Comparison of the elastic coefficients and Calculation Models of the Mechanical Behavior one- Dimensional Composites

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Abstract

In this paper, we present the mechanical models that are devoted to the elastic properties of one-dimensional composite. We have compared the equivalent coefficients of one-dimensional composite, resulting from different models. The validation of the results was made through effective experiments on a one-dimensional composite consisting of fibers of alumina and a matrix of aluminum. This study allows us to better assess the rigidity of composite structures, and the results of calculation of the mechanical behavior, resulting from each model. It appears that the finite element model is the best suited to the approach of a refined conception. For more insurance, we have chosen to make our calculations by finite element in the three-dimensional case, using the technique of homogenization by asymptotic development.

Keywords: *homogenization method, finite elements method, material composites*

1. Introduction

The composite materials are used in the majority of the high mechanical performances structures. Nowadays their presence in all the technical fields, justifies this importance. In this work we present the mechanical models are devoted to the elastic properties of composites. There is a basic and practical reason, for that to most of the applications structural of composite materials, instantaneous resistance of these materials is not the decisive criterion for the dimensioning, which must face-as well as behavior with the impact, of deterioration or ageing under static load. The other reason is that, while the elastic properties can be the subject of rather correct forecasts, the characteristics of the instantaneous ruptures and a fortiori the characteristics of the differed ruptures, behavior to the shock, deterioration or all characteristics strongly dependent on qualitative aspects, like the conditions of manufacture and environment, cannot be the subject of sufficiently general quantitative precise details. The objective of this work is initially to locate the state of progress the work of the mechanical modeling of the composite structures and to determine the models available with their possibilities, their limitations [4, 11, 18]. It provides a catalogue of concrete results, either resulting from the bibliography or calculated by our method starting from the theoretical concepts which make it possible to define models of increasing complexity which will be useful, bases for the modeling of other types of composites.

2. Principle of the equivalent medium

It is completely unimaginable to consider a direct calculation which requires a discretization of the whole composite structure. Generally, the shape of the composite, the geometric complexity of its basic constituents and the tedious nature of the mesh by finite element do not allow taking consideration of the composite geometry. To circumvent these difficulties, many researchers have proposed models that appeal to rational micromechanical notions. Thus, we can predict the mechanical performance of composites and their ultimate resistance. This is done through the nature and the provision of basic components. In this case, the composite area is considered as homogeneous but anisotropic: this is the principle of equivalent homogeneous area. This principle is based on replacing a real, highly heterogeneous composite material by equivalent homogeneous material. The approach is based on the notion of the details macroscopic behavior, neglecting the influence of the microscopic or those inherent in the composition of the composite. To conduct this analysis of behavior, it is necessary to define an elementary sample, represent a volume of the structure and its heterogeneity, on a microscopic scale. This representative volume plays the classic role of continuous mechanics of volume element. This idea involves a concept of statistics average, in which the actual constitution of the material is idealized in considering a continuous medium. Once the model of continuity is admitted, the concept of homogeneity is deducted. However, as close as the disparity between the different components is too large or that geometric continuity of heterogeneity is not acquired, it will be required have homogenization models of percolation or the methods of self consistent [4, 8, 16]. The mechanical characteristics of the material and analysis properties are defined from a basic of elementary volume and representative, small enough to account for microscopic composite to study, but large enough as to the fiber diameters and their separating distances. This representative volume sets the state of local stress, induced by external displacement of loads and boundary conditions [19]. This state represents requires of the volume, placed inside the real composite. The solution of the problem is made through the analysis of the stresses and deformations in the elementary volume, to establish relations of elastic behavior. Generally, an exact solution requires a complex procedure which is not always applicable without introducing simplifying assumptions. This is what limits the scope of this type of analysis prediction.

3. Empirical methods

We observe, in the literature, two types of models interested in the prediction of the composite materials elastic behavior. The first models are global and allow estimating the mechanical performance on a global scale. These models are limited to the determination of equivalent elasticity coefficients. In contrast, the second family of models is local and allows, not only, to determine the equivalent elasticity coefficients, but it also allows to go back on a microscopic scale to locate the possible interactions that can exist between the fibers and the matrix and identify zone of damage. We take into account the work of Hashin-Shtrikman [9,10], [13, 14] Hill, Hashin-Rosen [11], Christensen [5] and Hashin [12] who have token an approach to the fibrous composite case, in which fibers can have sections of circular, elliptical or square shapes and can be distributed randomly or following a hexagonal and identical diameters. The developed models are based on a variational method using the elastic strain energy theorems, in a representative elementary volume of the composite. Through other works in this framework, there are many semi-empirical methods in which weights coefficients, are introduced to adapt to a restricted application environment. In this context, it was noted the models developed by Tsai [21] [17, 18] which are related to the composite Puck reinforced one-directional fiber of square or circular and distributed sections following repetitive rectangular or hexagonal shapes. To account for potential irregularities distribution or alignment of the fibers, Tsai introduced a factor of contiguity of the fibers. However the empirical determination of these factors limits the scope of certain models in calculation of forecast. The distinction between the models is located in the strict framework of their assumptions or on their adjustment more and less empirical by correcting coefficients, so that results are consistent with the experience.

4. Homogenization of periodic structures composites.

The 1970s knew a new orientation of the homogenization technique of composite materials. The first work generated mathematical progress on the side. In particular, the use of functional analysis for the formulation of the mechanics heterogeneous environments problems. The work of Sanchez - Palencia [20], Duvaut [6], chaffoui [7] showed that the theory of homogenization can provide an excellent mathematical response for the determination of the mechanical properties of composites with periodic structure [1, 2, 3]. By supposing that the period is small compared of dimensions studied area, going to the extremities allows to calculate macroscopic and microscopic sizes exactly which are often inaccessible to the experiment. We noted that this technique is well adapted to calculations by finite elements.

5. Applications

It is about comparing the equivalent coefficients of a one-dimensional composite, resulting from the various models. The validation of the results was made through experimental tests carried out on a one-dimensional constituted an aluminum of fibers alumina of matrix. The rate of impregnation out of matrix is 50 %. The fibers are parallel to the longitudinal direction of the E_3 module and their sections are circular of ray $R = 5{,}75\ \mu m$. Mechanical characteristics of the components are:

- For the matrix: $E_m = 70\ Gap$, $\nu_m = 0{,}33$
- For the fibre: $E_f = 380\ GPa$, $\nu_f = 0{,}25$.

For more insurance, we have chosen to make our calculations of finite elements in the three-dimensional case, by using the technique of homogenization by asymptotic development, where the basic period form parallelepiped (Fig. 5. c), and the two-dimensional case which form the base period a square (Fig. 5. b). However geometrical symmetries compared to y_1 and y_2, lead to orthotropic isotropic and make it possible to reduce the field of the useful mesh for calculation, with the quarter of the period.(Fig. 6, a. 2D problem) et (Fig. 6. b. 3D problem).

Table 1 presents a comparison of the elastic modules resulting from the tests by ultrasonic evaluation non destructive (test 1) and from the mechanical tests (test 2), with the results, resulting from the empirical formulations, on the other hand, and with calculations of homogenization by finite elements We constant in the analysis of these results that the model calculations of homogenization by finite elements is the best performing, with compared the empirical methods.

6. Influence basic materials.

To determine the influence of the mechanical characteristics of the materials that constitute the composite, we varied the module of elasticity of fiber (fig.1-2) and of the matrix (fig.3-4). The variations of the elastic modules show the same observations mentioned above with knowing, the influence of coefficient of adjacency in the formulated of (Tsai) and the behavior of the results of the calculations by finite elements compared to the empirical formulae. Nevertheless, we note that the slopes of the curves are significantly different and depend on the module of the component that has been varied (E_f for fiber or E_m for the matrix).

Table 1: Comparison of the elastic coefficients.

	E_3 *(GPa)*	$E_1 = E_2$ *(GPa)*	G_{12} *(GPa)*	$G_{13} = G_{23}$ *(GPa)*	ν_{12}	$\nu_{13} = \nu_{23}$
Test 1	233,3	141,9	51,8	54,6	0,368	0,291
Test 2	223,0	138,0	- - - - - - -	52,0	- - - - - - -	0,310
Puck	225,0	181,8	- - - - -	73,9	- - - - -	0,290
Hashin	225,0	125,4	- - - - - - -	54,9	- - - - - - -	0,290
Tsai	213,8	154,2	- - - - - - -	72,9	- - - - - - -	O,278
Withney	225,1	154,2	- - - - - - -	54,9	- - - - - - -	0,291
F. E. M.	223,5	140,5	50,1	55,1	0,371	0,286

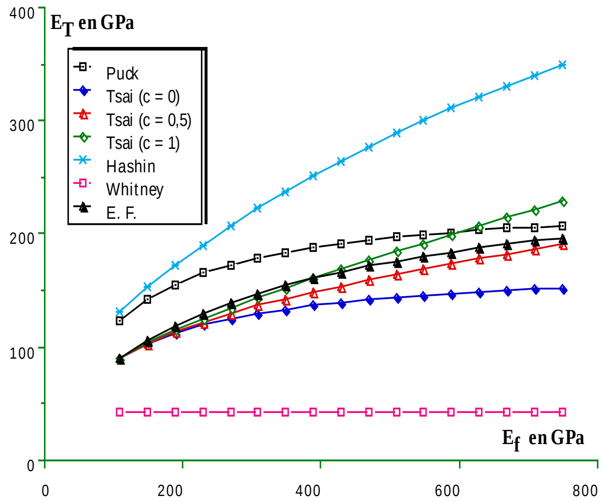

Fig. 1 *Variation of the module* E_L *in function* E_f

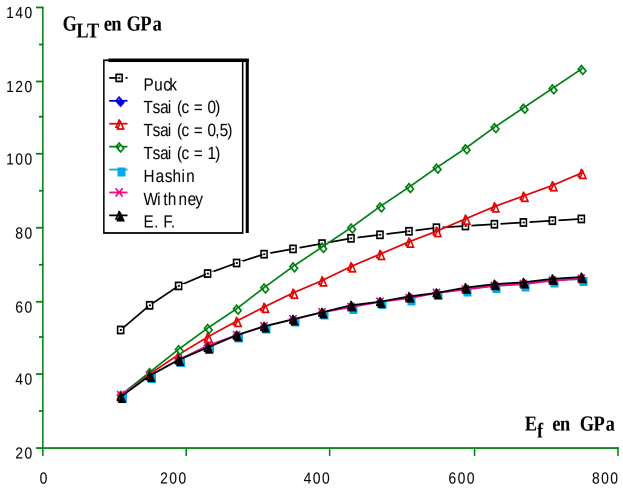

Fig. 2 *Variation of the module* G_{LT} *in function* E_f

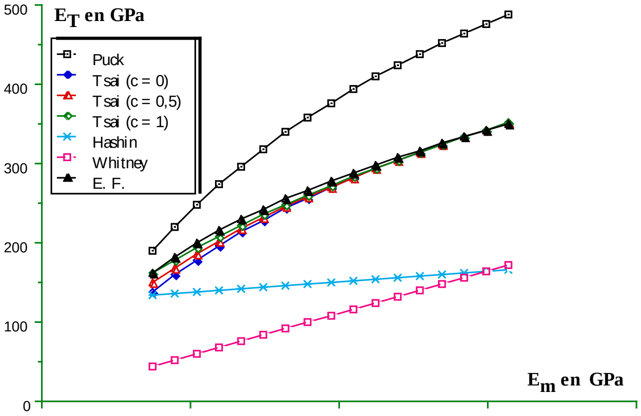

Fig. 3. *Variation of the module* E_T *in function* E_m

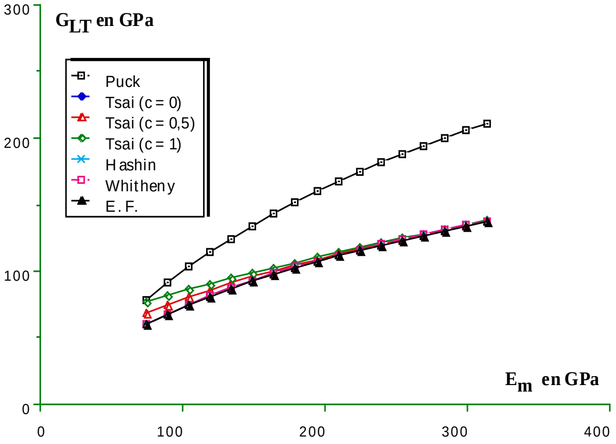

Fig. 4. *Variation of the module* G_{LT} *in function* E_m

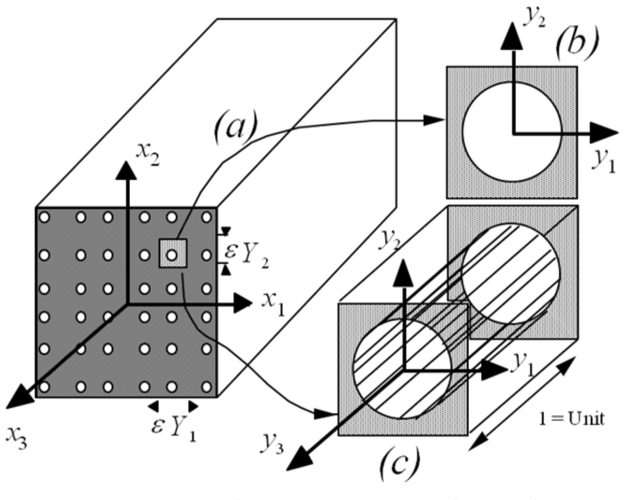

Fig.5 Geometry composite of one-dimensional and definition of the period

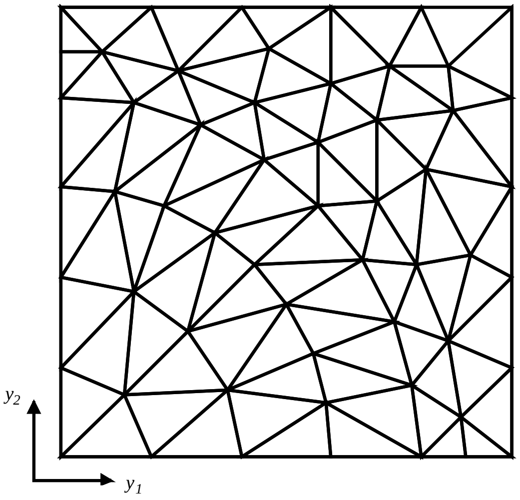

Fig. 6. a Mesh quarter of the period (2D problem)

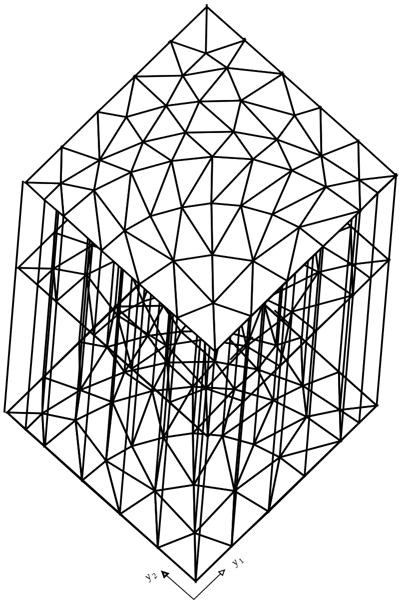

Fig. 6. b Mesh quarter of the period (3D problem)

7. Conclusions

This study allows to better assessing the composite rigidity of the structures, and the results calculation of the mechanical behavior of each model forecasting. It appears that the finite element model the best adapted to the approach of a refined conception. We constant in the analysis of these results that the model of calculations of homogenization by finite elements is the best, compared with the empirical methods.

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