


Figure 2: Simplified interpretative geological section Appearance of the piezometry: dotted curve in blue

## 2.2 Boundary conditions for the flow

The piezometry observed with the upstream (zone of base to the upstream) and with the downstream (sedimentary zone with the downstream of Doubaré) of the flow is prescribed as a limit with imposed potential. The lower limit of the field constitutes a limit with null flow. We must also take into account the recharge induced by rainfall in the upper part.

### 2.3 Responses required for flow modeling

The mathematical modeling and numerical simulation must bring answers to the following questions:

- What is the role of the fractures on the stair appearance observed of the piezometry (Figure 2)?
- What is the impact of the concentration of surface water on the evolution of the piezometry?

To provide some answers to these questions, we will do a simulation in steady state level of the water in two dates (1960 and 2000).

# 3 Mathematical model and numerical solution

## 3.1 Mathematical model

We consider the steady flow of an incompressible and monophasic fluid in a saturated porous medium Ω. The flow is governed by the law of conservation of mass and Darcy's law[3][7]:

$$
\begin{aligned}
div(u) &= f && \text{in} \quad \Omega \\
u &= -K\nabla p && \text{in} \quad \Omega \\
p &= d && \text{on} \quad \Gamma^0 \\
u.\nu &= -K\frac{\partial p}{\partial n} = g && \text{on} \quad \Gamma^1
\end{aligned}
\qquad (1)
$$

where u(x) is the Darcy velocity, p(x) is the hydraulic potential, f(x) the source term, K(x) the hydraulic conductivity, Ω is a bounded open of $\mathbb{R}^2$, d(x) is the Dirichlet boundary conditions and g(x) the Neumann boundary conditions.
Remark: in our case g(x) = 0 (no-flux), d(x) and K(x) are piecewise constant functions.
Subdivide our domain into several sub-domains $\Omega_i$,

(i = 1,2,.. . , 13) (see Figure 3). The

fractures are treated as porous media. We notice

$$\Gamma_i = \Omega_i \cap \Omega_{i+1},\ i = 1,2,\dots,12$$

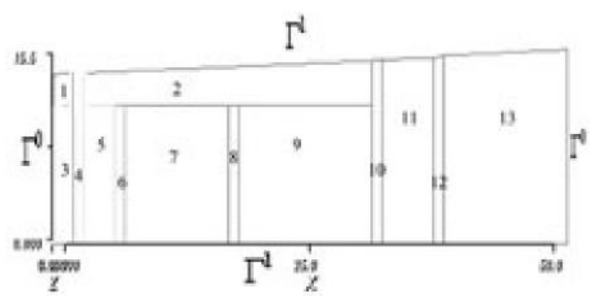


Figure 3: Subdivided domain

We introduce below the variational formulation of problem (1), which is equivalent to the following transmission equations [8]:

$$\begin{array}{rll} div(u_i) = f_i & \text{in} & \Omega_i \\ u_i = -K_i \nabla p_i & \text{in} & \Omega_i \\ p_i = d_i & \text{on} & \Gamma_{Di} \\ u_i.\nu_i = 0 & \text{on} & \Gamma_{N_i} \\ p_i = p_{i+1} & \text{on} & \Gamma_i \\ u_i.\nu_i = u_{i+1}.\nu_{i+1} & \text{on} & \Gamma_i \end{array} \quad (2)$$

These equations reflect the exchange between the aquifers.
By eliminating u in equation (1), we have formally:

$$\begin{array}{rll} -div(K\nabla p) = f & \text{in} & \Omega \\ p = d & \text{on} & \Gamma^0 \\ -K\frac{\partial p}{\partial n} = g & \text{on} & \Gamma^1 \end{array} \quad (3)$$

## 3.2 Problem solving

Let V be the space be defined by

V = {v ∈ $H^1(\Omega)$, v = 0 on $\Gamma^0$}.

Since mes($\Gamma^0$) > 0, according to [9], we can choose

$$\|v\|_V = \left(\int_\Omega |\nabla v|^2\right)^{1/2} \text{ as norm on } V.$$

Let v ∈ V be a test function. On multiplying (3) by v and integrating by parts, the variational formulation associated to the problem (3) is:

$$\left\{ \begin{array}{l} \text{Find } p \in H^1(\Omega) \text{ such that } p = d \text{ on } \Gamma^0 \text{ and such that} \\ \int_\Omega K\nabla p.\nabla v = \int_\Omega fv - \int_{\Gamma^1} gv, \ \forall v \in V \qquad (4) \end{array} \right.$$

We denote by γ0 the trace operator. Let $r_d$ ∈$H^1(\Omega)$ such as γ0($r_d$) = d and we denote $p_0 = p - r_d$. The variational formulation becomes:

$$\left\{ \begin{array}{l} \text{Find } p_0 \in V \text{ such as} \\ \int_\Omega K\nabla p_0.\nabla v = \int_\Omega K\nabla r_d.\nabla v + \int_\Omega fv - \int_{\Gamma^1} gv, \ \forall v \in V \end{array} \right. \quad (5)$$

The function K(x) is constant in each domain Ωi, i = 1, 2, ... ,13. We have $K(x) = \sum_{i=1}^{13} K_i\, I_i(x)$ where $K_i$ >0 and where the function $I_i(x)$ is the characteristic functions of the domain Ωi for i = 1, 2, ... , 13.
We have then 0 < min(Ki) < K(x) < max(Ki). We assume that $g$ ∈$L^2(\Gamma^1)$ and $f$ ∈$L^2(\Omega)$.
Let the bilinear form $\alpha : V\mathrm{x}V \to \mathbb{R}$ be defined by:

$$\alpha(p_0, v) = \int_\Omega K\nabla p_0.\nabla v$$

Let the linear form $L : V \to \mathbb{R}$ be defined by:

$$L(v) = \int_\Omega K\nabla r_d.\nabla v + \int_\Omega fv - \int_{\Gamma^1} gv$$

The space V is a Hilbert space for the Hilbertian norm $\|.\|_V$.
The bilinear form $\alpha$ is continuous, coercive and the linear form L is also continuous. Thus the theorem of Lax-Milgram [10] ensures the existence and uniqueness of a solution to the variational problem (5) and consequently the existence and uniqueness of a solution of (3).
Let $T_h$ be a triangulation of Ω. Let $P_1$ denote the space of continuous, piewise affine function in Ω i.e the space of continuous functions which are affine in x, y on each triangle of $T_h$. We pose $V_h = P_1 \cap V$. $V_h$ is a linear vector space of finite dimension. We denote N its dimension and $\phi_1, \ldots, \phi_N$ a basis. The approximated problem is:

$$\text{find } p_h \in V_h, \text{ such that } \alpha(p_h, v_h) = L(v_h) \text{ for all } v_h \in V_h \quad (6)$$

Let

$$p_h(x, y) = \sum_{i=1}^{N} p_i \phi_i(x, y)$$

and take $v_h = \phi_i$ for i = 1,..., N; equation (6) is equivalent to

$$\alpha(\sum_{j=1}^{N} p_j\phi_j, \phi_i) = L(\phi_i) \ , \ i = 1, ..., N \qquad (7)$$

This gives the system $Ax = b$, where:

$$A_{ij} = \int_\Omega K\nabla\phi_i.\nabla\phi_j = \sum_{T\in T_h} \int_T K\nabla\phi_i.\nabla\phi_j$$

$$b_i = \int_\Omega f\phi_i + \int_\Omega K\nabla r_d.\nabla\phi_i - \int_{\Gamma^1} g\phi_i$$

$$= \sum_{T\in T_h} \int_T f\phi_i + \sum_{T\in T_h} \int_T K\nabla r_d.\nabla\phi_i - \sum_{T\in T_h} \int_{T\cap\Gamma^1} g\phi_i$$

# 4 Results and discussions

## 4.1 Problem test

Before applying our code to our case, we compared our results to a reference case obtained by the method of domain decomposition [11]. The length is 2cm, the width is 1cm and the opening of the fracture is d = 0.01. Hydraulic conductivity at the fracture $K_f$ = 100 and that of the rock is 1. The results are shown in Figure 4.
**Remark:** We multiplied the size of the area by 10.

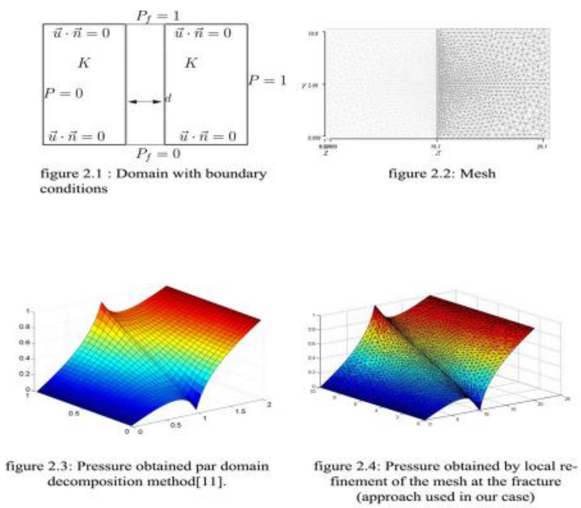


Figure 4: test case

## 4.2 Application to our problem

We used a triangular mesh. To have a high degree of accuracy we chose an average step of 50 m (horizontally) and 10 m (vertically) (see Figure 5). The data used for the realization of the simulations are from the thesis of Mr Youssouf Koussoubé[1]. We have

K3 = K5 = K7 = K9 = K11 and K1 = K2 because sub-domains 3, 5, 7, 9 and 11 are the same geological formation, and so are sub-domains 1 and 2. So we have a total of 8 hydrodynamic parameters.

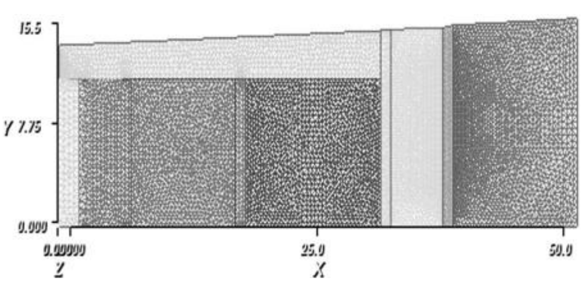


Figure 5: Mesh

### 4.2.1 Simulation 1: level of 1960

**Hydrodynamic parameters**

K1 = $10^{-6}$m/s, K2 = $55.10^{-4}$m/s, K3 = $10^{-6}$m/s, K4 = $10^{-8}$m/s, K5 = $10^{-8}$m/s, K6 = $10^{-8}$m/s, K7 = $5.10^{-8}$m/s, K8 = $2.10^{-6}$m/s.

**Boundary conditions and source term**

The potential imposed on the upstream is 222 m and 199 m on downstream. The source term is the groundwater recharge by rainfall that is 120 mm/year.

Figure 6 represents the result of this simulation by plotting the potential as a function of the space variables x and y.

### 4.2.2 Simulation 2: level of 2000

The hydrodynamic parameters values are the same than 1960. The potential imposed on the upstream is 287 m and 211 m on downstream. The source term is the groundwater recharge by rainfall that is 100 mm/year. The graphics of the Figure 7 represent the results of the simulation.

### 4.2.3 Role of fractures and surface water on the rise of piezometry

To see the impact of the fractures and surface water on the piezometry, we realised two simulations.

**Simulation 3 (Figure 8)**

The boundary conditions are those of 2000. In addition to the value of the source term of 2000, we added between fracture 4 and fracture 5, the contributions of surface water is 2600 million $m^3$ of water. We note a rise of piezometry compared to that of 2000 (see Figure 10).

**Simulation 4(Figure 9)**

In addition to the preceding test conditions, we modified the hydrodynamic properties of the fractures.

K1 = $10^{-6}$m/s, K2 = $55.10^{-4}$m/s, K3 = $10^{-6}$m/s, K4 = $10^{-3}$m/s, K5 = $10^{-3}$m/s, K6 = $10^{-6}$m/s, K7 = $5.10^{-7}$m/s, K8 = $2.10^{-6}$m/s. We note that the groundwater level has not increased despite the addition of the source term (see Figure 10). That shows the influence of the hydrodynamic properties of fractures on the piezometry.

### 4.2.4 Role of the fractures on the pace observed of piezometry

To see the role of the barriers on piezometry in stair observed we conducted two simulations by modifying the properties of the fractures. The other values are those of 1960.

**Simulation 5 (Figure 11)**

K1 = $10^{-6}$m/s, K2 = $55.10^{-4}$m/s, K3 = $10^{-3}$m/s, K4 = $10^{-3}$m/s, K5 = $10^{-3}$m/s, K6 = $10^{-3}$m/s, K7 = $5.10^{-3}$m/s, K8 = $2.10^{-6}$m/s

**Simulation 6(figure12)**

K1 = $10^{-6}$m/s, K2 = $55.10^{-4}$m/s, K3 = $10^{-3}$m/s, K4 = $10^{-3}$m/s, K5 = $10^{-3}$m/s, K6 = $10^{-8}$m/s, K7 = $5.10^{-8}$m/s, K8 = $2.10^{-6}$m/s.

Simulations 1 to 4 make it possible to conclude that the rise of the piezometry is the result of two factors: the contribution of surface water, between fractures 4 and 5, as well as the low permeability of the fractures which prevents the flow.

As for the stair appearance observed of the piezometry, seems to be the result of the hydrodynamic property of the fractures which cause jumps of pressures, i.e they cause discontinuities of the pressure. This is shown to us by the simulations 5 and 6.

For order to better see the impact of the hydrodynamic parameters on the piezometry, we will carry out an analysis of sensitivity.

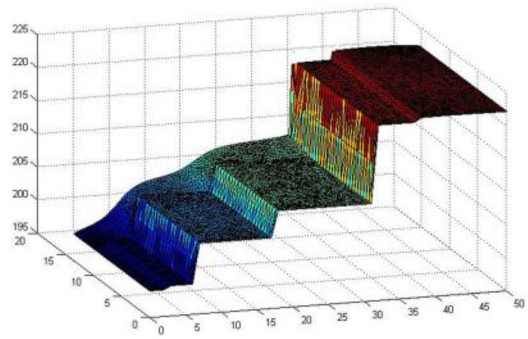

Figure 6: Simulation 1

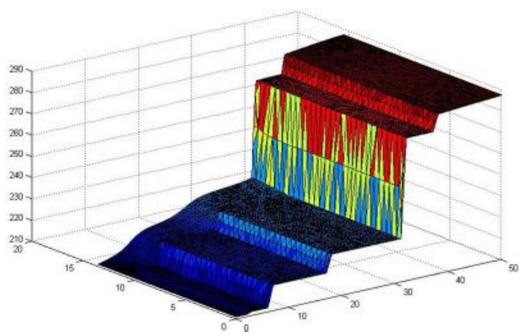

Figure 7: Simulation 2

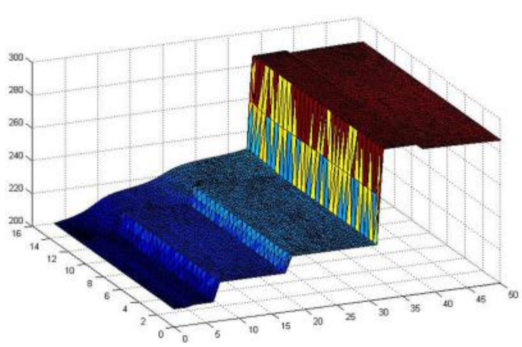

Figure 8: Simulation 3

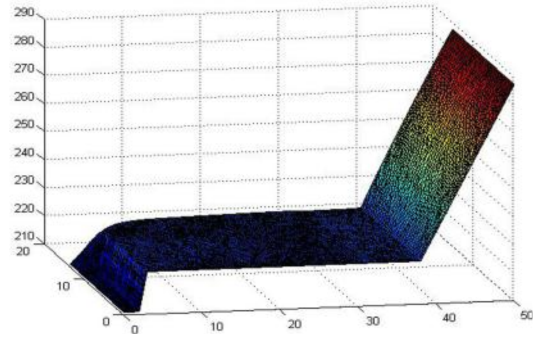

Figure 9: Simulation 4

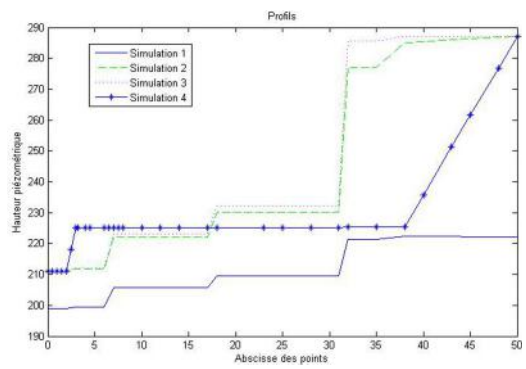


Figure 10: Profiles of simulations 1,2,3,4

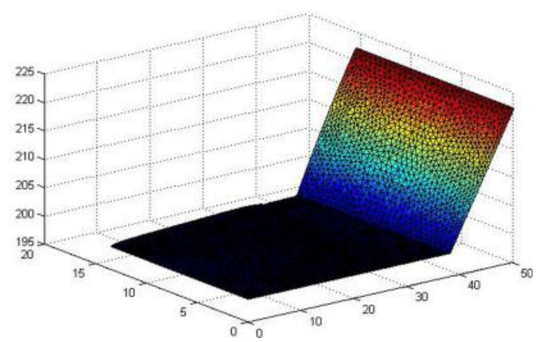

Figure 11: Simulation 5

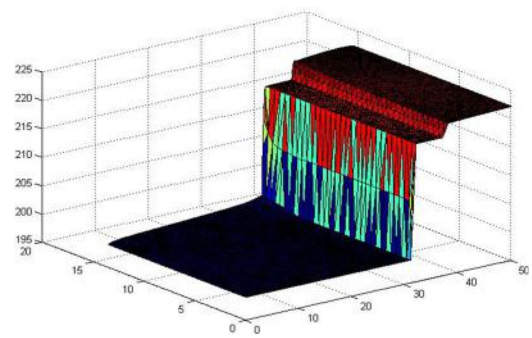

Figure 12: Simulation 6

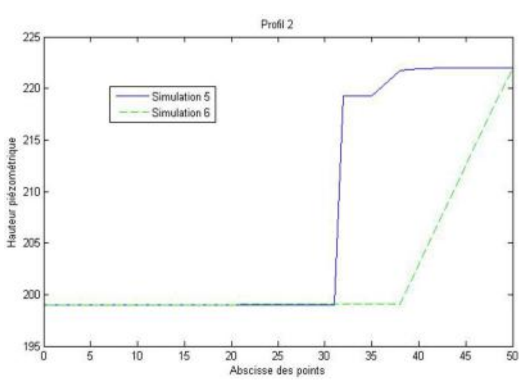


Figure 13: Profiles of simulation 5 and 6

# 5 Analysis of sensitivity of hydrodynamic parameters

## 5.1 Interest of the analysis of sensitivity

A mathematical model is a simplified or skewed representation, more or less realistic of the variable of state which it simulates. It is particularly the case for the models of flow, since one does not know exact equations governing the laws of their variables of state. In fact, to know uncertainty on the outputs of the model is essential. When a model is used, it is difficult to guess which parameters will have the most weight in the model, on which to pay more attention in term of precision, on which a disturbance would generate a consequent difference at exit. Given a model, one may ask what parameters must be estimated prioritarily, how a small change of control will impact the output. And even for economic reasons or practices, we may wish to consider the impact of the frequency of these observations on the results of estimating model parameters. Sensitivity analysis which is by definition the study of the impact of control variables on the output; it can provide some answers to these concerns. It is possible to group the methods of sensitivity analysis in three classes [12]: the methods of screening, which consist in a qualitative analysis of the sensitivity of the output variable to input variables, local methods analysis (based on the calculation of the derivative), which assess quantitatively the impact of a small variation around a given value of the inputs, and finally the methods of global sensitivity analysis (based on the analysis Statistics), interested in the variability of the model output in its entire range of variation. In this work we will conduct a sensitivity analysis to see the local impact of each control variable on the state variable.

## 5.2 Definition

A sensitivity analysis involves:

- a model

$$F(X, K) = 0 \tag{8}$$

  where $X$ is the state variable ; $K$ are the control variables of the model; $F$ a differential operator that is non-linear a priori, finite-dimensional.

  Given $K$, we suppose that the system (8) has a unique solution.

- a response function G, function of (X, K) with scalar value, which expresses (in a certain way adapted to the situation) one or several outputs of the model of which one tries to assess the sensitivity.

It then seeks to determine the sensitivity of G with respect to K. Mathematically the sensitivity S of G over K is defined as the gradient of G with respect to K [13]:

$$S = \nabla_K G \qquad (9)$$

In our case the mathematical model is given by equation (3). The variable of state is the pressure p and the variables of control are given by the vector $K = (K_1, \dots, K_8)$.

## 5.3 Definition of the function G

Let $p_r$ be a reference solution of equation (3) corresponding to a reference parameter vector $K_r$. We seek the impact of the disruption $K_r$ on $p_r$. For this we take the functional J defined below as the criterion.

$$J(K) = \|p(K) - p_r\|^2_{L^2(\Omega)} \qquad (10)$$

By definition calculate the sensitivity S is to calculate the gradient of J at point $K(\nabla_K J)$.
To do this we will go through the adjoint state method.

## 5.4 Calculating the gradient by the adjoint equation

We have:

$$J(K) = \int_\Omega (p(K) - p_r, p(K) - p_r) d\Omega$$

**Calculation of the directional derivative:**

By definition, we have:

$$J(K)[k] = \lim_{\beta \to 0} \frac{J(K + \beta k) - J(K)}{\beta}$$

After calculation, we have:

$$J(K + \alpha k) - J(K) =$$

$$\int_\Omega (p(K + \beta k) + p(K) - 2p_r).(p(K + \beta k) - p(K)$$

Dividing by β and taking the limit, we have:

$$J(K)[k] = \int_\Omega 2(p(K) - p_d).\hat{p} d\Omega$$

where

$$\hat{p} = \lim_{\beta \to 0} \frac{p(K + \beta k) - p(K)}{\beta}$$

**Tangent linear model:**
As p(K + βk) and p(K) are solutions of the equation (3) we show that $\hat{p}$ is solution of

$$\begin{array}{lll} -div(K\nabla\hat{p}) = div(k\nabla p) & \text{in} & \Omega \\ \hat{p}(x) = 0 & \text{on} & \Gamma^0 \\ \frac{\partial \hat{p}}{\partial n} = 0 & \text{on} & \Gamma^1 \end{array} \qquad (11)$$

**Adjoint model**
By multiplying the tangent linear model by a function q and integrating by parts twice, we have:

$$\int_\Omega -div(K\nabla q)\hat{p} = \int_\Omega -div(k\nabla p) q$$

we consider

$$\begin{array}{lll} -div(K\nabla q) = 2(p - p_r) & \text{in} & \Omega \\ q = 0 & \text{on} & \Gamma^0 \\ \frac{\partial q}{\partial n} = 0 & \text{on} & \Gamma^1 \end{array} \qquad (12)$$

We have then

$$\int_\Omega 2(p - p_r).\hat{p} d\Omega = -\int_\Omega k\nabla p.\nabla q$$

as

$$(2(p - p_r), \hat{p}) = (\nabla J(K), k)$$

We have

$$\nabla J_K(k) = \int_\Omega -k\nabla p.\nabla q \qquad (13)$$

Finally

$$\frac{\partial J}{\partial K_i}(k) = -\int_{\Omega_i} k\nabla p.\nabla q, \; i = 1, \dots, 8 \qquad (14)$$

In summary, to calculate the sensitivity, we proceed as follows for each set of parameters and for each parameter:

1. Solve the direct problem (3)
2. Solve the adjoint problem (12)
3. Calculate $S_i$ of parameter $K_i$ using equation (14)

As can be seen, the calculation of sensitivities of different parameters, requires several time the solving of problems (3) and (12). This requires enormous computing capacity if the number of parameters and the number of data sets are very high. To solve this problem of resources, we ran the program on the Moroccan grid (magrid).

## 5.5 Grid Computing

### 5.5.1 Definition

According to FOSTER [14], a grid computing is a system that:

- coordinates the resources which are not under the control of a central system,
- uses standards, opened and generic protocols and interfaces,
- provides multiple high quality services.

The ingenuity of the concept of the grid lies in its ability to virtualize resources. With this virtualization, we see the overall system as a super virtual machine. Grid computing allows you to have great computer's resources for the realization of heavier calculation.

### 5.5.2 Grid computing operation:

To use a grid, you must have a certificate that allows you to attach a virtual organization which is a dynamic group of entities that choose to share resources and to define the conditions and roles of sharing them. Then we must create a proxy to submit the application called "job" written in a language called "job description language (JDL)". Very briefly, the elements involved in the care of a job submitted to a grid.
The Workload Management System (WMS) is consisted of the following elements:

- User Interface (UI): interface through which users access to the grid
- Computing Element (CE): represent the access point unified to resources of calculation, of the worker nodes which will be used by the grid for the execution of the jobs. The Computing Element is responsible for the management of jobs assigned to it. the Computing Element maintains a list of jobs to submit (batch queue). Storage Element (SE) : manages the storage of information.
- Information System (IS): set of resource information indicating the characteristics and condition of the Computing Element (CE) and Storage Elements (SE)
- Resource Broker (RB) or Workload Manager (WM): matches the needs of users with the resources available on the grid.
- Worker Nodes (WN): group of machines on which jobs will be executed. It is also on the worker nodes that are stored the data from the Storage Element. This is usually a cluster of several computers.

For more details on the use of grid computing, see [15],[16].

## 5.6 Distributed programing and execution of our application on magrid

To launch our application on magrid, we started by installing Freefem++ on the grid with the help of researchers from the National Center for Scientific Research and Technology of morocco. Then we prepared our job for submission. To exploit the distributed architecture of the grid, we used a parametric job. Run a Parametric job is to run N jobs differing only by the value of a parameter which can indicate PARAM values. As we have two simulations, only the input file parameter changes. So PARAM shows the data file to use(see sensitivity.jdl of appendicies). To see [17] for magrid using.

## 5.7 Numerical results

To realize the sensitivity analysis, we randomly generated 10000 parameter set Ki, i = 1, ..., 8 following a uniform distribution on the interval [ai, bi] (range of parameter variations Ki, i = {1, ..., 8}). The reference values are those used in the simulation of groundwater level in 2000. We repeated two times the simulations. The estimated execution time on a personal computer hp intel (R) Core (TM) i5 CPU 540@2.53GHz M 2.53 GHz and 2 GB of RAM was 49 hours. On the grid, this time was reduced to 19 hours. The results are given in Figures 14 and 15.

By observing the sensitivity curves, we can see that four elements strongly influence the level of the water. These are the parameters 3 to 6 ie fractures 3 to 6. And particularly the fractures 3 and 6 (see Figure 1) whose the sensitivities are too high.

# 6 Conclusion

This work answers questions posed in the thesis of Mr Koussoubé[1]; it treats the numerical simulation of a flow in fractured porous media whose hydraulic conductivities are lower than those of the rock. We refined the mesh at the fractures that were treated as porous media. Our simulations have helped provide some answers on a real problem posed in the thesis of Mr Koussoubé[1]. A sensitivity analysis allowed identifying the parameters that influence a lot the solution. Collaboration with researchers of the National Center for Scientific Research and Technology of Morocco

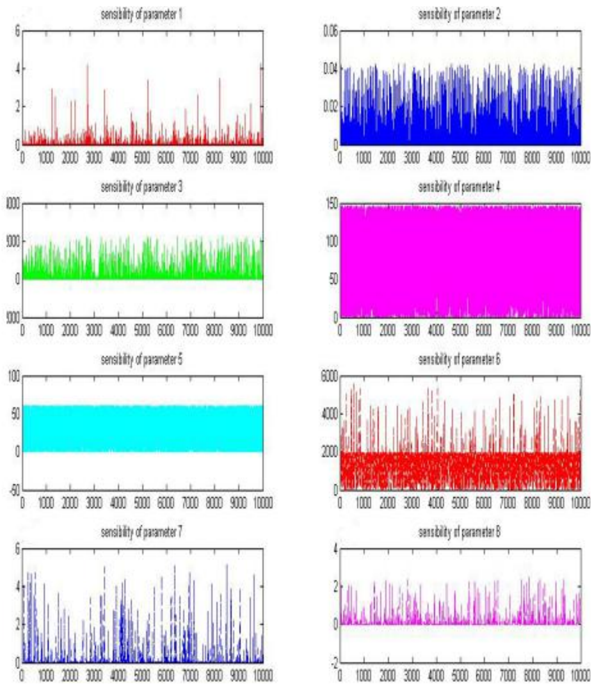


Figure 14: Analysis 2

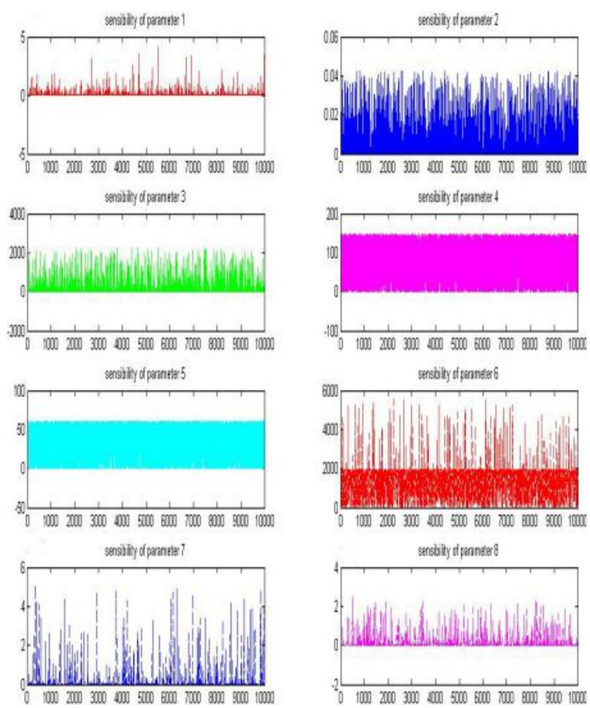


Figure 15: Analysis 3

and of the Meraka Institute of South Africain in the framework of project "brain gain" of the UNESCO-HP has allowed us to install the software for numerical Freefem++ on the Moroccan grid (margrid) and the South African grid (sagrid). This gave us more substantial computing resources for our calculations.
We are planning thereafter to parallelize our computer code to perform simulations on the transfer of pollutants in the water. This will undoubtedly help install bore wells in order drinking supply water of the population.

## Appendix

**freefem.sh**

```
#!/bin/bash
source $VO_MAGRID_SW_DIR/setenv_freefempp_v3.14.sh
FreeFem++ -nw $1
tar -czf output$2.tar.gz s1.dat s2.dat
s3.dat s4.dat s5.dat s6.dat s7.dat s8.dat
```

**sensitivity.jdl**

```
Executable="freefem.sh";
JobType = "Parametric";
Parameters = 3;
ParameterStart = 1;
ParameterStep = 1;
Arguments="sensitivity_PARAM_.edp _PARAM_";
InputSandbox={"freefem.sh","sensitivity_PARAM_.edp",
"parameter_PARAM_.dat"}; StdOutput="sensitivity_PARAM_.out";
StdError="sensitivity_PARAM_.err";
OutputSandbox={"sensitivity_PARAM_.out",
"sensitivity_PARAM_.err","output_PARAM_.tar.gz"};
MyProxyServer="myproxy.ct.infn.it";
```

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## References

[1] Y. Koussoubé, Hydrogéologie des séries sédimentaires de dépression piézométrique de Gondon(Bassin du Sourou)-I

Faso,Thèse de doctorat de l'université Pierre et Marie Curie, Spécialité: Hydrogéologie, juillet 2010.

[2] F. Hetch, O. Pironneau, FreeFem++, http://www.freefem.org.

[3] E. Ledoux, Modèles mathématiques en hydrogéologie, Centre d'Informatique Géologique ́Ecole Nationale Supérieure des Mines de Paris, 2003.

[4] C. Alboin, J. Jaffré, J. E. Roberts, Domain decomposition for some transmission problems in flow in porous media , In Numerical treatment of multiphase flows in porous media, (Beijing, 1999), volume 552 of Lecture Notes in Phys., pages 2234. Springer, Berlin, 2000.

[5] T. Arbogast, The double porosity model for single phase flow in naturally fractured reservoirs , In M.F. Wheeler., editor, Numerical simulation in oil recovery, volume 11 of the IMA volumes in Mathematics and its Applications, pages 23-45. Springers Verlag 1988.

[6] H. MUSTAPHA Simulation numérique de l'écoulement dans des milieux fracturés tridimensionnels, Thèse de l'Universitéde Rennes 1, 2005.

[7] G. de Marsily, Cours d'hydrogéologie ,Université Paris VI, Septembre 2004.

[8] C. Alboin, Deux outils mathématiques pour modéliser l'écoulement et le transport de polluants dans un milieux poreux saturé, Thèse de l'UniversitéParis IX Dauphine, 2000.

[9] P. G. Ciarlet , Introduction à l'Analyse Numérique Matricielle et à l'Optimisation, Masson, Paris, 1982.

[10] P. G. Ciarlet , The Finite Element Method for Elliptic Problems, North-Holland,1980.

[11] V. Martin, Simulations multidomaines des écoulements en milieu poreux, Thèse de l'UniversitéParis IX Dauphine,2004.

[12] C. Lauvernet , Assimilation variationnelle d'observations de télédétection dans les modèles de fonctionnement de la végétation: utilisation du modèle adjoint et prise en compte de contraintes spatiales, Doctorat de l'UniversitéJoseph Fourier-Grenoble 1, Avril 2005.

[13] H. E. Ngodock, Assimilation de données et Analyse de sensibilité: Une application à la circulation océanique., Doctorat de l'UniversitéJoseph Fourier-Grenoble 1, Avril 2005..

[14] I. Foster, What is the Grid ? A three point check-list, Chicago, Argonne National Laboratory and University of Chicago, 2002 .

[15] GILDA , gLite users tutorial, https ://grid.ct.infn.it/twiki/bin/view/GILDA/ UserTutorials.

[16] B. Jacob, Brown, K. Fukui, N. Trivedi, Introduction to Grid computing, Redbooks, 2005.

[17] CNRST Rabat, Utilisation Magrid , http://wiki.marwan.ma.

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