A new method for classification of Brachiopods based on the radon transformation

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Abstract

Brachiopods have a lateral outline which is quite important in systematic studies. It is often assessed by a qualitative evaluation and linear measurements, which are not clear enough and precise for describing the shape of the shell and its changes

In this paper we propose a new method for classification of fossils based on the radon transform from their greyscale image. We take the case of brachiopods which has Complex shapes. We use an adaptation of Radon transform called R-transform which is invariant to common geometrical transformations. Each shape is described by R_{3D} transform. We consider the grayscale image as a set of cuts obtained from successive binarization for each gray level in image, and for each segmentation we compute the R-transform then we obtained the R_{3D} transform. The advantages of the proposed method are robustness to noise, and invariant to common geometrical transformations scale, translation and rotation.

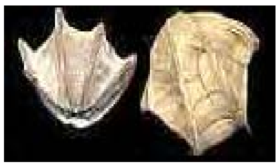
Keywords: Brachiopod, *Fossil, Classification, Radon transform, R-transform, Grayscale image, R_{3D} transform, Shape recognition.*

1. Introduction

Technological advances and development gigantic of storage capacity push geologists thinking of automating the task of classification of fossil species. That an important interest in palaeontological studies. On the one hand, they make it possible to understand the biodiversity in its morphological dimension. On the other hand, they highlight the morphological transformations undergone during the biological evolution. Historically, the form of was encircled by a purely descriptive approach based on the qualitative evaluations of the morphological change starting from simple images. This approach was replaced gradually by the biometric methods having leads to the populated design of the fossil species. The variables used in such methods are linear dimensions, angles, surfaces and ratio or combination of dimensions. But, these biometric descriptors are insufficiently informative since they give only one approximate quantitative representation of the form and its changes [1], [2], [3], [4].Then we used the Fourier analysis which consists in approximating the shape by a goniometrical function defined by a sum of terms of sine and cosine [5], [6], [7], [8], [9], [10]. This function is broken up into a series of amplitude of harmonics and phases or into a series of coefficient of Fourier being useful like variables for the quantitative analyses. But this method is valid right for the forms regular, indeed when morphologies become complex, it is not more possible to use such descriptors. In this paper, we propose a new method for classification of Brachiopods based on the Radon transform [11], [12], [13], [14] from their grayscale image. We use an adaptation of Radon transform called R-transform [15], [16] which is invariant to common geometrical transformations. Each shape is described by R_{3D} transform. We considered the grayscale image as a set of cuts obtained from successive binarization for each gray level in image, and for each segmentation we compute the R-transform then we obtained the R_{3D} transform. The advantages of the proposed method are robustness to noise, and invariant to common geometrical transformations scale, translation and rotation. This article is organized in the following way: In the first section we defined the concept of the Radon transform. The R-transform and 2D R-transform are described in section 3. The R_{3D} transform is defined in section 4 and the measure of similarity between two shapes (Brachiopod) is defined in section 5.

Cyclothyris vespertilio

Anathyris ezquerai



Cheirothyris fleuriausa

Fig.1 Images of some Brachiopod families

2. Radon transform

The Radon transform of an image $f(x,y)$ is determined by a set of projections of the image along lines taken at different angles θ. The resulting projection is the sum of the intensities of the pixels in each direction. The result is a new image $R_f(\rho,\theta)$. Therefore, the Radon transform can be written as [17]:

$$R_f(\rho,\theta) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\delta(\rho - x\cos(\theta) - y\sin(\theta))dxdy \qquad (1)$$

Where $\delta(.)$ is the Dirac delta-function ($\delta(t)$ = 1 if t = 0 and 0 elsewhere), $\theta \in [0,\pi[$ and $\rho \in]-\infty,+\infty[$. In other words, R_f is the integral of f over the line $L_{(\rho,\theta)}$ defined by $\rho = x\cos(\theta) + y\sin(\theta)$. As show in the figure (2).

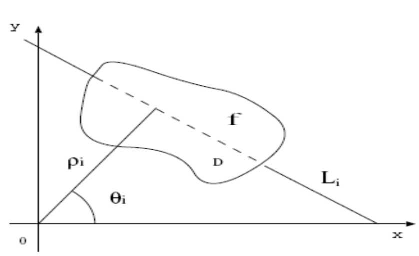


Fig. 2 Radon transform

There are two distinct Radon transforms. The source can either be a single point or it can be an array of sources as shown in (Figure 3). The method discussed in this paper uses an array of sources.

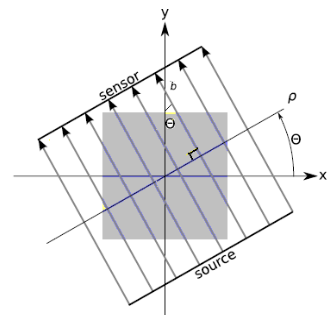


Fig. 3 Radon transforms uses an array of sources

The source and sensor are rotated about the center of the object. For each angle θ the density of the pixel the ray from the source passes through is accumulated at the sensor. This is repeated for a given set of angels, usually from $\theta \in [0,180[$. The angel 180 is not included since the result would be identical to the angel 0. The equation of the summation line is given as $y = ax + b$. As can be seen by using trigonometry, the inclination is $a = -\cos(\theta)/\sin(\theta)$ and the intersection with the y axis is $b = \rho/\sin(\theta)$. These parameters are determined for each combination of θ and ρ . In order to reduce the number of calculations necessary the maximum and minimum of either x or y are determined. ρ_{max} is the size of the diagonal of the image. The value of x can be real, to resolve the problem we used a linear interpolation. So the algorithm is:

```
For k from 1 to 180
  θ = kπ/180, Compute a = −cos(θ)/sin(θ)
   For ρ from 1 to ρ max
    Compute b = ρ/sin(θ) ,Determine  ymin and  ymax
      For y from ymin  to  ymax
         Compute x = (y − b)/a , Rf(ρ,θ) += f(x,y)
      End
   End
End
```

We tested our algorithm on a binary image shown in Figure (4). The figure (5) present the sinogram of Radon transform used an array of sources. We compared our implementation with the result obtained using the Radon transform found in MATLAB as show in figure (6). This is nearly identical the difference is presumably due to using a different interpolation.

Fig. 4 Binary image

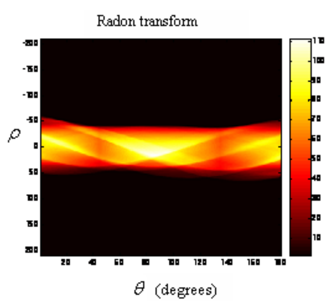


Fig. 5 result obtained with our implementation

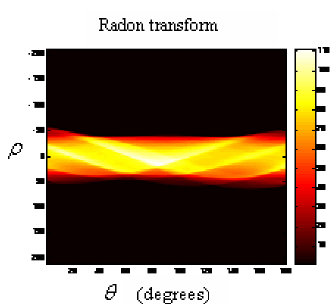


Fig. 6 result obtained with Radon transform found in MATLAB

Radon transform has some interesting properties relating to the application of affine transformations. We can compute the Radon transform of any translated, rotated or scaled image, knowing the Radon transform of the original image and the parameters of the affine transformation applied to it. This is a very interesting property for shape recognition because it permits to distinguish between transformed objects, but we can also know if two objects are related by an affine transformation by analyzing their Radon transforms. Let's see these properties:

Periodicity

$$R_f(\rho,\theta) = R_f(\rho,\theta + 2k\pi) \qquad (2)$$

For any integer k the period is 2π.

Symmetry

$$R_f(\rho,\theta) = R_f(-\rho,\theta \pm \pi) \qquad (3)$$

Translation of a vector $u_0 = (x_0, y_0)$: $R_f(\rho - x_0\cos\theta - y_0\sin\theta, \theta)$. A translation of f results in the shift of its transform in the variable ρ by a distance equal to the projection of the translation vector on the line $\rho = x\cos(\theta) + y\sin(\theta)$.

Rotation by angle θ_0 **:** $R_f(\rho, \theta + \theta_0)$. Implies a shift of the radon transform in the variable θ .

Scaling of α : $\frac{1}{|\alpha|}R_f(\alpha\rho,\theta)$. A scaling of f results in a scaling of both the ρ coordinates and the amplitude of the transform.

3. R-transform

R-transform has been developed by Tabbone in [15], [16], [18]. Its principle is simple: It consists to do for a given value of theta (i.e. within of the same of column the Radon matrix), the sum of squared elements. Can be written as:

$$R_T = \int_{-\infty}^{+\infty} R_f^{\,2}(\rho,\theta)\,d\rho \qquad (4)$$

Where R_f is the Radon transform of the function f .

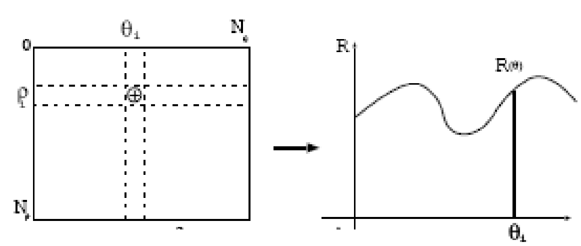


Fig. 7 definition of the R-transform

We applied here a Radon transform used an array of sources. The study of this R-transform allows us to define the following properties:

- **Periodicity:**

$$R_T(\theta \pm \pi) = \int_{-\infty}^{+\infty} R_f{}^2(\rho, \theta \pm \pi) d\rho = R_T(\theta) \qquad (5)$$

The R-transform is periodic and the period is π.

- **Rotation:**

For a rotation of the shape by an angle θ_0, the R-transform is:

$$\int_{-\infty}^{+\infty} R_f{}^2(\rho, \theta + \theta_0) d\rho = R_T(\theta + \theta_0) \qquad (6)$$

So a rotation of the shape by an angle θ_0 implies a translation of the R-transform of θ_0.

- **Translation:**

For a translation of vector $u_0 = (x_0, y_0)$ the R-transform is:

$$\int_{-\infty}^{+\infty} R_f{}^2((\rho - x_0\cos(\theta) - y_0\sin(\theta)), \theta) d\rho = \int_{-\infty}^{+\infty} R_f{}^2(\nu, \theta) d\nu \qquad (7)$$
$$= R_T(\theta)$$

We set $\nu = \rho - x_0\cos(\theta) - y_0\sin(\theta)$

The R-transform is invariant under a translation of f by a vector $u_0 = (x_0, y_0)$

- **Scaling:**

For a scaling $\alpha > 0$ the R-transform is :

$$\frac{1}{\alpha^2}\int_{-\infty}^{+\infty} R_f{}^2(\alpha\rho, \theta) d\rho = \frac{1}{\alpha^3}\int_{-\infty}^{+\infty} R_f{}^2(\nu, \theta) d\rho \qquad (8)$$
$$= \frac{1}{\alpha^3} R_T(\theta)$$

we set $\nu = \alpha\rho$

A zoom (before or back) of α generates a zoom of α^3 for R-transform.

4. R_{3D} - transform

The principle of 2D R-transform is applied to the binary image a distance transforms (After binarization of the grayscale image). A distance transform of a binary image specifies the distance from each pixel to the nearest non-zero pixel. The result of the distance transform is a gray level image and we capture both the internal structure and the boundaries of the shape. There are different families of distance transformation see [19], [20] for more information.

Given the distance transform of a shape, the distance image is segmented into n equidistant levels to keep the segmentation isotropic. For each distance level, pixels having a distance value superior to that level are selected and at each level of segmentation, an R-transform is computed [15]. The Figure (9) present the result obtained The X-axis present the angle θ in the Radon transform and the Y-axis reports the number of cuts in the distance transform.

Fig. 8 binary shape and his Chamfer distance transform

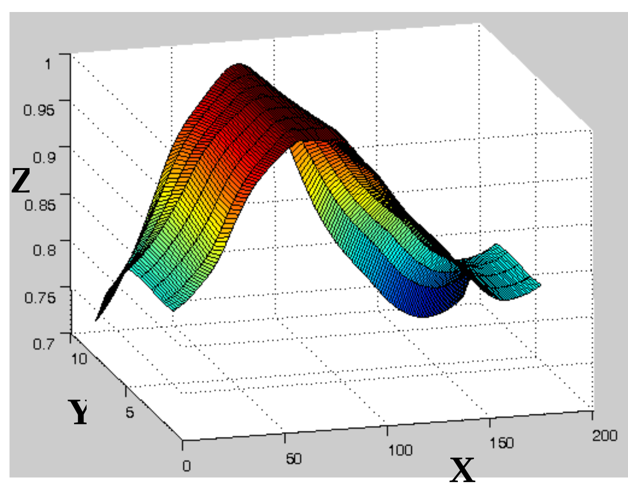


Fig. 9 surface visualisation of the 2D R-transform

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Generally 2D R-transform is sufficient to obtain accurate results by considering only the shape of the object. It is poorly adapted for greyscale objects recognition.

To overcome this problem we used a transform called R_{3D} transform. We considered the grayscale image as a set of cuts obtained from successive binarization for each gray level in image.
Set O an object composed of k gray level and I_i is a binary threshold i cut applied to the object O. We have $R_{3D} = \bigcup \{R_{I_i}\}_{i=1,\ldots,K}$ with R_{I_i} is the R- transform computed on the binary image I_i. By definition an R_{3D}-transform is a set of R-transform checking separately the properties of R-transform described previously
and as the photometry is invariant to rotation, translation and scale, R_{3D} transform preserve the properties of a conventional R-transform ie the invariance to rotation, translation and scale. We normalized by the volume of each R_{3D} transform.

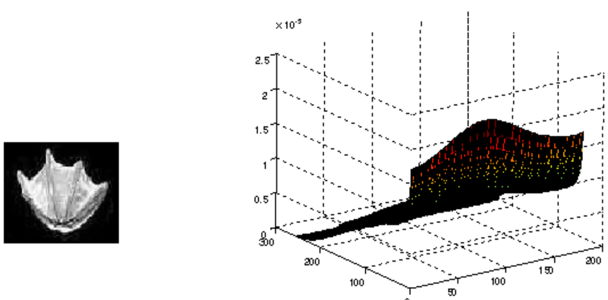

Fig. 10 Cheirothyris fleuriausa image and his R_{3D} transform

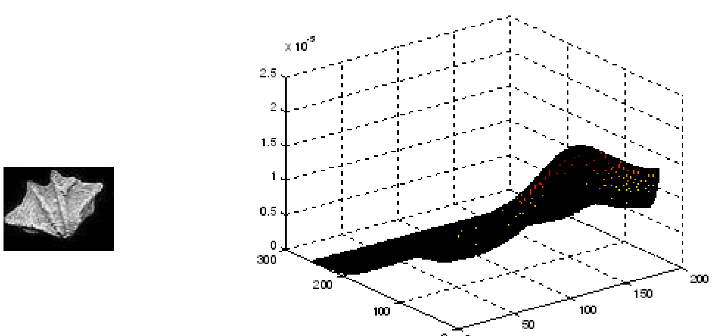

Fig. 11 Anathyris ezquerai image and his R_{3D} transform

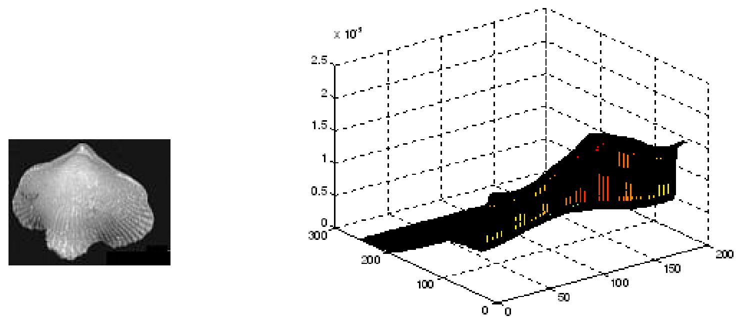

Fig. 12 Cyclothyris vespertilio image and his R_{3D} transform

5. Classification of the Brachiopod

Each shape (Brachiopod) is described by 3D R-transform. Consider two shapes A et B, R^A_{3D} and R^B_{3D} are the normalized R_{3D}. To measure the similarity between two shapes we used the ratio of similarity RS, the Shift of θ cyclyque are applied to R^B_{3D}, and RS is obtained by maximizing the index (min divided by max). that is insensitive to object rotation and scaling. This ratio is expressed as a percentage calculated between R^A_{3D} and R^B_{3D} is defined as:

$$RS = 100 \max_{x \in [0,p[} \left\{ \frac{\sum_{\theta=0}^{p} m^{AB}(\theta, x)}{\sum_{\theta=0}^{p} M^{AB}(\theta, x)} \right\}$$

With p is the nomber of orientations

and $$m^{AB}(\theta, x) = \sum_{i=1}^{k} \min(R^A_{I_i}(\theta), R^B_{I_i}(\theta + x))$$

$$M^{AB}(\theta, x) = \sum_{i=1}^{k} \max(R^A_{I_i}(\theta), R^B_{I_i}(\theta + x))$$

6. Conclusion

In this paper we presented a new method for classification of the Brachiopods based on the Radon transform from their grayscale image. We used an adaptation of Radon transform called R-transform, we considered the grayscale image as a set of cuts obtained from successive binarization for each gray level in image. The proposed method is robustness to noise, and invariant to common geometrical transformations scale, translation and rotation. The proofs of transformation invariance including translation, scaling and rotation are provided. Our method gives satisfactory and encouraging results.

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