A Low-complexity Power and Bit Allocation Algorithm for Multiuser MIMO-OFDM Systems

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Abstract

In this paper, we present a low-complexity bit and power allocation algorithm for multiuser MIMO-OFDM downlink transmission. In order to minimize the total transmit power under the condition that users'QoS requirements are satisfied, a novel resource allocation scheme is proposed to exploit the multiuser diversity gain. The proposed algorithm involves adaptive subcarrier allocation, adaptive modulation and eigen beamforming and achieves significant improvement in overall system performance. Simulation results shows that the proposed algorithm offers a similar performance and a lower complexity than previous algorithms.

Keywords: **Multiuser MIMO OFDM, SVD, bit and power allocation.**

1. Introduction

With the increasing requirements for high-data-rate multimedia services, multiple-input multiple-output (MIMO) and orthogonal frequency division multiplexing (OFDM) techniques have received more and more interest. MIMO-OFDM is a very promising technology in future wireless communication systems. However, it introduces new problems relating how to utilize systems spatio-temporal-spectral and power resources appropriately. With an efficient dynamic resource allocation scheme high data rate can be provided and different users' QoS requirement can be guaranteed [1]. In order to obtain optimal subcarrier power or bit allocations the greedy algorithm is usually applied. One has to note that this algorithm is of high computational complexity and yields one bit optimal solution. Most of the existing algorithms are based on greedy algorithm and require an iterative procedure for their implementation, which delays obtaining an optimal solution and affects the quality of service [2].

In MIMO-OFDM systems, the MIMO channel can be decomposed to a parallel scalar eigenmode subchannels by singular value decomposition (SVD) without crosstalk from one scalarchannel to the other. The results have shown that the subcarrier andbit allocation achieved significant reduction in total transmitpower. Most of the existing algorithms only use one ortwo of the largest eigenmode subchannels to transmit data and neglected the other spatial subchannels. In fact, more eigen subchannels can be exploited to transmit data [3,4]. In this paper, a low-complexity adaptive bit and power allocation algorithm for downlink MIMO-OFDM systems is investigated. We assume that the CSI is perfectly knowen at both the transmitter and receiver. A group of parallel singular value subchannels are first generated by singular value decomposition (SVD) to the MIMO-OFDM channel. In order to efficiently utilize the spatial resources, the proposed algorithm extends the data transmission to all the non-zero spatial subchannels. The rest of this paper is organized as follows. Section 2 describes the system model and definitions. In Section 3, the proposed algorithm is explained and in Section 4 the performance obtained from simulations results is presented. Finally, some conclusions are drawn.

2. System Description

In this paper, we consider a multiusers MIMO-OFDM system with *K* users and *N* subcarriers. The base station (BS) has N_t transmit antennas and each user has N_r receive antennas. The downlink system diagram is shown in Fig 1. We assume that the channel state information (CSI) is perfectly known to the receiver and the transmitter, and the channel changes little during the transmission [5].

At the transmitter, we assume that user *k* has a data-rate requirement of R_k bits per OFDM symbol. In each symbol duration a data stream composed of R_k bits is fed into a subcarrier and bit allocation block. The proposed algorithm is applied to assign different subcarriers to different users. Then the mapped data stream is load to corresponding subcarriers. Transmit precoding matrix V is derived from singular value decomposition (SVD) for every subcarrier, which changes the spatial channel into a series of parallel subchannels with no crosstalk from each other. After precoding, the data stream is sent to inverse fast-Fourier-transformation (IFFT) module to do OFDM modulation for every transmit antenna, the cyclic prefix (CP) is added to every OFDM symbol and then transmitted.

At the receiver, the similar adverse process is taken.

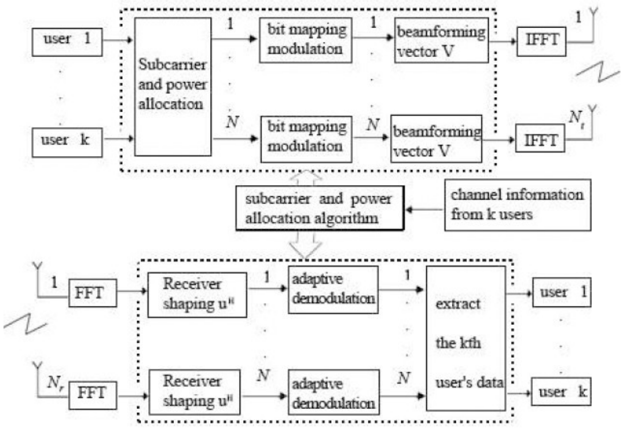

Figure 1: The system model of downlink multiuser MIMO-OFDM.

Let $H_{k,n}$ denotes the $N_r\times N_t$ channel matrix of user *k* on subcarrier *n*. By SVD, the channel matrix can be decomposed into

$$H_{k,n}=U_{k,n}\Lambda_{k,n}V_{k,n}^H=\sum_{i=1}^{M}u_{k,n}^i\lambda_{k,n}^i(v_{k,n}^i)^H \tag{1}$$

where $(.)^H$ represents the complex conjugate transpose of a matrix. $U_{k,n}=[u_{k,n}^1,u_{k,n}^2\cdots u_{k,n}^{N_r}]$ and $V_{k,n}=[v_{k,n}^1,v_{k,n}^2\cdots v_{k,n}^{N_t}]$ are the singular vectors, $\Lambda_{k,n}$ is the diagonal matrix with singular value of $H_{k,n}$, and $M=rank(H_{k,n})$ is the rank of $H_{k,n}$ The stream data over subcarrier *n* is demultiplexed into *M* substream. Let $S=[s_1,s_2\cdots s_M]^T$ denotes the transmitted symbol of *M* substream. The corresponding transmit power diagonal matrix is $P=diag(p_1,p_2\cdots p_M)$. By precoding the transmitted symbol vector S *with* $V_{k,n}^1=[v_{k,n}^1,v_{k,n}^2\cdots v_{k,n}^M]$, *the transmitted* signal vector can be written as:

$$X=V_{k,n}^1P^{\frac{1}{2}}S=\sum_{j=1}^{M}v_{k,n}^j\sqrt{p_j}s_j \tag{2}$$

$$r_{k,n}=[r_1,r_2\cdots r_{N_r}]^T=H_{k,n}X+n \tag{3}$$

Where *n* is the complex white Gaussian noise vector with every dimension a variance of σ^2.

At the receiver, by decoding the receive symbol vector $r_{k,n}$ by $(u_{k,n}^j)^H$, we get the received data symbol on spatial subchannel *j*.

$$y_j=(u_{k,n}^j)^H r_{k,n}=(u_{k,n}^j)^H(H_{k,n}X+n)$$

$$y_j=(u_{k,n}^j)^H\left(\sum_{j=1}^{M}u_{k,n}^j\lambda_{k,n}^j(v_{k,n}^j)^H\right)\left(\sum_{j=1}^{M}v_{k,n}^j\sqrt{p_j}s_j\right)+(u_{k,n}^j)^H n$$

$$y_j=\lambda_{k,n}^j(v_{k,n}^j)^H\left(\sum_{j=1}^{M}v_{k,n}^j\sqrt{p_j}s_j\right)+(u_{k,n}^j)^H n$$

$$y_j=\lambda_{k,n}^j\sqrt{p_j}s_j+(u_{k,n}^j)^H n \tag{4}$$

With precoding and decoding the transmit symbol vector respectively by $V_{k,n}$ and $U_{k,n}$, we can notice from equation (4) that the MIMO channel is transformed into M parallel single-input single-output (SISO) subchannels without crosstalk when the CSI is perfectly known at the transmitter and the receiver.

3. Resource allocation algorithm

In this section a resource allocation algorithm is presented for downlink multiuser MIMO-OFDM system. To avoid severe co-channel interference (CCI), we do not allow more than one user to share the same subcarrier, we assume that $p^{i}_{k,n}$ is the required power to transmit $b^{j}_{k,n}$ bits on *i*th spatial subchannel over *n*th subcarrier of user *k*. $S_k \subset \{1,2,\ldots,N\}$ denote the set of subcarriers of user *k*, and BER_{target} is the objective bit error rate, the optimization problem can be formulated as:

$$\text{Minimize} \quad P_T = \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{i=1}^{M} p^{i}_{k,n} \tag{5}$$

$$\begin{aligned} \text{Subject to} \quad & \sum_{i=1}^{M} p^{i}_{k,n} = R_{k,n} \\ & BER^{i}_{k,n} = BER_{Target} \\ & S_i \cap S_j = \emptyset \qquad \forall i \neq j \\ & S_1 \cup S_2 \cup \cdots \cup S_K = \{1,2,\ldots,N\} \\ & M = rank(H_{k,n}) \end{aligned}$$

When $S_1 \ldots S_K$ are disjoint, the system can be viewed as a single user system on each subcarrier. So, we can transform the problem of minimizing the total transmit power to a problem of minimizing the power required on each subcarrier [6], then the optimization problem in (5) can be rewritten as:

$$\text{Minimize} \quad \sum_{i=1}^{M} p^{i}_{k,n} \tag{6}$$

$$\begin{aligned} \text{Subject to} \quad & \sum_{i=1}^{M} p^{i}_{k,n} = R_{k,n} \\ & BER^{i}_{k,n} = BER_{Target} \\ & S_i \cap S_j = \emptyset \qquad \forall i \neq j \\ & S_1 \cup S_2 \cup \cdots \cup S_K = \{1,2,\ldots,N\} \\ & M = rank(H_{k,n}) \end{aligned}$$

Denote $f_k(c)$ be the required transmit power to transmit c bits satisfying target bit error rate (BER_k) when channel gain is unity. In the case of M-ary Quadrature Amplitude Modulation (MQAM), $f_k(c)$ can be represented as [7]

$$f_k(c) = \frac{N_0}{3}\left[Q^{-1}\left(\frac{BER_k}{4}\right)\right]^2 (2^c - 1) \tag{7}$$

where $\frac{N_0}{2}$ denotes the variance of the Additive White Gaussian Noise (AWGN) and $Q(x)$ is the Q-function [8].

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{\frac{-t^2}{2}}\, dt$$

In order to guarantee users QoS requirements, the required power, to transmit $b^{i}_{k,n}$ bit on the *i*th spatial subchannel over *n*th subcarrier for user *k*, is given by [9]

$$p^{i}_{k,n} = \frac{f_k(b^{i}_{k,n})}{(\lambda^{i}_{k,n})^2} \tag{8}$$

Let $\Delta P^{i}_{k,n}$ denote the additional power needed for transmitting one additional bit on the *i*th spatial subchannel over *n*th subcarrier for user *k*. It is given by

$$\Delta P^{i}_{k,n} = \frac{f_k(b^{i}_{k,n}+1) - f_k(b^{i}_{k,n})}{(\lambda^{i}_{k,n})^2} \tag{9}$$

We define the term G_k as follows

$$G_k = \sum_{n=1}^{N}\sum_{i=1}^{M} \frac{(\lambda^{i}_{k,n})^2}{N_0} \tag{10}$$

To solve the problem of minimization the total transmit power, we present our approach in two steps: the first step is to allocate the subcarriers to the user that has the largest G_k. In the second step we assign the bits and power to user k over all subcarrier in S_k on the subchannel that requires the least additional power.

Let $Ne(k)$ be the number of subcarriers for user k

$$Ne(k) = floor(N/K) \tag{11}$$

We assume that the data rate $R_{k,n}$ for user k on subcarrier n is constant, so $R_{k,n}$ can be expressed as

$$R_{k,n} = round\left(\frac{R_k}{Ne(k)}\right) \tag{12}$$

Our algorithm is described as follows:

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Initialization ;
calculate $G$ for all users according to equation (10);
sort $G$ by descending order $G_1 > G_2 > .... > G_K$;
$N_e(1:K) = floor(N/K)$;
$N_{rem} = N - K.floor(N/K)$;
$\hat{i} = \arg\min_{i \in \{1,...,K\}} G_i$;
$N_e(\hat{i}) = N_k + N_{rem}$;
$R_{k,n} = round(\frac{R_k}{N_e(k)})$        for k=1,...,K;
$b^i_{k,n} = 0$        $\forall k, \forall n, \forall i,$;
$S_{free} = \{1, 2, ..., N\}$        $N_{free} = \sharp S_{free}$ where $\sharp$
denote the number of element in $S_{free}$;
for $k \leftarrow 1$ to $K$ do
    Step 1
    for $t \leftarrow 1$ to $N_{free}$ do
        $\hat{n} = S_{free}(t)$;
        evaluate $\Delta P^i_{k,\hat{n}}(0)$;
        for $r \leftarrow 1$ to $R_{k,n}$ do
            $\hat{i} = \arg\min_{i \in \{1,...,M\}} \Delta P^i_{k,\hat{n}}$;
            $b^{\hat{i}}_{k,\hat{n}} \leftarrow b^{\hat{i}}_{k,\hat{n}} + 1$;
            update $\Delta P^{\hat{i}}_{k,\hat{n}}(b^{\hat{i}}_{k,\hat{n}})$ according to equation (9);
        end
        $P_{k,\hat{n}} = \sum_{i=1}^{M} \frac{f_k(b^{\hat{i}}_{k,\hat{n}})}{(\lambda^{\hat{i}}_{k,\hat{n}})^2}$;
    end
    $S_k = \phi$;
    for $j \leftarrow 1$ to $N_e(k)$ do
        $m = \arg\min_{n \in S_{free}} P_{k,n}$ ;
        $S_k \leftarrow S_k \cup \{m\}$;
        $S_{free} \leftarrow S_{free} - \{m\}$;
        $N_{free} \leftarrow N_{free} - 1$;
    end
    Step 2
    $\hat{b}^i_{k,n} = 0$        $\forall k, \forall n, \forall i,$;
    $\alpha = 0, \beta = 1$;
    while $[(\alpha < R_k) and (\beta \leq N_e(k))]$ do
        $r = 0, \hat{m} = S_k(\beta)$;
        while $[(\alpha < R_k) and (r \leq R_{k,n})]$ do
            $r \leftarrow r + 1$;
            $\hat{i} = \arg\min_{i \in \{1,...,M\}} \Delta P^i_{k,\hat{m}}$;
            $\hat{b}^{\hat{i}}_{k,\hat{m}} \leftarrow \hat{b}^{\hat{i}}_{k,\hat{m}} + 1$;
            update $\Delta P^{\hat{i}}_{k,\hat{m}}(\hat{b}^{\hat{i}}_{k,\hat{m}})$;
            $\alpha \leftarrow \alpha + 1$;
        end
        $\beta \leftarrow \beta + 1$;
    end
    $P_k = \sum_{n \in S_k} \sum_{i=1}^{M} \frac{f(\hat{b}^i_{k,n})}{(\lambda^i_{k,n})^2}$;
end
```

Algorithm 1: The proposed algorithm

4. Performance analysis

The performance of the proposed algorithm is investigated in this section. In our simulation system, the channel is modeled as Raleigh fading channel. The bandwidth of the system is 2.5MHz and the number of transmit data for each user is $R_k = 192\text{bits}$.

The proposed algorithm (PA) is compared with a novel resource allocation algorithm presented in [9] and dynamic subcarrier allocation with only the best eigen subchannel (DSA-BES) [10].

Figure 2 shows the total transmit power versus the number of users for $BER = 10-3$, number of subcarriers $N = 256$ and $N_t = N_r = 4$. It can be seen that the proposed algorithm gives almost the same results as Algorithm in [9] and gives better results compared to the DSA-BES especially when the number of users is large.

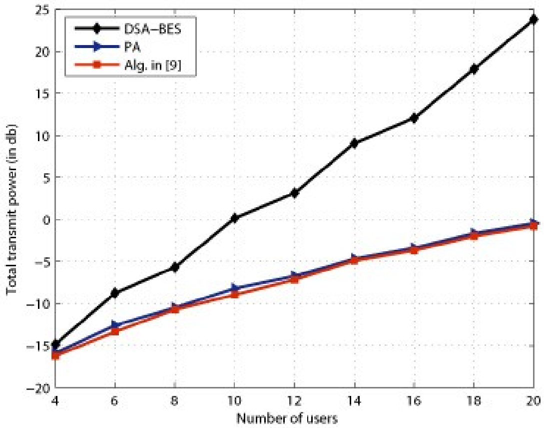

Figure 2: Total transmit power versus the number of users forK = 20,N = 256 $N_r = N_t = 4$ *and* $BER = 10^{-3}$.

Figure 3 shows the same simulation as Figure 2 except in this case the number of subcarriers is $N = 128$. When we compare the result in the Figure 2 with the result in the Figure 3, we can see that the total transmit power increases when the number of subcarriers in the system decreases. It can also see that proposed algorithm (PA) keeps the same performances that in Figure 2.

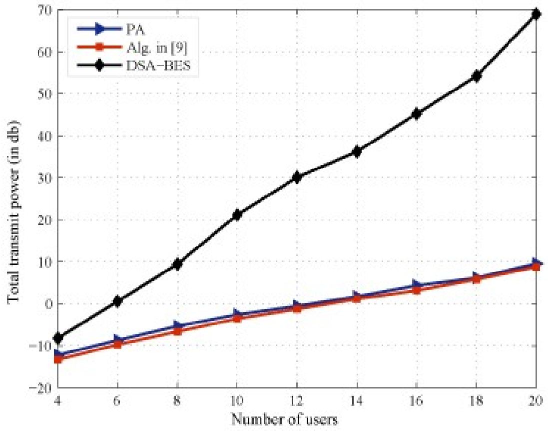

Figure 3: Total transmit power versus the number of users for K = 20, N = 128, $N_r=N_t=4$ *and* $BER=10^{-3}$

In order to investigates the impact of the number of antenna, Figure 4 shows the total transmit power versus the number of users for $BER=10^{-3}$, number of subcarriers N = 128, the number of receive antenna $N_r=2$ and the number of transmit antenna $N_t=4$. the simulation results demonstrate that the required transmit power for proposed algorithm (PA) and the algorithm in [9] is increased when the number of receive antennas is decreased . the reason is that the number of exploited spatial subchannels decreases.

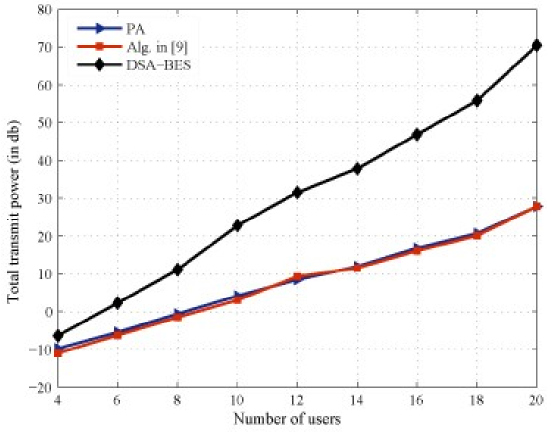

Figure 4: Total transmit power versus the number of users for K=20,N=128, $N_r=2$ $N_t=4$ *and* $BER=10^{-3}$

Figure 5 shows the total transmit power of the PA versus the number of users for different values of BER. Simulation results shows that the total transmit power is decreasing with the increase in the BER value.

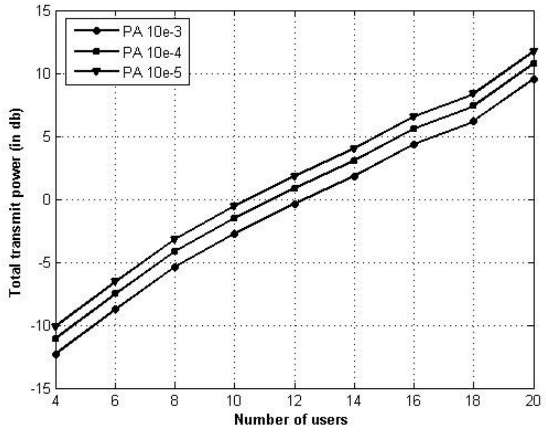

Figure 5: Total transmit power versus the number of users for K = 20,N = 128, $N_r=N_t=4$ *for different values of BER.*

In order to compare the computational complexity between the proposed algorithm and the algorithm in [9], we compare the needed CPU times for running each algorithm. Figure 6 shows the CPU times needed for running each algorithm versus the number of users for $K=20, N=128, N_t=N_r=4$ and $BER=10^{-3}$ It can be seen that our algorithm converge rapidly than the algorithm in [9] especially when the number of users is large.

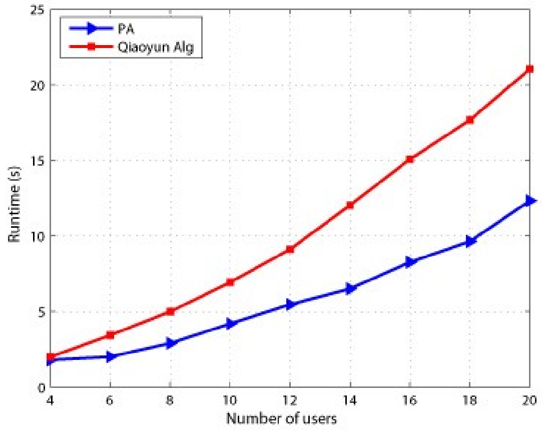

Figure 6: Total transmit power versus the number of users for K = 20,N = 128, $N_r=N_t=4$ *and* $BER=10^{-3}$.

4. Conclusion

In this paper, a low complexity algorithm for bit, subcarrier and power allocation for MIMO-OFDM downlink system has been presented. The proposed algorithm minimizes the total transmit power under the condition that users QoS requirements are satisfied. The simulation results demonstrate that the proposed algorithm offers almost the same required transmit power than the algorithm in [9]. Moreover, the proposed algorithm converge rapidly than the previous algorithms especially when the number of users is large.

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