An Adaptive Notch Filter For Noise Reduction and Signal Decomposition

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Abstract

Detection, estimation and filtering of desired signal in the presence of noise are some of the most common and practical problems in the analysis of time series. The analysis in the time domain often involves the comparison of two different signals and stochastic signals are usually more profitably analyzed in the time domain. Extraction of signal are considerably improves quality of signal. In many signal processing applications is to decompose an original signal into its primitive or fundamental constituents and to perform simple operations separately on each component, thereby accomplishing extremely sophisticated operations by a combination of individually simple operations. An adaptive notch filter is able extract the desired signal from noisy signal. In this paper proposed filter is parallely connected adaptive notch filter which is able to extract the fundamental frequency from the noise corrupted signal and each filter able decompose 'n' sinusoid which is harmonically related to its constituent component.

Keywords: *Adaptive notch filter, signal decomposition, estimation of fundamental frequency, steady state performance.*

1. Introduction

The term *estimator* or *filter* is commonly used to refer to a system that is designed to extract information about a prescribed quantity of interest from noisy signal. Fundamental frequency estimation has many practical applications in various branches of engineering. Among these applications are active noise and vibration control in helicopters, cancellation of periodic disturbances in control system, elimination of power line noise in ECG signals.

Notch filter is able to extract the desired sinusoid component of a given signal provided that the signal frequency remains constant. The method of suppressing a sinusoid interference which corrupting an information bearing signal is to use a fixed *notch filter* tuned to the frequency of interference. A fixed notch filter may eliminate the noise when it is centered exactly at the frequency for which the filter was designed. But what if the notch is required to be very sharp and the interfering sinusoid is known to drift slowly? Clearly, then, we have a problem that calls for an adaptive solution. When the signal is non stationary that is fundamental frequency of the signal varies with time to time then the notch filter fails to remove noise from the signal. If any change in the input frequency then an adaptive notch filter which is capable of changing the notch frequency accordingly by tracking the frequency variations of the input signal. By such a system we mean one that is self designing in that the adaptive filter relies for its operation on a recursive algorithm, which makes it possible for the filter to perform satisfactorily in an environment where complete knowledge of the relevant signal characteristics is not available.

2. Motivation

The proposed adaptive notch filter in this paper is originated from Regali's algorithm. He proposed lattice-based discrete time adaptive IIR notch filter [1]. He proposed novel and efficient second order lattice structure. The filtered error and regressor signal necessary for an adaptive implementation which is available from single, numerically robust. The algorithm features favorable convergence properties, such as unbiased frequency estimation, improved tracking behavior compared to competitive designs, and improved reliability for extrimal input frequencies.

Regalia's algorithm was later adopted for continuous-time by Bodson [2]. He proposed two algorithms for rejection of sinusoidal disturbances with unknown frequency. The first is an indirect algorithm where the frequency of the disturbance is estimated, and the estimate is used in another adaptive algorithm that adjusts the magnitude and phase of the input needed to cancel the effect of disturbance. The second is direct algorithm that uses the concept of a phase locked loop is also presented in which frequency estimation and disturbance cancellations are performed simultaneously.

The modified version of Bodson's algorithm [3] was proposed by Hsu *et al.* He solved the problem of global frequency estimation. The proposed algorithm is a scaled and normalized ANF. The new ANF was analyzed in terms of its stability, convergence and tuning. Despite a quite formidable complexity of its dynamical behavior, some conclusive results were established. Some preliminary results about the integration of the new ANF in noise cancellation systems are reported.

Hsu's stability analysis is guaranteed only when the forcing signal is pure sinusoidal. In some practical applications, such as periodic disturbances in active noise control, and voltage and current harmonics in the presence of nonlinear loads in power systems. The strength of this analysis is that it permits the presence of harmonic components in the input signal and is not limited to pure sinusoidal signals.

3. Frequency estimation and harmonic extraction

The dynamic behavior of the ANF of is characterized by the following set of differential equations:

$$\begin{aligned} \ddot{x} + \theta^2 x &= 2\zeta(\theta^2 y(t) - \theta\dot{x}) \\ \dot{\theta} &= -\gamma x(\theta^2 y(t) - \theta\dot{x} \end{aligned} \qquad (1)$$

In equation (1), $y(t)$ is the input signal, θ represents the estimated frequency, ζ real positive number which determines "depth of notch", γ real positive number which determines "adaptation speed". In the modification in the given equations are the input signal $y(t)$ is scaled by θ instead of θ^2. Thus the ANF equation (1) changes to,

$$\begin{aligned} \ddot{x} + \theta^2 x &= 2\zeta\theta(y(t) - \dot{x}) \\ \dot{\theta} &= -\gamma x\theta(y(t) - \dot{x}) \end{aligned} \qquad (2)$$

Very often, when it exhibits some periodicity, a signal is modeled by a single or a combination of multiple sinusoids given by,

$$y(t) = \sum_{i=1}^{n} A_i \sin(i\omega_0 t + \varphi_i) = \sum_{i=1}^{n} y_i(t) \qquad (3)$$

where A_i, φ_i and ω_0 are real unknown parameters. In such a modelling, signal characteristics may also vary with time. Estimation of frequency and extraction of the individual sinusoidal components of such a signal provide useful information about the signal and therefore, provide means for signal analysis. Introducing an algorithm which capable of decomposing such a signal into its individual frequency components. A close observation of the filter dynamics in equation (2) is a resonator $\ddot{x} + \theta^2 x = 0$ that is forced with the error signal $e(t) = y(t) - \dot{x}$. The regressor signal $x(t)$ and the error signal $e(t)$ incorporate in the θ update law in equation (3). The term θ in both equations is for scaling. Also, the steady state error signal tends to zero and $\dot{x}$ is an estimate for the input signal $y(t)$. These key ideas are used for a new structure to estimate the fundamental frequency of periodic signal and extract its individual constituting harmonics as follows.

The ith componentof the signal in equation (3) satisfies,

$$\ddot{x}_i + i^2\omega_0^2 x_i = 0$$

Thus, a filter dynamic to extract ith component may be proposed as,

$$\ddot{x}_i + i^2\theta^2 x_i = 2\zeta_i\theta[y(t) - \textstyle\sum_{l=1}^{n} \dot{x}_l], \quad i = 1,2,\dots n \qquad (4)$$

where, θ is an estimate of ω_0. The error signal which forms the force function is redefined as,

$$e(t) = y(t) - \sum_{l=1}^{n} \dot{x}_l$$

The ω_0 is the frequency of the first component, i.e. x_1. Therefore, x_1 is used as the regressor signal and the update law for frequency estimation is proposed as,

$$\dot{\theta} = -\gamma x_1\theta[y(t) - \textstyle\sum_{l=1}^{n} \dot{x}_l] \qquad (5)$$

Rewriting the equation set (4) and (5) in terms of the redefined error signal $e(t)$ yields the resultant equations for the proposed signal analysis method as follows:

$$\begin{aligned} \ddot{x}_i + i^2\theta^2 x_i &= 2\zeta_i\theta e(t), \qquad i = 1,2,\dots n \\ \dot{\theta} &= -\gamma x_1\theta e(t) \end{aligned} \qquad (6)$$

In equation (6), ζ_i and γ are real positive numbers and they determine the behavior of the ith filter and the θ update law in terms of (steady-state) accuracy and (transient) convergence speed. Figure. 1. and Figure. 2. shows that a general configuration of the proposed dynamics of equation (6). The detailed implementation block diagram of the ith filter path is shown in Figure. 2. where the error signal $e(t)$ is applied to each filter and the update law of equation (6) is employed to force the error signal to zero. Corresponding output of the ith filter is $\dot{x}_i$. Both hardware and software implementation of the proposed parallel structure are feasible and can take advantages of both pipelining and parallel computing. For the ith filter dynamic and in the steady-state, the output is that is the ith component of the input signal.

$$\dot{x}_i = A_i \sin(i\omega_0 t + \varphi_i)$$

This means that the Figure. 1. separates the input signal components in a manner that the ith component is made available by the ith filter. This feature desirable for a variety of real-time applications, for example, extraction and subsequent elimination of one or multiple harmonic components of the input signal in active noise cancellation schemes or active power filtering applications.

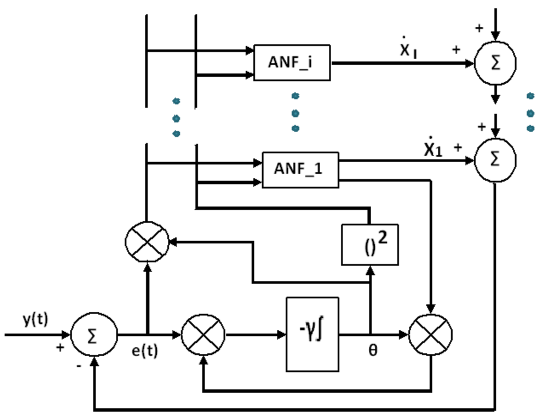

Figure.1. Block diagram of proposed algorithm.

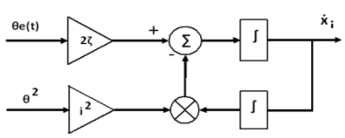

Figure. 2. Details of the ith parallel filter.

4. Performance Evaluation

The performance of ANF based analysis simulated on computer using Matlab/Simulink software. The initiatory performance and tracking features are examined in the simulation. A structure shown in Figure 1 with $n = 8$ is simulated and constructed in Matlab/Simulink that is, to extract the first eight constituting components of the signal as well as the fundamental frequency ω_0. A set of values are $\zeta_i = 0.5, i = 1,2, \ldots 8, \gamma = 250$ are choosen for simulatons. The input signal consists of fundamental frequency with third, fifth and seventh harmonics as,

$$y(t) = \sin(\omega_0 t + \phi_1) + 0.5\sin(3\omega_0 t + \phi_3) + 0.2\sin(5\omega_0 t + \phi_5) + 0.4\sin(7\omega_0 t + \phi_7) \quad (1)$$

in which $\omega_0 = 30\, rad/sec$ and the initial phase angle ϕ_i's are selected randomly between 0 and $2\pi\, rad$. The initial conditions of all the integrators are set to zero except the one associated with the fundamental frequency which is set to its nominal value, i.e., $\omega_0 = 30\, rad/sec$. The responses of system to the input signal of (7) are shown in Figure. 3. The eight extracted constituting components are shown in Figure. 4. a. to Figure 4. h respectively. The error signal $e(t) = y(t) - \sum_{n=1}^{6} y_i(t)$ is shown in Figure. 5. a. and the estimated fundamental frequency is shown in Figure. 5. b.

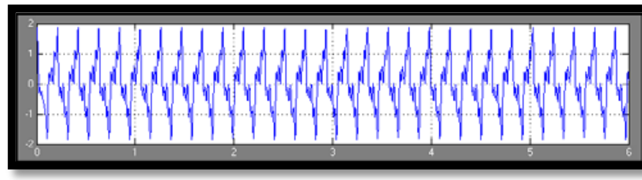

Figure. 3. Sketch of input signal.

a) Extracted signals:

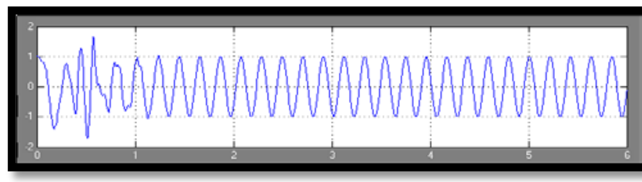

Figure. 4. *a*. Fundamental frequency.

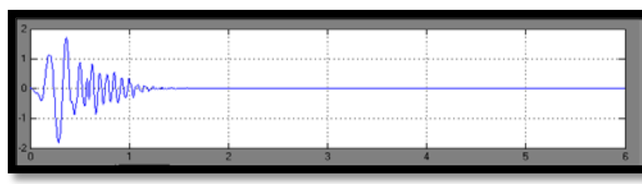

Figure. 4. *b*. Second component.

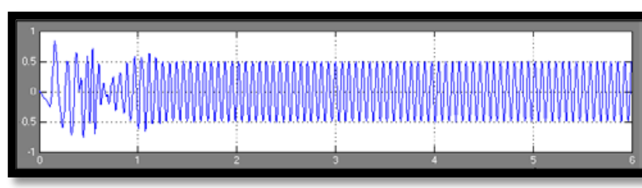

Figure. 4. *c*. Third component.

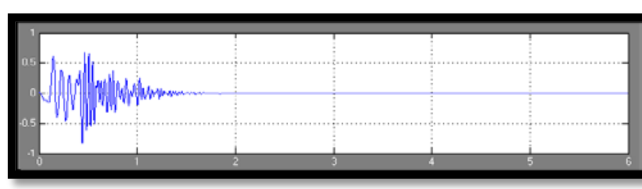

Figure. 4. *d*. Fourth component.

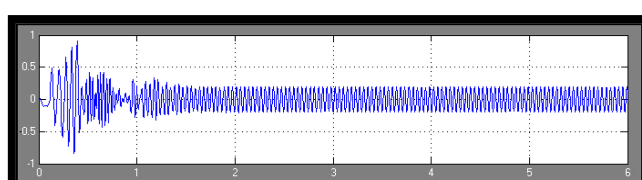

Figure. 4. *e*. Fifth component.

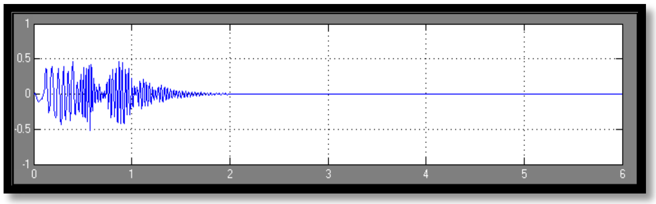

Figure. 4. *f*. Sixth component.

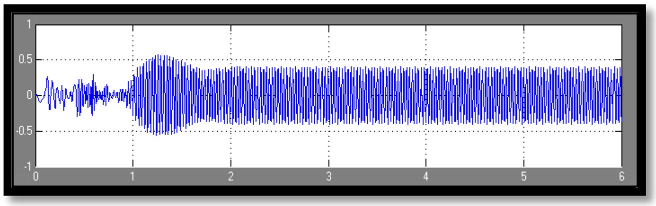

Figure 4. *g*. Seventh Component.

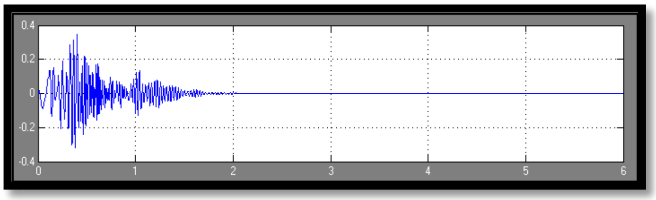

Figure 4. *h*. Eighth Component.

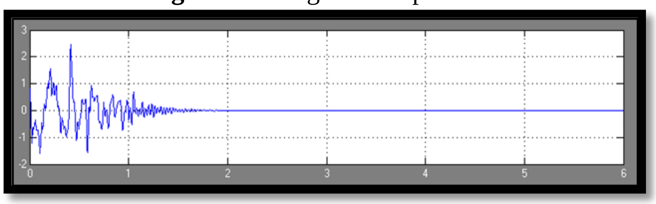

Figure. 5. *a*. Error signal.

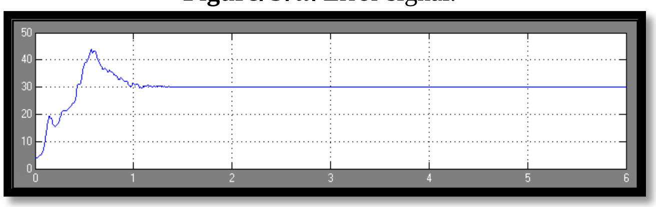

Figure. 5. *b*. Estimated frequency.

b) Tracking Performance:

Tracking performance of proposed algorithm is examined. If a step change from 30 *to* 40 *rad/sec* in the fundamental frequency of the input signal, then the step change is faithfully detected. The error signal, the actual and the estimated signals are shown in Figure. 6.

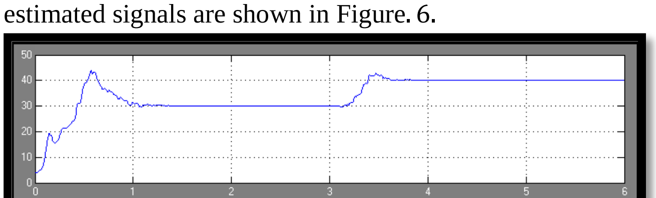

Figure. 6. *a*. Tracking: estimated signal.

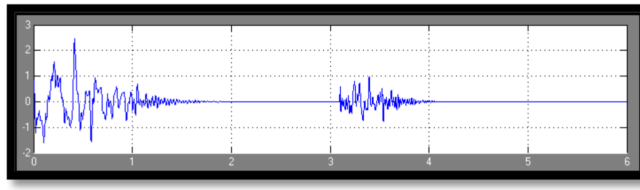

Figure. 6. *b*. Error signal.

c) Tracking multiple changes in parameters of input signal:

The third case study of this section verifies the capability of the presented method in tracking multiple simultaneous changes in the parameters of the input signal. The changes are following,

1) The amplitude of the fundamental component changes from 1 to 0.8
2) The amplitude of the third component changes from 0.5 to 0.3
3) A fourth component with amplitude 0.2 (and random phase) is introduced.
4) The amplitude of the fifth component changes from 0.2 to 0.4
5) The amplitude of the seventh component changes from 0.4 to 0.6
6) The fundamental frequency changes from 30 to 28 rad/s.

The extracted constituting components are shown in Figure 7 and they confirm desired tracking of the variables. The estimated signal and the error signal are also shown in Figure 8.

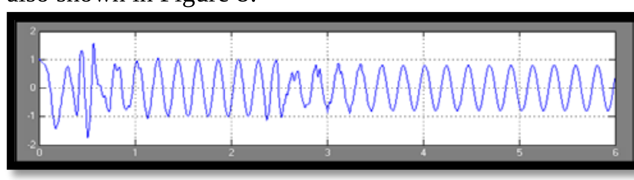

Figure 7.a. Fundamental frequency.

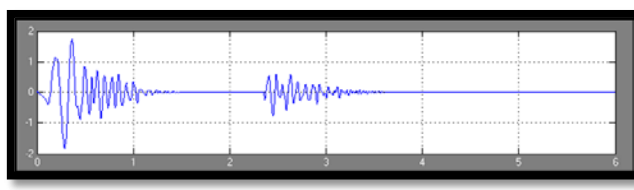

Figure 7.b Second Component.

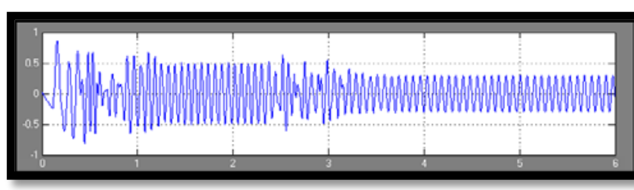

Figure 7.c Third Component.

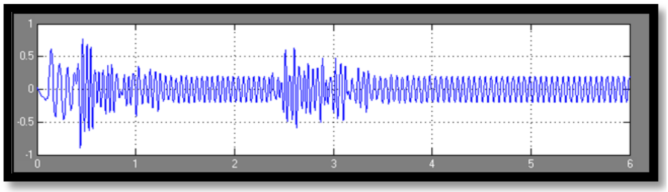

Figure 7.d Fourth Component.

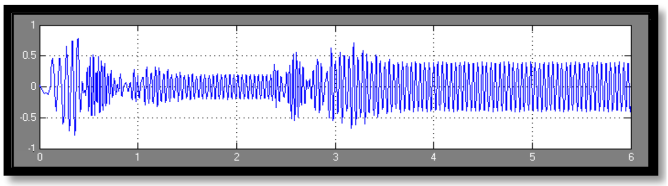

Figure 7.e Fifth Component.

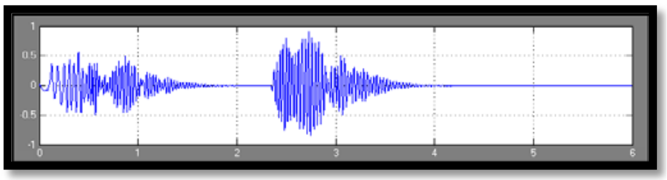

Figure 7.f Sixth Component.

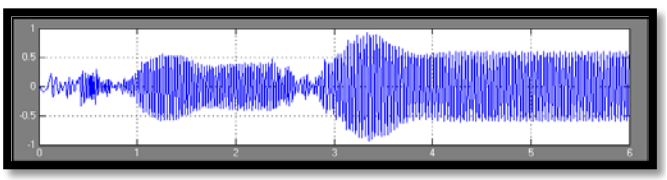

Figure 7.g Seventh Component.

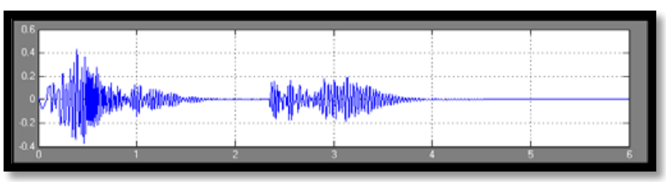

Figure 7.h Eighth Component.

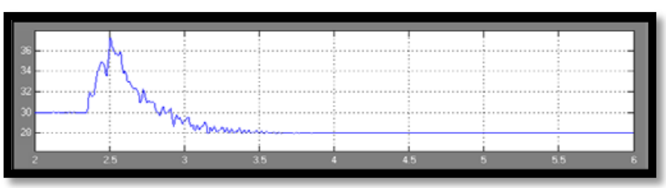

Figure 8.a Estimated Signal.

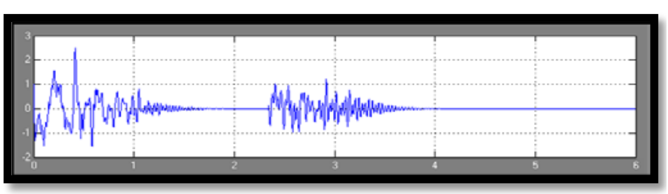

Figure 8.b Error Signal.

d) Noise characteristics:

To conduct a thorough simulation study for noise in the Matlab/Simulink environment, dynamics of the antialiasing filter must also be included. We consider a digital antialiasing filter which is implemented at a high sampling rate (such as 10 kHz). The input signal is sampled at 10 kHz and is passed through this antialiasing filter before being down-sampled to 1 kHz. Then, the low-frequency sampled signal (at 1 kHz) is forwarded to the proposed system. Figure 9 shows this mechanism.

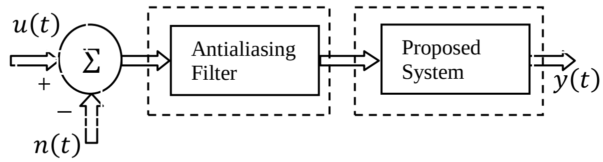

Figure 9 Block Diagram for Noise Study.

The input signal comprises a fundamental component at 30 rad/s with unity amplitude which is corrupted with a white Gaussian noise with a variance of $\sigma^2 = 0.5$ and zero mean. This signal, shown in Figure 5.12.a, is applied to the digital antialiasing filter, as shown in Figure 9. The output of the antialiasing filter, shown in Figure 5.12.b. This signal is down-sampled and forwarded to the proposed system. The output of the proposed system is shown in Figure5.12.c. This is confirms the desirable noise rejection of the presented system.

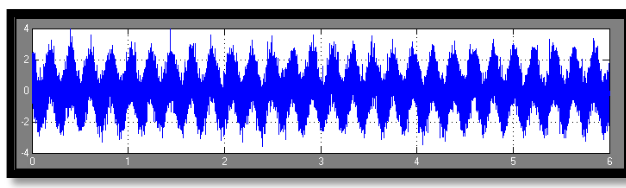

Figure 5.12.a Input Signal Corrupted by Gaussian Noise.

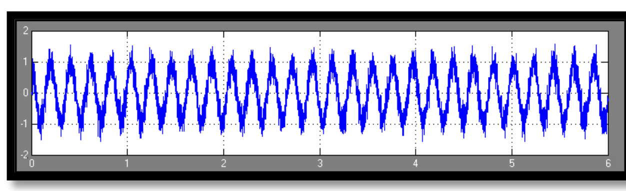

Figure 5.12.b Output of Anti-aliasing Filter.

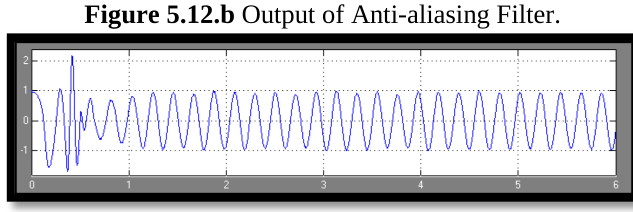

Figure 5.12.c Output of Presented System.

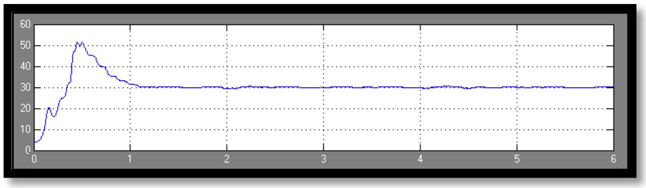

Figure 5.12.d Estimated Frequency.

5. Conclusions

An algorithm presented in this paper is capable for estimating the fundamental frequency of input signal which is composed of n harmonically related sinusoids and for separating or extracting input signal into its constituent components. The structure of the presented algorithm is composed of n second-order Notch Filters and each of which extracts one constituting component. An update law estimates the fundamental frequency of input signal and forwards it to the notch filters. All simulations are performed on computer in Matlab/Simulink software. The desirable initiatory performance, tracking features, and noise characteristics of the presented algorithm are examined. In the initiatory performance we set the filter parameters for optimizing damping and adaptation speed. Then we observe the filter responses for the input signal along with noise. The result of initiatory performance shows that the filter can extracts the fundamental frequency and harmonically related components from the input signal. Tracking performance results are indicates that, if fundamental frequency changes then presented Adaptive Notch Filter tracks faithfully the change in the fundamental frequency. In the noise characteristic case study, fundamental frequency corrupted by white Gaussian noise, the presented filters is capable of extracting fundamental frequency from the noise.

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