Cost-Benefit Analysis of a Redundant System with Inspection and Priority Subject to Degradation

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Abstract

This paper deals with the cost-benefit analysis of a system of two identical units-one is operative and the other is kept as cold standby. There is a single server who attends the system immediately whenever needed. The unit becomes degraded after repair. The server inspects the degraded unit at its failure to see the feasibility of repair. If the repair of the degraded unit is not feasible, it is replaced by new unit which gets priority in operation as well as in repair over the degraded unit. The system is considered in up-state if either of new/degraded unit is operative. The distributions of failure time of the units are taken as negative exponential while that of inspection and repair times are taken as arbitrary such as exponential distribution, Erlang distribution and Weibull distribution etc. Various reliability measures of system effectiveness are obtained by using semi-Markov process and regenerative point technique. The behavior of mean time to system failure (MTSF), availability and profit of the system have also been studied through graphs.

Keywords: *Redundant System, Inspection, Priority, Degradation and Cost-Benefit Analysis.*

1. Introduction

Recently, two unit standby systems have widely been studied because of their importance in modern business and industries. Various reliability engineers and scholars including the authors of References 2, 3, 5 and 6 have discussed such operating systems under these assumptions that each unit works as new after repair and there is no need to give priority to one unit over the other in operation and repair.

In fact, however, these assumptions cannot be imposed on every system due to different operating and repair characteristics. And, the unit may have increased failure rate after its repair by an ordinary server. In such a situation the unit becomes degraded after repair. Also, in some cases, the repair of the degraded unit is neither possible nor economical to the system due to its excessive use. Under such conditions, the degraded, now failed unit may be replaced by new one and this can be revealed by inspection. Malik et al. [2008] analyzed a system with two types of inspection subject to degradation. Further, the availability of a system can be increased by giving priority in operation and repair to one unit over the other. Chander [2005] analyzed reliability models introducing the concept of priority.

By considering all these facts, here a reliability model for a two-unit cold standby is under take for study. There is a single server who attends the system immediately whenever needed. The unit becomes degraded after repair. The server inspects the degraded unit at its failure to see the feasibility of repair. If the repair of the degraded unit is not feasible, it is replaced by new one. The new unit gets priority in operation as well as in repair over the degraded unit. The system is considered in up-state if either of new/degraded unit is operative. The distributions of failure time of the units are taken as negative exponential while that of inspection and repair times are taken as arbitrary. Various reliability measures of system effectiveness such as mean sojourn times, mean time to system failure (MTSF), steady state availability, busy period of the server, expected number of visits by the server and profit are obtained by using semi-Markov process and regenerative point technique. The behavior of MTSF, availability and profit of the system have also been studied through graphs drawn for a particular case.

The system of power generators in an air craft can be cited as a good example of the present system model.

2. Notations

E : Set of regenerative states

No : The unit is new and operative

Do : The unit is degraded and operative

NCs /DCs : The new/degraded unit in cold standby

p/q : Probability that repair of degraded unit is feasible/not feasible

λ/λ_1: Constant failure rate of new/degraded unit

$g(t)/G(t)$, $g_1(t)/G_1(t)$: pdf/cdf of repair time for new/degraded unit h(t)/H(t) : pdf/cdf of inspection time of the degraded unit

$NF_{ur}/NF_{UR}/NF_{wr}$: New unit is failed and under repair / under continuous repair from previous state /waiting for repair/continuously waiting for repair from previous state

$DF_{ur}/DF_{UR}/DF_{wr}$: Degraded unit is failed and under repair/under repair continuously from previous state/waiting for repair.

DF_{ui}/DF_{wi} /DF_{UI} /DF_{WI} : Degraded unit is failed and is under inspection/waiting for inspection/under inspection continuously from the previous state/ waiting for inspection continuously.

$q_{ij}(t), Q_{ij}(t)$: pdf and cdf of first passage time from regenerative state *i* to a regenerative state *j* or to a failed state *j* without visiting any other regenerative state in (0,t].

$q_{ij.kr}(t), Q_{ij.kr}(t)$: pdf and cdf of first passage time from regenerative state *i* to a regenerative state *j* or to a failed state *j* visiting state k,r once in (0,t].

$M_i(t)$: P[system up initially in state $S_i \in E$ is up at time *t* without visiting to any other regenerative sate]

$W_i(t)$: P[server is busy in the state S_i up to time *t* without making any transition to any other regenerative state or returning to the same via one or more non- regenerative states]

m_{ij} : Contribution to mean sojourn time in state $S_i \in E$ and non regenerative state if occurs before transition to $S_j \in E$.

®/© : Symbols for Stieltjes convolution/Laplace convolution

~|* : Symbols for Laplace Stieltjes Transform(LST)/Laplace Transform (LT)

'(desh) : Symbol for derivative of th function

The following are the possible transition states of the system model

S_0 = (No, NCs), S_1 = (No, NF_{ur}),
S_2 = (NF_{wr} ,NF_{UR}), S_3 = (No, DCs),
S_4 = (Do, NF_{ur}), S_5 = (DF_{wi}, NF_{UR}),
S_6 = (Do, DCs), S_7 = (Do, DF_{ui}),
S_8 = (Do, DF_{ur}), S_9 = (DF_{wi}, DF_{UI}),
S_{10} = (DF_{wi}, DF_{UR}), S_{11} = (DF_{ur}, DF_{WI}),
S_{12} = (No, DF_{ui}), S_{13} = (No, DF_{ur}),
S_{14} = (NF_{ur}, DF_{wi}), S_{15} = (NF_{ur},DF_{wr})
(1)

The states S_0, S_1, S_3, S_4, S_6, S_7, S_8, S_{12}, S_{13}, S_{14} and S_{15} are regenerative states while S_2, S_5, S_9, S_{10} and S_{11} are non-regenerative states. Thus E = {S_0, S_1, S_3, S_4, S_6, S_7, S_8, S_{12}, S_{13}, S_{14}, S_{15}}.The possible transition between states along with transition rates for the model is shown in Fig.-1.

3. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

$p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)\,dt$ as

$p_{01} = p_{34} = p_{67} = p_{14,7} = p_{15,8}$, $p_{13} = g^*(\lambda)$,
$p_{12} = 1-g^*(\lambda) = p_{14.2}$, $p_{46} = g^*(\lambda_1)$,
$p_{47.5} = 1-g^*(\lambda_1) = p_{45}$, $p_{7,3} = qh^*(\lambda_1)$,
$p_{7,8} = p\,h^*(\lambda_1)$, $p_{7,9} = 1-h^*(\lambda_1)$,
$p_{7,12.9} = [1-h^*(\lambda_1)]q$, $p_{7,7.9,11} = p[1-h^*(\lambda_1)]$,
$p_{8,6} = g_1^*(\lambda_1)$, $p_{8,10} = 1-g_1^*(\lambda_1) = p_{8,7.10}$,
$p_{12,13} = ph^*(\lambda)$, $p_{12,0} = qh^*(\lambda)$,
$p_{12,14} = 1-h^*(\lambda)$, $p_{13,3} = g_1^*(\lambda)$,
$p_{13,15} = 1-g_1^*(\lambda)$
(2)

For these transition probabilities, it can be verified that

$$p_{01}=p_{34}=p_{67}=p_{14,7}=p_{15,8}=p_{12}+p_{13}=p_{14.2}+p_{13}=p_{45}+p_{46}$$
$$=p_{46}+p_{47.5}=p_{7,3}+p_{7,8}+p_{7,9}=p_{7,3}+p_{7,8}+p_{7,12.9}+p_{7,7.9,11}$$
$$=p_{86}+p_{8,10}=p_{86}+p_{8,7.10}=p_{12,13}+p_{12,0}+p_{12,14}$$
$$=p_{13,3}+p_{13,15}=1$$
(3)

The mean sojourn times μ_i in state S_i are given by

$\mu_0 = \frac{1}{\lambda} = \mu_3$, $\mu_1 = \frac{1}{\lambda}[1-g^*(\lambda)]$,

$\mu_4 = \frac{1}{\lambda_1}[1-g^*(\lambda_1)]$, $\mu_6 = \frac{1}{\lambda_1}$

$\mu_7 = \frac{1}{\lambda_1}[1-h^*(\lambda_1)]$, $\mu_8 = \frac{1}{\lambda_1}[1-g_1^*(\lambda_1)]$,

$\mu_{12} = \frac{1}{\lambda}[1-h^*(\lambda)]$, $\mu_{13} = \frac{1}{\lambda}[1-g_1^*(\lambda)]$

(4)

The unconditional mean time taken by the system to transit from any state S_i
when time is counted from epoch at entrance into state S_j is stated as:

$m_{ij} = \int t\,dQ_{ij}(t) = -q_{ij}^{*\prime}(0)$ and

$$\mu_i = E(T) = \int_0^\infty P(T>t)\,dt = \sum_j m_{ij} \qquad (5)$$

where T denotes the time to system failure.
Thus

$m_{01} = \mu_0$, $m_{12} + m_{13} = \mu_1$,
$m_{13} + m_{14.2} = \mu_1^1$ (say), $m_{34} = \mu_3$,
$m_{45} + m_{46} = \mu_4$, $m_{46} + m_{47.5} = \mu_4^1$ (say),

$m_{67}=\mu_6$, $m_{7,8}+m_{7,3}+m_{7,9}=\mu_7$,
$m_{7,8}+m_{7,3}+m_{7,12.9}+m_{7,7.9,11}=\mu^1{}_{7}$,
$m_{86}+m_{8,10}=\mu_8$, $m_{86} + m_{8,7.10} = \mu^1{}_8$ (say),
$m_{12,13} + m_{12,0}+ m_{12,14}= \mu_{12}$
$m_{13,3}+m_{13,15}=\mu_{13}$,
(6)

4. Mean Time to System Failure

Let $\phi_i(t)$ be the cdf of the first passage time from regenerative state *i* to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:

$$\phi_i(t)=\sum_j Q_{i,j}(t)Ⓢ\phi_j(t)+\sum_k Q_{i,k}(t) \quad (7)$$

where *j* is an operative regenerative state to which the given regenerative state *i* can transit and *k* is a failed state to which the state *i* can transit directly.

Taking LST of Eq. (7) and solving for $\tilde{\phi}_0(s)$.
Using this, we have

$$R^*(s) = (1-\tilde{\phi}_0(s))/s \quad (8)$$

The reliability R(t) can be obtained by taking Laplace inverse transform of Eq. (8).
The mean time to system failure can be given by

$$\text{MTSF}(T_1) = \lim_{s\to 0} R^*(s) = \frac{N_{11}}{D_{11}} \quad (9)$$

Where

$N_{11}=(p_{78}m_{86}+p_{78}p_{86}m_{67}+m_{78}p_{86})+(p_{46}m_{73}+p_{46}p_{73}m_{34}$
$+m_{46}p_{73}+p_{46}m_{67}p_{73})+m_{01}[(p_{13}p_{45}+p_{12})(1-p_{78}p_{86})$
$-p_{12}p_{46}p_{73}+p_{46}p_{13}(p_{79}+p_{78}p_{810})] +[(p_{45}p_{13}+p_{12})$
$(-m_{86}p_{78}-p_{86}m_{78}p_{86}m_{67}p_{78})+(1-p_{86}p_{78})(m_{45}p_{13}$
$+m_{34}p_{45}p_{13}+p_{45}m_{13}+m_{12})+p_{12}p_{46}m_{73}+p_{73}p_{12}p_{46}m_{67}$ $+p_{73}p_{12}m_{46}$
$+p_{12}p_{46}m_{34}p_{73}+m_{12}p_{46}p_{73}+p_{13}p_{46}(m_{79}$
$+p_{8,10}m_{78}+m_{8,10}p_{78})+(p_{79}+p_{78}p_{8,10})(p_{13}p_{46}m_{67}+p_{13}m_{46}$
$+p_{13}p_{46}m_{34}+m_{13}p_{46})]$

and

$D_{11}=1-p_{46}p_{73}-p_{78}p_{86}$

5. Availability Analysis

Let $A_i(t)$ be the probability that the system is in up-state at instant *t* given that the system entered regenerative state *i* at *t=0*. The recursive relations for $A_i(t)$ are given by :

$$A_i(t)=M_i(t)+\sum_j q_{i,j}^{(n)}(t)©A_j(t) \quad (10)$$

where *j* is any successive regenerative state to which the regenerative state *i* can transit through $n\geq 1$ (natural number) transitions.
We have,

$M_0(t)=e^{-\lambda t}=M_3(t)$, $M_1(t)=e^{-\lambda t}\overline{G}(t)$,
$M_4(t)=e^{-\lambda_1 t}\overline{G}(t)$, $M_6(t)=e^{-\lambda_1 t}$,
$M_7(t)=e^{-\lambda_1 t}\ \overline{H}(t)$, $M_8(t)=e^{-\lambda_1 t}\overline{G}_1(t)$,
$M_{12}(t)=e^{-\lambda t}\ \overline{H}(t)$, $M_{13}(t)=e^{-\lambda t}$ $\overline{G}_1(t)$

(11)

Taking LT of Eq. (10) and solving for $A_0{}^*(s)$.
The steady-state availability of the system can be given by

$$A_0(\infty)=\lim_{s\to 0} s\,A_0^*(s)=\frac{N_{12}}{D_{12}} \quad (12)$$

Where

$N_{12}=p_{7,12.9}p_{12,0}(\mu_0+\mu_1)+[1-p_{7,7.9,11}-p_{12,14}p_{7,12.9}(p_{78}$
$+p_{7,12.9}p_{12,13}p_{13,15})](p_{13}\mu_3+\mu_4+p_{1,4.2}\mu_3(p_{73}$
$+p_{7,12.9}p_{12,13}p_{13,3})-\mu_6[(p_{46}p_{8,7.10}-p_{4,7.5}p_{8,6})(p_{78}$
$+p_{7,12.9}p_{12,13}p_{13,15})-p_{46}(1-p_{7,7.9,11}-p_{12,14}p_{7,12.9})]$
$+[\mu_7+p_{7,12.9}(\mu_{12}+\mu_{13}p_{12,13}+\mu_8(p_{78}+p_{7,12.9}p_{12,13}p_{13,15})]$

$D_{12}=\mu^1{}_7+\mu_{12}p_{7,12.9}+m_{14,7}p_{12,14}p_{7,12.9}+(\mu_8{}^1+p_{86}\mu_6)(p_{78}$
$+p_{7,12.9}p_{12,13}p_{13,15})+m_{15,8}p_{12,13}p_{7,12.9}p_{13,15}$
$+\mu_{13}p_{7,12.9}p_{12,13}+(\mu^1{}_{4}+p_{46}\mu_6)[(p_{73}+p_{7,12.9}p_{12,13}p_{13,3})$
$+p_{12,0}p_{7,12.9}]+\mu_3(p_{73}+p_{7,12.9}p_{12,13}p_{13,3})$
$+\mu_0p_{12,0}p_{7,12.9}+p_{12,0}p_{7,12.9}\ (\mu^1{}_1+p_{1,3}\mu_3)$

6. Busy Period Analysis for Server

Let $B_i(t)$ be the probability that the server is busy at an instant *t* given that the system entered regenerative state *i* at *t* = 0. The following are the recursive relations for $B_i(t)$

$$B_i(t)=W_i(t)+\sum_j q_{i,j}^{(n)}(t)©B_j(t) \quad (13)$$

where *j* is a subsequent regenerative state to which state *i* transits through $n\geq 1$ (natural number) transitions.
We have,

$W_1(t)=[e^{-\lambda t}+(\lambda e^{-\lambda t}©1)]\overline{G}(t)$,
$W_4(t)=e^{-\lambda_1 t}\overline{G}(t)+[(\lambda_1 e^{-\lambda_1 t}©1)]\overline{G}(t)$,
$W_7(t)=e^{-\lambda_1 t}\overline{H}(t)\ +[(\lambda_1 e^{-\lambda_1 t}©1)]\overline{H}(t)\ +(\lambda_1 e^{-\lambda_1 t}$
$©ph(t)©1)\overline{G}_1(t)$,
$W_8(t)=e^{-\lambda_1 t}\overline{G}_1(t)+[(\lambda_1 e^{-\lambda_1 t}©1)]\overline{G}_1(t)$,
$W_{12}(t)=e^{-\lambda t}\overline{H}(t)$,
$W_{13}(t)= e^{-\lambda t}\ \overline{G}_1(t)$,

$W_{14}(t)=$ $\overline{G}(t)=$ $W_{15}(t)$

(14)

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Taking LT of Eq. (13) and solving for $B_0^*(s)$ and using this, we can obtain the fraction of time for which the repairman is busy in steady state

$$B_0 = \lim_{s\to 0} s B_0^*(s) = \frac{N_{13}}{D_{12}} \quad (15)$$

$N_{13}=p_{7,12.9}p_{12,0}W_1^*(0)+W_4^*(0)[1-p_{7,7.9,11}-p_{12,14}p_{7,12.9}-(p_{78}+p_{7,12.9}p_{12,13}p_{13,15})]+[W_7^*(0)+p_{7,12.9}(W_{12}^*(0)+W_{13}^*(0)p_{12,13}+W_{14}^*(0)p_{12,14}+W_{15}^*(0)p_{12,13}p_{13,15})+W_8^*(0)(p_{78}+p_{7,12.9}p_{12,13}\,p_{13,15})]$

and D_{12} is already mentioned.

7. Expected Number of Visits

Let $N_i(t)$ be the expected number of visits by the server in (0,t] given that the system entered the regenerative state *i* at *t=0*. We have the following recursive relations for $N_i(t)$:

$$N_i(t) = \sum_j Q_{i,j}(t) \circledR [\delta_j + N_j(t)] \quad (16)$$

Where j is any regenerative state to which the given regenerative state *i* transits and δ_i=1, if *j* is the regenerative state where the server does job afresh, otherwise $\delta_i = 0$.

Taking LST of Eq. (16) and solving for $\tilde{N}_0(s)$.

The expected number of visits per unit time are given by

$$N_0 = \lim_{s\to 0} s\tilde{N}_0(s) = \frac{N_{14}}{D_{12}} \quad (17)$$

$N_{14}=p_{7,12.9}p_{12,0}+[1-p_{7,7.9,11}-p_{12,14}p_{7,12.9}-(p_{78}+p_{7,12.9}p_{12,13}p_{13,15})]p_{13}+p_{1,4.2}(p_{73}+p_{7,12.9}p_{12,13}p_{13,3})-[(p_{46}p_{8,7.10}-p_{4,7.5}p_{8,6})(p_{78}+p_{7,12.9}p_{12,13}p_{13,15})-p_{46}(1-p_{7,7.9,11}-p_{12,14}p_{7,12.9})]$

8. Cost-Benefit Analysis

Profit incurred to the system model in steady state is given by

$$P_1=K_1A_0-K_2B_0-K_3N_0$$

where K_1 = Revenue per unit up time of the system

K_2 = Cost per unit time for which server is busy

K_3 = Cost per visit by the server

9. Particular Case

In real life, various systems have constant failure and repair rates. Also, to improve the importance of results and to study the system graphically, here we assume inspection and repair times as exponentially distributed that is

Let us take $g(t)=\theta e^{-\theta t}$, $g_1(t)=\theta_1 e^{-\theta_1 t}$ and $h(t)=\alpha e^{-\alpha t}$

By using the non-zero elements p_{ij}, we get the following results:

MTSF$(T_1)=N_{11}/D_{11}$,

Availability$(A_0)=N_{12}/D_{12}$

Busy Period$(B_0)=N_{13}/D_{12}$,

Expected no. of visits$(N_0)=N_{14}/D_{12}$

Where

$D_{11}=[(\theta+\lambda_1)(\alpha+\lambda_1)(\lambda_1+\theta_1)-q\theta\alpha(\lambda_1+\theta_1)-p\theta_1\alpha(\lambda_1+\theta)]\;\lambda\lambda_1(\theta+\lambda_1)(\alpha+\lambda_1)(\lambda_1+\theta_1)(\theta+\lambda)^2$

$N_{11}=p\alpha\theta_1\lambda(\lambda_1+\theta)^2(\theta+\lambda)^2[(\alpha+\lambda_1)(2\lambda_1+\theta_1)+\lambda_1(\lambda_1+\theta_1)]+\alpha q\theta(\lambda_1+\theta_1)^2(\theta+\lambda)^2[\lambda\lambda_1(\alpha+2\lambda_1+\theta)+(\theta_1+\lambda_1)(\alpha+\lambda_1)(\lambda+\lambda_1)]+\lambda_1(\theta+\lambda_1)(\theta+\lambda)(\alpha+\lambda_1)(\lambda_1+\theta_1)[(\theta\lambda_1+\lambda(\lambda_1+\theta))((\alpha+\lambda_1)(\lambda_1+\theta_1)-p\alpha\theta_1)-q\alpha\theta\lambda(\lambda_1+\theta_1)+\theta^2(\lambda_1(\theta_1+\lambda_1)+p\alpha\lambda_1)]+(\theta_1+\lambda_1)^2(\alpha+\lambda_1)^2\lambda_1[\theta\lambda_1(\lambda+\lambda_1+\theta)(\lambda+\theta)+\theta\lambda_1\lambda(\lambda_1+\theta)+(\theta+\lambda_1)^2\lambda^2]+\lambda q\alpha\theta(\theta_1+\lambda_1)^2[\lambda\lambda_1(\theta+\lambda)(\alpha+2\lambda_1+\theta)+(\lambda+\theta)(\theta+\lambda_1)(\alpha+\lambda_1)(\lambda+\lambda_1)+\lambda\lambda_1(\theta+\lambda_1)(\alpha+\lambda_1)]+\lambda\lambda_1^2\theta^2(\theta+\lambda_1)(\lambda+\theta)[p\alpha(\alpha+\theta_1+2\lambda_1)+(\lambda_1+\theta_1)^2]+\theta^2(\alpha+\lambda_1)(\theta_1+\lambda_1)[(\lambda_1(\theta_1+\lambda_1)+p\alpha\lambda_1)(\theta+\lambda)(\lambda_1+\theta)(\lambda+\lambda_1)+(2\theta+\lambda_1+\lambda)\lambda_1\lambda]$

$D_{12}=[A\lambda_1\theta_1\theta\lambda(\alpha+\lambda)^2(\alpha+\lambda_1)^2(\theta+\lambda)(\theta_1+\lambda)^2(\theta+\lambda_1)(\theta_1+\lambda_1)+\lambda_1\theta_1\lambda^2(\alpha+\lambda_1)(\theta+\lambda)(\theta_1+\lambda)^2(\theta+\lambda_1)(\theta_1+\lambda_1)[\lambda_1\theta q+(\alpha+\lambda)\{B\theta(\lambda_1+\alpha)+q\lambda_1\}]+\lambda\theta p\alpha(\alpha+\lambda)(\alpha+\lambda_1)(\theta+\lambda)(\theta_1+\lambda)(\theta+\lambda_1)[\lambda_1(\theta_1+\lambda_1)+\theta_1^2][(\alpha+\lambda)(\theta_1+\lambda)+\lambda q\lambda_1]+p\lambda_1\theta_1\lambda\alpha(\theta+\lambda)(\theta_1+\lambda_1)(\theta+\lambda_1)[\theta(\alpha+\lambda)(\theta_1+\lambda)\{(\alpha+\lambda)(\theta_1+\lambda)+B\lambda(\alpha+\lambda_1)^2\}+\theta\lambda\lambda_1 q(\alpha+\lambda_1)(\theta_1+2\lambda+\alpha)+\lambda q\lambda_1(\alpha+\lambda)(\alpha+\lambda_1)(\theta_1+\lambda)]+q\lambda\theta_1\alpha(\alpha+\lambda)(\alpha+\lambda_1)(\theta+\lambda)(\theta_1+\lambda)(\theta_1+\lambda_1)[\theta^2+\lambda_1(\theta+\lambda_1)][(\alpha+\lambda+q\lambda_1)(\theta_1+\lambda)+p\lambda_1\theta_1]+q\lambda_1\theta_1\alpha\theta(\alpha+\lambda)(\alpha+\lambda_1)(\theta+\lambda)(\theta_1+\lambda)(\theta+\lambda_1)(\theta_1+\lambda_1)[p\lambda_1\theta_1+(\lambda+\alpha)(\theta_1+\lambda)]+\lambda_1\theta_1\lambda\theta(\theta_1+\lambda_1)(\theta+\lambda_1)(\theta+\lambda)[(\alpha+\lambda)^2(\theta_1+\lambda)^2 q\alpha+\alpha Bp\theta_1(\alpha+\lambda)(\alpha+\lambda_1)^2(\theta_1+\lambda)+\alpha pq\lambda_1\theta_1(\alpha+2\lambda+\theta_1)(\alpha+\lambda_1)]+\lambda_1\theta_1\theta(\alpha+\lambda_1)(\theta_1+\lambda)^2(\theta+\lambda_1)(\theta_1+\lambda_1)(\theta+\lambda)[\alpha q^2\lambda_1\lambda+\alpha q(\alpha+\lambda)\{B\lambda(\alpha+\lambda_1)+q\lambda_1\}]+\lambda_1\theta_1(\alpha+\lambda_1)(\theta_1+\lambda)^2(\theta+\lambda_1)(\theta_1+\lambda_1)(\alpha+\lambda)[\alpha q^2\lambda_1(\theta^2+\lambda(\theta+\lambda))]]/[\lambda_1\theta_1\theta\lambda(\alpha+\lambda)^2(\alpha+\lambda_1)^2(\theta+\lambda)(\theta_1+\lambda)^2(\theta+\lambda_1)(\theta_1+\lambda_1)]$

$N_{12}=[q^2\lambda_1^2\alpha(\lambda+\theta_1)(\theta+2\lambda)(\theta+\lambda_1)(\theta_1+\lambda_1)+\lambda_1(\theta_1+\lambda_1)[q(\alpha+\lambda)(\alpha+\lambda_1)(\theta_1+\lambda)-q\lambda\lambda_1(p\alpha+\lambda+\theta_1)][\theta(\theta+\lambda_1)+(\theta+\lambda)\lambda]+\lambda\alpha q\lambda_1(\theta+\lambda_1)(\theta_1+\lambda_1)[(\alpha+\lambda)(\theta_1+\lambda)+\lambda_1 p\theta_1]-\lambda(\theta+\lambda)[\lambda_1\alpha p(\theta-\theta_1)\{(\alpha+\lambda)(\theta_1+\lambda)+\lambda_1\lambda q\}-\theta(\theta_1+\lambda)(\theta_1+\lambda_1)\{(\alpha+\lambda)(\alpha+\lambda_1-p\lambda_1)-q\lambda_1\lambda\}]+\lambda\lambda_1(\theta+\lambda)(\theta+\lambda_1)[(\alpha+\lambda)(\theta_1+\lambda)(\theta_1+\lambda_1)+q\lambda_1(\theta_1+\lambda+p\alpha)(\theta_1+\lambda_1)$

$+p\alpha\{(\alpha+\lambda)(\theta_1+\lambda)+\lambda q\lambda_1\}]]/[\lambda_1\lambda(\alpha+\lambda)(\alpha+\lambda_1)(\theta+\lambda)(\theta_1+\lambda)(\theta+\lambda_1)(\theta_1+\lambda_1)]$

$N_{13}=[\theta_1 q^2\lambda_1\alpha^2(\lambda+\theta_1)(\alpha+\theta_1)(\theta_1+\lambda_1)+\theta_1 q\alpha(\alpha+\theta_1)(\theta_1+\lambda_1)[(\alpha+\lambda)(\alpha+\lambda_1)(\theta_1+\lambda)-\lambda\lambda_1(\theta_1+\lambda+p\alpha)]+[\theta_1\theta(\alpha+\lambda)(\alpha+\lambda_1)(\theta_1+\lambda)\{(\theta_1+\lambda_1)(\alpha+\theta_1)+p\alpha\lambda_1\}+p\alpha^2\theta(\alpha+\theta_1)(\theta_1+\lambda_1)\{(\alpha+\lambda)(\theta_1+\lambda)+q\lambda\lambda_1\}+\theta_1 q\lambda_1\alpha(\lambda+\theta)(\alpha+\theta_1)(\theta_1+\lambda_1)\{(\lambda+\theta_1)+p\alpha\}]]/[\theta\theta_1\alpha(\lambda+\theta_1)(\alpha+\theta_1)(\theta_1+\lambda_1)(\alpha+\lambda)(\alpha+\lambda_1)]$

$N_{14}=[q^2\lambda_1\alpha(\theta+\lambda)(\theta_1+\lambda)(\theta+\lambda_1)(\theta_1+\lambda_1)+q\theta(\theta+\lambda_1)(\theta_1+\lambda_1)[(\alpha+\lambda)(\alpha+\lambda_1)(\theta_1+\lambda)-\lambda\lambda_1(\theta_1+\lambda+p\alpha)]+\alpha q\lambda(\theta+\lambda_1)(\theta_1+\lambda_1)[(\alpha+\lambda)(\theta_1+\lambda)+p\lambda_1\theta_1]-(\theta+\lambda)[\lambda_1 p\alpha(\theta-\theta_1)\{(\alpha+\lambda)(\theta_1+\lambda)+q\lambda\lambda_1\}-\theta(\theta_1+\lambda_1)(\theta_1+\lambda)\{(\alpha+\lambda)(\alpha+\lambda_1-p\lambda_1)-q\lambda\lambda_1)\}]]/[(\alpha+\lambda)(\alpha+\lambda_1)(\theta+\lambda)(\theta_1+\lambda)(\theta+\lambda_1)(\theta_1+\lambda_1)]$

$A=[p(\alpha+\theta_1)(\alpha+\lambda_1)^2-\alpha^2 p(\alpha+\theta_1+\lambda_1)]/[(\theta_1\alpha(\alpha+\lambda_1)^2]$

and $B=[q\lambda_1(2\alpha+\lambda_1)]/[\alpha(\alpha+\lambda_1)^2]$

10. Conclusion

Fig.-2 shows that mean time to system failure decreases rapidly with increase the failure rates λ and λ_1. But it increases with increase the repair rates θ and θ_1 for fixed values of other parameters. The behavior of availability and profit of the system model are shown in Fig.-3 and 4 respectively. From these figures it can be seen that availability and profit of the system model decrease with the increase of failure rates λ and λ_1. However, their values increase if repair rates θ and θ_1 increase. It is also observed that system becomes more available to use and thus profitable if the degraded unit at its failure is replaced by new one. Hence, on the basis of the results obtained for a particular case it is concluded that the concepts of priority for operation and repair to new unit over the degraded unit and replacement of the degraded unit at its failure are economically beneficial to use.

11. References

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State Transition Diagram

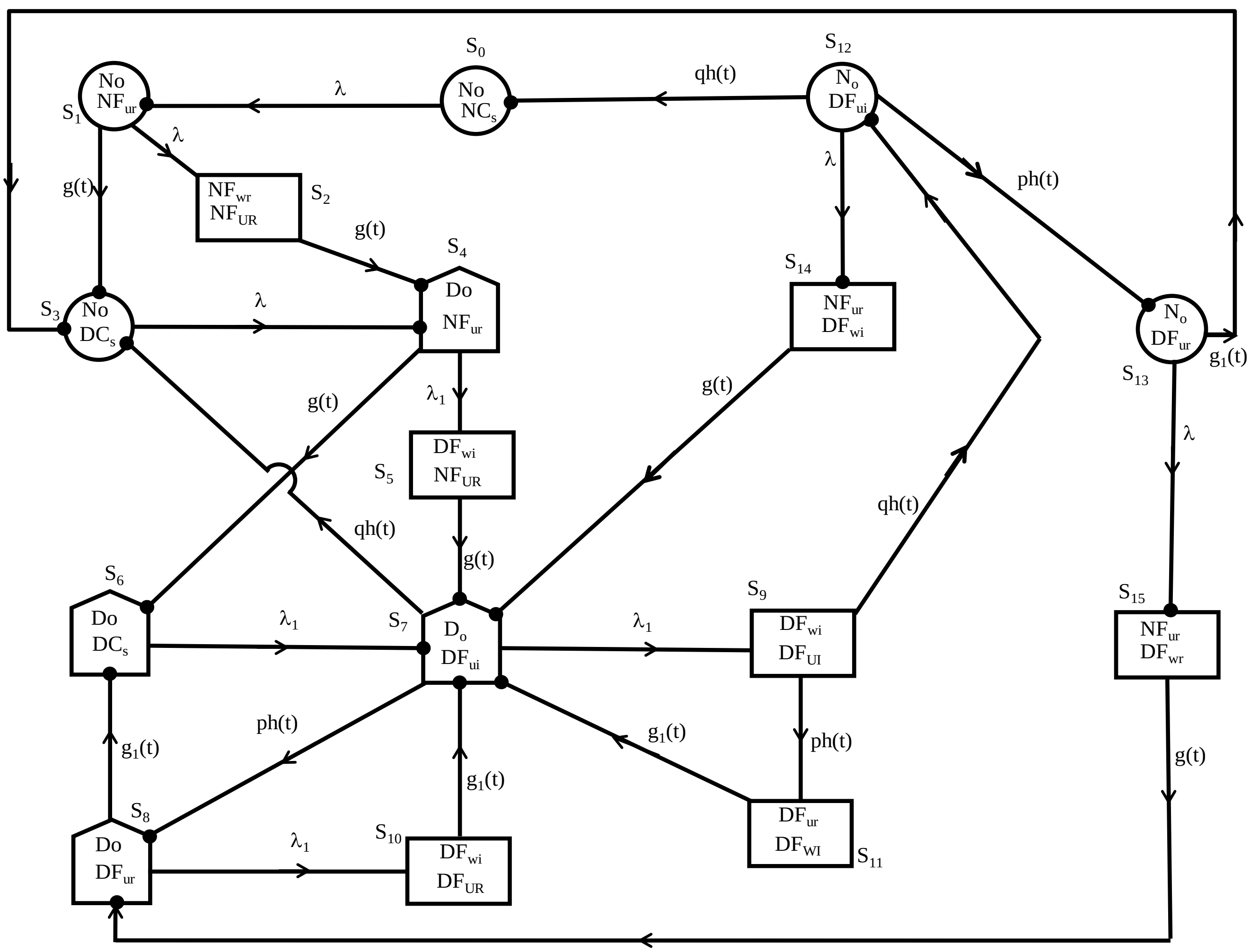
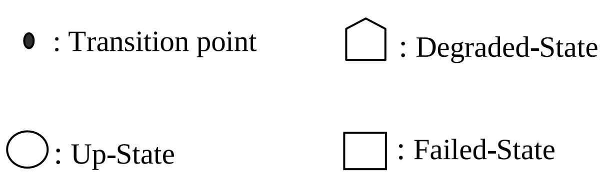
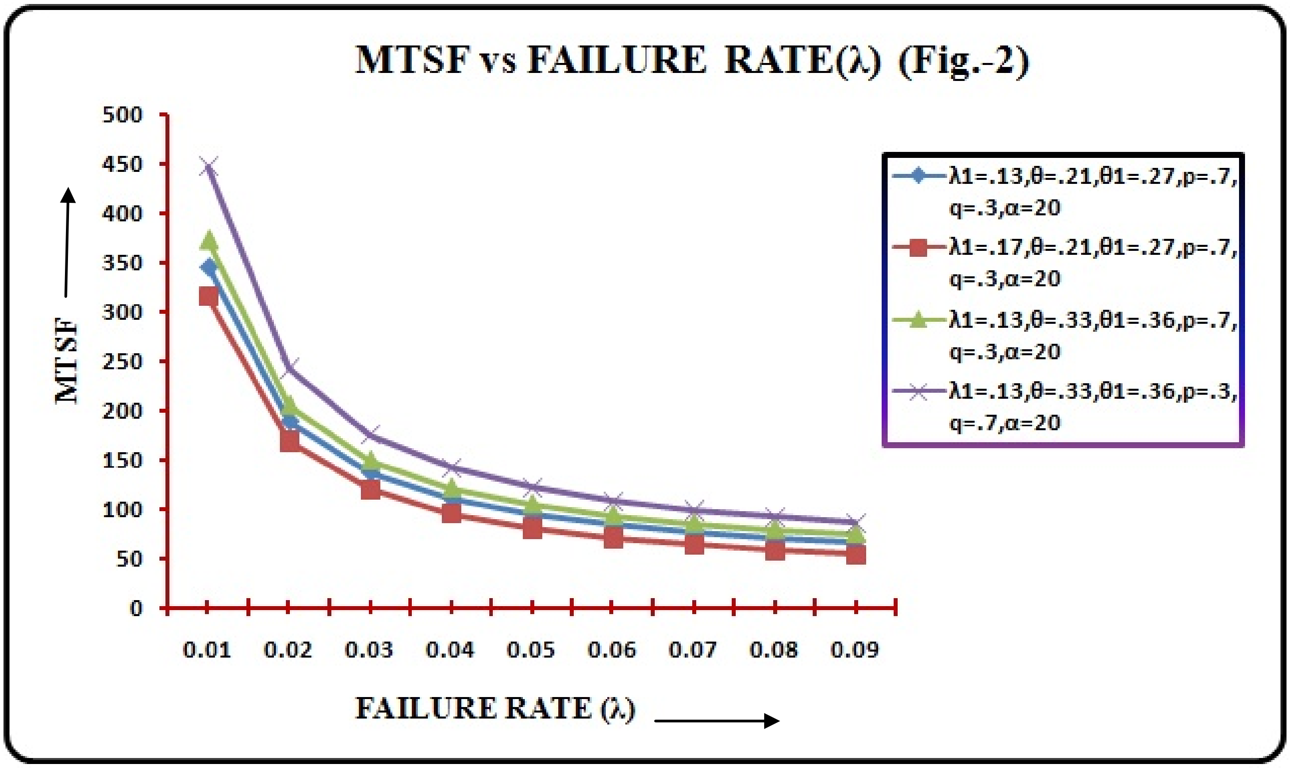
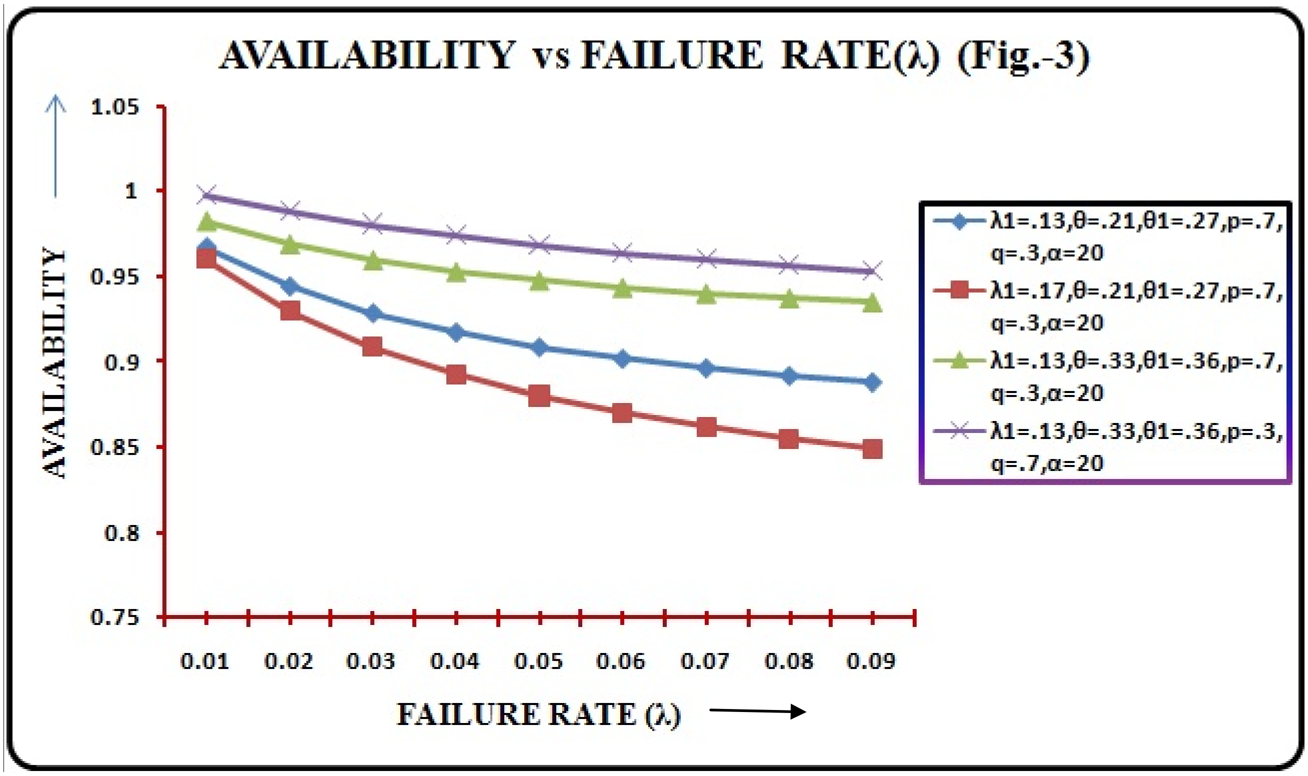
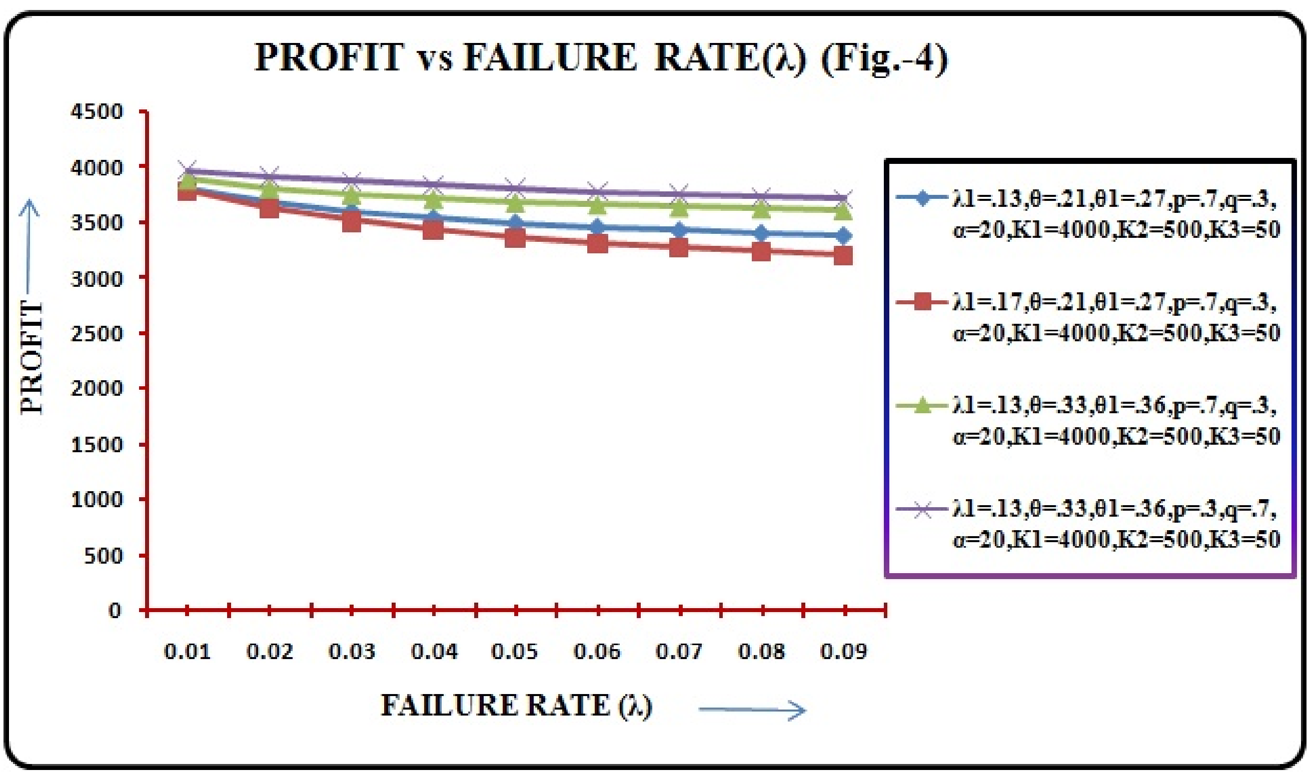


Fig.-1




MTSF vs FAILURE RATE(λ) (Fig.-2)
MTSF
500
450
400
350
300
250
200
150
100
50
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
FAILURE RATE (λ)
λ1=.13,θ=.21,θ1=.27,p=.7, q=.3,α=20
λ1=.17,θ=.21,θ1=.27,p=.7, q=.3,α=20
λ1=.13,θ=.33,θ1=.36,p=.7, q=.3,α=20
λ1=.13,θ=.33,θ1=.36,p=.3, q=.7,α=20


AVAILABILITY vs FAILURE RATE(λ) (Fig.-3)
AVAILABILITY
1.05
1
0.95
0.9
0.85
0.8
0.75
0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09
FAILURE RATE (λ)
λ1=.13,θ=.21,θ1=.27,p=.7, q=.3,α=20
λ1=.17,θ=.21,θ1=.27,p=.7, q=.3,α=20
λ1=.13,θ=.33,θ1=.36,p=.7, q=.3,α=20
λ1=.13,θ=.33,θ1=.36,p=.3, q=.7,α=20


PROFIT vs FAILURE RATE(λ) (Fig.-4)
PROFIT
4500
4000
3500
3000
2500
2000
1500
1000
500
0
0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09
FAILURE RATE (λ)
λ1=.13,θ=.21,θ1=.27,p=.7,q=.3, α=20,K1=4000,K2=500,K3=50
λ1=.17,θ=.21,θ1=.27,p=.7,q=.3, α=20,K1=4000,K2=500,K3=50
λ1=.13,θ=.33,θ1=.36,p=.7,q=.3, α=20,K1=4000,K2=500,K3=50
λ1=.13,θ=.33,θ1=.36,p=.3,q=.7, α=20,K1=4000,K2=500,K3=50