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Active Fault Tolerant Control (FTC) Design for Takagi-Sugeno Fuzzy Systems with Weighting Functions Depending on the FTC

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Abstract

In this paper the problem of active fault tolerant control design for noisy systems described by Takagi-Sugeno fuzzy models is studied. The proposed control strategy is based on the known of the fault estimated and the error between the faulty system state and a reference system state. The considered systems are affected by actuator and sensor faults and have the weighting functions depending on the fault tolerant control. A mathematical transformation is used to conceive an augmented system in which all the faults affecting the initial system appear as actuator faults. Then, an adaptive proportional integral observer is used in order to estimate the state and the faults. The problem of conception of the proportional integral observer and of the fault tolerant control strategy is formulated in linear matrices inequalities which can be solved easily. To illustrate the proposed method, It is applied to the three tanks systems.

Keywords: *fault estimation, active fault tolerant control, proportional integral observer, nonlinear Takagi-Sugeno fuzzy models, actuator faults, sensor faults.*

1. Introduction

State observers are always used to estimate system outputs by the known of the system model and some measures of the system control and output [27]. This estimation is compared to the measured value of the output to generate residuals. The residuals are used as reliable indicators of the process behavior. They are equal to zero if the system is not affected by faults. The residuals depend of faults if they are present. There are three categories of faults detection methods: sensors faults detection, actuators faults detection and system faults detection.

In most cases, processes are subjected to disturbances which have as origin the noises due to its environment and the model uncertainties. Moreover, sensors and/or actuators can be corrupted by different faults or failures. Many works are dealing with state estimation for systems with unknown inputs or parameter uncertainties. In [37], Wang *et al.* propose an observer able to entirely reconstruct the state of a linear system in the presence of unknown inputs and in [28], to estimate the state, a model inversion method is used. Using the Walcott and Zak structure observer [36], Edwards *et al.* [7] and [8] have also designed a convergent observer using the Lyapunov approach.

In the context of nonlinear systems described by Takagi-Sugeno fuzzy models, some works tried to reconstruct the system state in spite of the unknown input existence. This reconstruction is assured via the elimination of unknown inputs [10]. Other works choose to estimate the unknown inputs and system state simultaneously [1], [12], [18], [23] and [30]. Unknown input observers can be used to estimate actuator faults provided they are assumed to be considered as unknown inputs. This estimation can be obtained by using a proportional integral observer [15], [17], [21-23]. That kind of observers gives some robustness property of the state estimation with respect to the system uncertainties and perturbations [4], [31].

Faults affecting systems have harmful effects on the normal behavior of the process and their estimation can be used to conceive a control strategy able to minimize their effects (named fault tolerant control (FTC)). A control loop can be considered fault tolerant if there exist adaptation strategies of the control law included in the closed-loop that introduce redundancy in actuators [38]. Fault Tolerant Control (FTC) is, relatively, a new idea in the research literature [5] which allows having a control loop that fulfils its objectives when faults appear [11], [16], [19] and [20]

There are two main groups of control strategies: the active and the passive techniques. The passive techniques are control laws that take into account the faults appearance as system perturbations [38]. Thus, within certain margins, the control law has inherent fault tolerant capabilities, allowing the system to cope with the fault presence [38]. This kind of control is described in [5], [6] [25], [26], [32] and [33]. The active fault tolerant control techniques consist on adapting the control law using the information given by the FDI block [5], [16], [19], [20] and [40]. With this information, some automatic adjustments are done trying to reach the control objectives [38].

In this paper, an active FTC strategy inspired from that given in [38] is proposed. In [38] Witczak *et al.* designed a FTC strategy for the class of discrete systems. This FTC is conceived using the error between the faulty and the reference system states. However, in real cases the faulty system state is unknown. The main contribution in this work is to conceive the FTC for the case of non linear systems described by Takagi-Sugeno fuzzy models with weighting functions depending on the fault tolerant control. This case is not treated enough in the literature [11]. It is important to consider this system class because if the weighting functions are depending on the system input and if the system input changes because of the action of the fault affecting the system, the weighting functions must depend on the new system input. State and faults estimation is made using an adaptive proportional integral observer. A mathematical transformation is used to conceive an augmented system in which the sensor fault affecting the initial system appears as an actuator fault. The actuator fault is considered as an unknown input. Once the fault is estimated, the FTC controller is implemented as a state feedback controller. In this work the observer design and the control implementation can be made simultaneously.

The paper is organized as follows. Section 2 recalls an elementary background about the Takagi-Sugeno fuzzy models (named also multiple models). In section 3 the proposed method of fault tolerant control design is presented. The application of the proposed control to the three tanks system is the subject of section 4.

2. On the Takagi-Sugeno fuzzy systems

Takagi–Sugeno fuzzy models are non linear systems described by a set of if–then rules which gives local linear representations of an underlying system [1], [12], [14] and [39] Such models can approximate a wide class of non linear systems [39]. They can even describe exactly some non linear systems [38] and [39].
Each non linear dynamic system can be simply, described by a Takagi-Sugeno fuzzy model [35] and [34]. A Takagi-Sugeno fuzzy model is the fuzzy fusion of many linear models [1-3], [12] and [30] each of them represents the local system behavior around an operating point. A Takagi-Sugeno model is described by fuzzy IF-THEN rules which represent local linear input/output relations of the non-linear system [38]. It has a rule base of M rules, each having p antecedents, where the i^{th} rule is expressed as:

$$R^i: \text{ IF } \xi_1 \text{ is } F_1^i \text{ and ... and } \xi_p \text{ is } F_p^i$$
$$\text{THEN}: \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} \quad (1)$$

in which $i = 1 \ldots M$, $F_j^i (j = 1 \ldots p)$ are fuzzy sets and $\xi = \begin{bmatrix} \xi_1 & \xi_2 & \ldots & \xi_p \end{bmatrix}$ is a known vector of premise variables [23] which may depend on the state, the input or the output.
The final output of the normalized Takagi-Sugeno fuzzy model can be inferred as:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) C_i x(t) \end{cases} \quad (2)$$

The weighting functions $\mu_i(\xi(t))$ are non linear and depend on the decision variable $\xi(t)$.
The weighting functions are normalized rules defined as:

$$\mu_i(\xi(t)) = \frac{T_{j=1}^{p} \omega_i(\xi(t))}{\sum_{j=1}^{M} T_{j=1}^{p} \omega_j(\xi(t))} \quad (3)$$

where $\omega_i(\xi(t))$ is the grade of membership of the premise variable $\xi(t)$ and T denotes a t-norm. The weighting functions satisfy the sum convex property expressed in the following equations:

$$0 \le \mu_i(\xi(t)) \le 1 \quad \text{and} \quad \sum_{i=1}^{M} \mu_i(\xi(t)) = 1 \quad (4)$$

If, in the equation which defines the output, we impose that $C_1 = C_2 = \ldots = C_M = C$, the output of the model (2) is reduced to: $y(t) = Cx(t)$ and the Takagi-Sugeno fuzzy model becomes:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = Cx(t) \end{cases} \quad (5)$$

This model, known also as Takagi-Sugeno multiple model, has been initially proposed, in a fuzzy modeling framework, by Takagi and Sugeno [34] and in a multiple model modeling framework in [13] and [29]. This model has been largely considered for analysis [29], [34] and [9], modeling [13] and [41], control [21] and [9] and state estimation [1-3], [12], [22], [23] and [30] of non linear systems.

3. Active fault tolerant control design

A non linear system described by multiple model can be expressed as follow:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(u(t)) A_i x(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \quad (6)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^r$ is the input vector, $y(t) \in R^m$ the output vector and A_i, B and C are known constant matrices with appropriate dimensions. The scalar M represents the number of local models.

Consider the following nonlinear Takagi-Sugeno model affected by actuator and sensor faults and measurement noise:

$$\begin{cases} \dot{x}_f(t) = \sum_{i=1}^{M} \mu_i(u_f(t)) A_i x_f(t) + Bu_f(t) + Ef_a(t) \\ y_f(t) = Cx_f(t) + Ff_s(t) + Dw(t) \end{cases} \quad (7)$$

where $x_f(t) \in R^n$ is the state vector, $u_f(t) \in R^r$ is the fault tolerant control which will be conceived, $y_f(t) \in R^m$ is the output vector. $f_a(t)$ and $f_s(t)$ are respectively the actuator and sensor faults which are assumed to be bounded and $w(t)$ represents the measurement noise. E, F and D are respectively the faults and the noise distribution matrices which are assumed to be known.

Let us define the following states [15]:

$$\begin{aligned} \dot{z}(t) &= \sum_{i=1}^{M} \mu_i(u(t))(-\bar{A}z(t) + \bar{A}Cx(t)) \\ \dot{z}_f(t) &= \sum_{i=1}^{M} \mu_i(u_f(t))(-\bar{A}z(t) + \bar{A}Cx(t) + \bar{A}_i Ff_s(t) + \bar{A}Dw(t)) \end{aligned} \quad (8)$$

where $-\bar{A}$ is a stable matrix with appropriate dimension.

Defining the two augmented states $X(t)$ and $X_f(t)$ as:

$X(t) = \begin{bmatrix} x(t)^T & z(t)^T \end{bmatrix}^T$ and $X_f(t) = \begin{bmatrix} x_f^T(t) & z_f^T(t) \end{bmatrix}^T$,

these two augmented state vectors can be written:

$$\begin{cases} \dot{X}(t) = \sum_{i=1}^{M} \mu_i(u(t)) A_{ai} X(t) + B_a u(t) \\ Y(t) = C_a X(t) \end{cases} \quad (9)$$

and

$$\begin{cases} \dot{X}_f(t) = \sum_{i=1}^{M} \mu_i(u_f(t)) A_{ai} X_f(t) + B_a u_f(t) + E_a f(t) + D_a w(t) \\ Y_f(t) = C_a X_f(t) \end{cases} \quad (10)$$

with:

$$A_{ai} = \begin{bmatrix} A_i & 0 \\ -\bar{A}C & -\bar{A} \end{bmatrix},\ E_a = \begin{bmatrix} E & 0 \\ 0 & \bar{A}F \end{bmatrix},\ f = \begin{bmatrix} f_a \\ f_s \end{bmatrix},\ B_a = \begin{bmatrix} B \\ 0 \end{bmatrix},$$

$$D_a = \begin{bmatrix} 0 \\ \bar{A}D \end{bmatrix} \text{ and } C_a = \begin{bmatrix} 0 & I \end{bmatrix}, \quad (11)$$

A proportional integral observer is used to estimate the augmented state $X_f(t)$ and the generalized fault $f(t)$. It is given by the following equations:

$$\begin{cases} \dot{\hat{X}}_f(t) = \sum_{i=1}^{M} \mu_i(u_f(t))(A_{ai}\hat{X}_f(t) + K_i\tilde{Y}_f(t)) + B_a u_f(t) + E_a \hat{f}(t) \\ \dot{\hat{f}}(t) = \sum_{i=1}^{M} \mu_i(u_f(t)) L_i \tilde{Y}_f(t) \\ \hat{Y}(t) = C_a \hat{X}(t) \end{cases} \quad (12)$$

where $\hat{X}_f(t)$ is the estimated system state, $\hat{f}(t)$ represents the estimated fault, $\hat{Y}_f(t)$ is the estimated output, K_i are the proportional gains of the local observers and L_i are their integral gains to be computed and $\tilde{Y}_f(t) = Y_f(t) - \hat{Y}_f(t)$.

The fault tolerant control $u_f(t)$ is conceived on the base of the strategy described by the following expression [38].

$$u_f(t) = -S\hat{f}(t) + G(X(t) - \hat{X}_f(t)) + u(t) \quad (13)$$

where S and G are two constant matrices with appropriate dimensions.

Let us define $\tilde{X}(t)$ the error between the states $X(t)$ and $X_f(t)$, $\tilde{X}_f(t)$ the estimation error of the state $X_f(t)$ and $\tilde{f}(t)$ the fault estimation error :

$$\begin{aligned} \tilde{X}(t) &= X(t) - X_f(t) \\ \tilde{X}_f(t) &= X_f(t) - \hat{X}_f(t) \\ \tilde{f}(t) &= f(t) - \hat{f}(t) \end{aligned} \quad (14)$$

Choosing the matrix S verifying $E_a = B_a S$, the dynamics of $\tilde{X}(t)$ is given by:

$$\begin{aligned} \dot{\tilde{X}}(t) &= \dot{X}(t) - \dot{X}_f(t) \\ &= \sum_{i=1}^{M} \mu_i(u(t))(A_{ai} - B_a G)\tilde{X}(t) - E_a \tilde{f}(t) - B_a G \tilde{X}_f(t) + \Delta_1(t) \end{aligned} \quad (15)$$

with :

$$\Delta_1(t) = \sum_{i=1}^{M} \mu_i(u_f(t) - \mu_i u(t)) A_{ai} \tilde{X}_f(t) - D_a w(t) \quad (16)$$

The dynamic of $\tilde{X}_f(t)$ can be written:

$$\dot{\tilde{X}}_f(t) = \dot{X}_f(t) - \dot{\hat{X}}_f(t) = \sum_{i=1}^{M} \mu_i(u(t))(A_{ai} - K_i C_a)\tilde{X}_f(t) + E_a \tilde{f}(t) + \Delta_2(t) \quad (17)$$

with :

$$\Delta_2(t) = \sum_{i=1}^{M} \mu_i(u_f(t) - \mu_i u(t))(A_{ai} - K_i C_a)\tilde{X}_f(t) + D_a w(t) \quad (18)$$

The dynamic of the fault error estimation is:

$$\dot{\tilde{f}}(t) = \dot{f}(t) - \dot{\hat{f}}(t) = -\sum_{i=1}^{M} \mu_i(u(t)) L_i C_a \tilde{X}_f(t) + \Delta_3(t) \quad (19)$$

with :

$$\Delta_3(t) = \sum_{i=1}^{M} \mu_i(u_f(t) - \mu_i u(t)) L_i C_a \tilde{X}_f(t) + D_a w(t) + \dot{f}(t) \quad (20)$$

The equations (15), (17) and (19) can be rewritten:

$$\dot{\varphi}(t) = A_m \varphi(t) + \varepsilon(t) \quad (21)$$

where :

$$\varphi(t) = \begin{bmatrix} \tilde{X}(t) \\ \tilde{X}_f(t) \\ \tilde{f}(t) \end{bmatrix}, \ \varepsilon(t) = \begin{bmatrix} \Delta_1(t) \\ \Delta_2(t) \\ \Delta_3(t) \end{bmatrix} \text{ and } A_m = -\sum_{i=1}^{M} \mu_i(u(t)) A_{mi} \quad (22)$$

where

$$A_{mi} = \begin{bmatrix} A_{ai} - B_a G & -B_a G & B_a \\ 0 & A_{ai} - K_i C_a & B_a \\ 0 & L_i C_a & 0 \end{bmatrix} \quad (23)$$

Considering the Lyapunov function $V(t) = \varphi(t)^T P \varphi(t)$, the generalized error vector $\varphi(t)$ converges to zero if $\dot{V}(t) < 0$, $\dot{V}(t) < 0$ if $A_{mi}^T P + P A_{mi} < 0 \quad \forall i \in \{1...M\}$.

The problem of robust state and faults estimation and of the fault tolerant control design is reduced to find the gains K and L of the observer and the matrix G to ensure an asymptotic convergence of the generalized error vector $\varphi(t)$ toward zero if $\varepsilon(t) = 0$ and to ensure a bounded error in the case where $\varepsilon(t) \neq 0$, i.e.:

$$\begin{aligned} &\lim_{t\to\infty} \varphi(t) = 0 && \text{for } \varepsilon(t) = 0 \\ &\|\varphi(t)\|_{Q_\varphi} \le \lambda \|\varepsilon(t)\|_{Q_\varepsilon} && \text{for } \varepsilon(t) \neq 0 \end{aligned} \quad (24)$$

where $\lambda > 0$ is the attenuation level. To satisfy the constraints (13), it is sufficient to find a Lyapunov function $V(t)$ such that:

$$\dot{V}(t) + \varphi(t)^T Q_\varphi \varphi(t) - \lambda^2 \varepsilon(t)^T Q_\varepsilon \varepsilon(t) < 0 \quad (25)$$

where Q_φ and Q_ε are two positive definite matrices.

The inequality (25) can be written:

$$\begin{bmatrix} \varphi(t) \\ \varepsilon(t) \end{bmatrix}^T \Phi \begin{bmatrix} \varphi(t) \\ \varepsilon(t) \end{bmatrix} < 0 \quad (26)$$

where:

$$\Phi = \begin{bmatrix} A_m^T P + P A_m + Q_\varphi & P \\ P & -\lambda^2 Q_\varepsilon \end{bmatrix} \quad (27)$$

Choosing $Q_\varphi = Q_\varepsilon = I$ and assume that the Lyapunov matrix P has the form: $diag(I, P_2, P_3)$, the matrix Φ is written :

$$\Phi = \sum_{i=1}^{M} \mu_i(u(t)) \Phi_i \quad (28)$$

where:

$$\Phi_i = \begin{bmatrix} \Phi_{11i} & -B_a G & B_a & I & 0 & 0 \\ -G^T B_a^T & \Phi_{22i} & \Phi_{23i} & 0 & P_2 & 0 \\ B_a^T & \Phi_{32i} & I_3 & 0 & 0 & P_3 \\ I & 0 & 0 & \lambda_1 I_{01} & 0 & 0 \\ 0 & P_2 & 0 & 0 & \lambda_2 I_{02} & 0 \\ 0 & 0 & P_3 & 0 & 0 & \lambda_3 I_{03} \end{bmatrix} \quad (29)$$

with:

$$\begin{aligned} \Phi_{11i} &= A_{ai} - B_a G + A_{ai}^T - G^T B_a^T + I_1 \\ \Phi_{22i} &= P_2 A_{ai} - P_2 K_i C_a + A_{ai}^T P_2 - C_a^T K_i^T P_2 + I_2 \\ \Phi_{23i} &= P_2 B_a + C_a^T L_i^T P_3 \\ \Phi_{32i} &= \Phi_{23}^T \end{aligned} \quad (30)$$

$\Phi < 0$ if $\Phi_i < 0 \quad \forall i \in \{1...M\}$, the inequalities $\Phi_i < 0$ are bilinear, they can be linearised using the changes of variables : $U_{2i} = P_2 K_i$ and $U_{3i} = P_3 L_i$. The observer gains are then computed using the equations:

$$\begin{aligned} K_i &= P_2^{-1} U_{2i} \\ L_i &= P_3^{-1} U_{3i} \end{aligned} \quad (31)$$

Summarizing the following theorem can be proposed:

Theorem:

The system (21) describing the evolution of the errors $\tilde{X}(t), \tilde{X}_f(t)$ and $\tilde{f}(t)$ is stable if there exist symmetric definite positive matrices P_2 and P_3 and matrices U_{3i}, U_{2i} and G , $i \in \{1...M\}$ so that the LMI $\Phi_i < 0$ are verified $\forall i \in \{1...M\}$ where :

$$\Phi_i = \begin{bmatrix} \Phi_{11i} & -B_a G & B_a & I & 0 & 0 \\ -G^T B_a^T & \Phi_{22i} & \Phi_{23i} & 0 & P_2 & 0 \\ B_a^T & \Phi_{32i} & I_3 & 0 & 0 & P_3 \\ I & 0 & 0 & \lambda_1 I_{01} & 0 & 0 \\ 0 & P_2 & 0 & 0 & \lambda_2 I_{02} & 0 \\ 0 & 0 & P_3 & 0 & 0 & \lambda_3 I_{03} \end{bmatrix} \quad (32)$$

and:

$$\begin{aligned}
\Phi_{11i} &= A_{ai} - B_a G + A_{ai}^T - G^T B_a^T + I_1 \\
\Phi_{22i} &= P_2 A_{ai} - P_2 U_{2i} + A_{ai}^T P_2 - C_a^T U_{2i}^T + I_2 \\
\Phi_{23i} &= P_2 B_a + C_a^T U_{3i}^T \\
\Phi_{32i} &= \Phi_{23}^T
\end{aligned} \tag{33}$$

The observer gains are obtained by:

$L_i = P_3^{-1} U_{3i}$ and $K_i = P_2^{-1} U_{2i}$

4. Application to the three tanks system

The main objective of this part is to show the robustness of the proposed method by its application to a hydraulic process made up of three tanks [3] and [34].

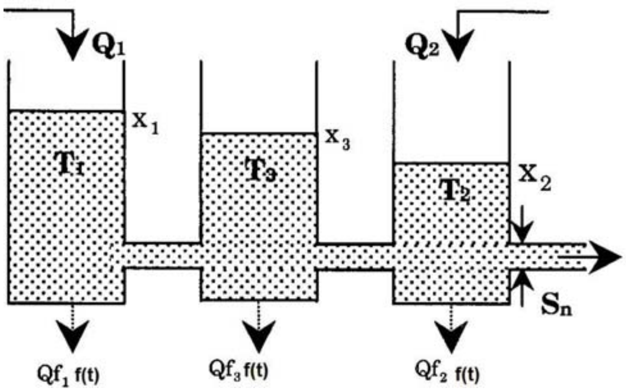

Fig. 1 Three tanks system

The considered system is affected simultaneously by sensor and actuator faults. The three tanks T_1, T_2, and T_3 with identical sections σ, are connected to each others by cylindrical pipes of identical sections S_n. The output valve is located at the output of tank T_2; it ensures to empty the tank filled by the flow of pumps 1 and 2 with respectively flow rates Q_1 and Q_2. Combinations of the three water levels are measured. The pipes of communication between the tanks are equipped with manually adjustable ball valves, which allow the corresponding pump to be closed or open. The three levels x_1, x_2 and x_3 are governed by the constraint $x_1 > x_3 > x_2$; the process model is given by the equation (33). Indeed, taking into account the fundamental laws of conservation of the fluid, one can describe the operating mode of each tank; one then obtains a non linear model expressed by the following state equations [3] and [41]

$$\begin{cases}
\sigma \dfrac{dx_1}{dt} = -\alpha_1 S_n (2g(x_1(t) - x_3(t)))^{1/2} + Q_1(t) + Qf_1 . f_a(t) \\
\sigma \dfrac{dx_2}{dt} = -\alpha_3 S_n (2g(x_3(t) - x_2(t)))^{1/2} \\
\qquad - \alpha_2 S_n (2g(x_2(t))^{1/2} + Q_2(t) + Qf_2 . f_a(t) \\
\sigma \dfrac{dx_3}{dt} = -\alpha_1 S_n (2g(x_1(t) - x_3(t)))^{1/2} + Qf_3 . f_a(t) \\
\qquad - \alpha_3 S_n (2g(x_3(t) - x_2(t)))^{1/2}
\end{cases} \tag{34}$$

where α_1, α_2 and α_3 are constants, $f_a(t)$ is the actuator fault regarded as an unknown input. $Qf / f_i, i \in \{1...3\}$ denote the additional mass flows into the tanks caused by leaks and g is the gravity constant. The multiple model, with $\xi(t) = u(t)$, which approximates the non linear system (34), is:

$$\begin{cases}
\dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + Bu(t) + Ef_a(t) + d_i) \\
y(t) = Cx(t) + Ff_s(t) + Dw(t)
\end{cases} \tag{35}$$

The matrices A_i, B_i, and d_i are calculated by linearizing the initial system (34) around four points chosen in the operation range of the system. Four local models have been selected in a heuristic way. That number guarantees a good approximation of the state of the real system by the multiple models [3] and [41]. The following numerical values were obtained:

$$A_1 = \begin{bmatrix} -0.0109 & 0 & 0.0109 \\ 0 & -0.0206 & 0.0106 \\ 0.0109 & 0.0106 & -0.0215 \end{bmatrix}, d_1 = 10^{-3} \begin{bmatrix} -2.86 \\ -0.38 \\ 0.11 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.0110 & 0 & 0.0110 \\ 0 & -0.0205 & 0.0104 \\ 0.0110 & 0.0104 & -0.0215 \end{bmatrix}, d_2 = 10^{-3} \begin{bmatrix} -2.86 \\ -0.34 \\ 0.038 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -0.0084 & 0 & 0.0084 \\ 0 & -0.0206 & 0.0095 \\ 0.0084 & 0.0095 & -0.0180 \end{bmatrix}, d_3 = 10^{-3} \begin{bmatrix} -3.7 \\ -0.14 \\ 0.69 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} -0.0085 & 0 & 0.0085 \\ 0 & -0.0205 & 0.0095 \\ 0.0085 & 0.0095 & -0.0180 \end{bmatrix}, d_4 = 10^{-3} \begin{bmatrix} -3.67 \\ -0.18 \\ 0.62 \end{bmatrix}$$

$$B_i = \frac{1}{\sigma} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

In the following, the functions Qf_1, Qf_2 and Qf_3 are constant, the numerical application are performed with:

$Qf_i = 10^{-4} \forall i \in \{1...4\}$ and $t \in [0, x[$, $g = 9.8$, $\alpha_1 = 0.78$, $\alpha_2 = 0.78$ and $\alpha_3 = 0.75$, $S_n = 5*10^{-5}$ and $\sigma = 0.0154$.

The two actuator faults signals $f_a(t) = [\, f_{a1}(t) \quad f_{a2}(t) \,]$ are defined as:

$$f_{a1}(t) = \begin{cases} \sin(0.4\pi t), & \text{for } 15s \le t \le 75\text{s} \\ 0, & \text{elswere} \end{cases} \quad \text{and}$$

$$f_{a2}(t) = \begin{cases} 0.3, & \text{for } 20s \le t \le 70\text{s} \\ 0.5, & \text{for } t > 70\text{s} \\ 0, & \text{elswere} \end{cases}$$

It is supposed that a sensor fault $f_s(t)$ is affecting the system. This fault is defined as follows: $f_s(t) = [\, f_{s1}(t) \quad f_{s2}(t) \,]$ with:

$$f_{s1}(t) = \begin{cases} 0, & \text{for } t < 35\text{s} \\ 0.6, & \text{for } t \ge 35\text{s} \end{cases} \quad \text{and}$$

$$f_{s2}(t) = \begin{cases} 0, & \text{for } t < 25\text{s} \\ \sin(0.6\pi t), & \text{for } t \ge 25\text{s} \end{cases}$$

The chosen weighting functions depends on the system input $u(t)$. They have been created on the basis of Gaussian membership functions. Figure (2) shows their time-evolution showing that the system is clearly nonlinear since $\mu_i, i \in \{1,...,4\}$ are not constant functions.

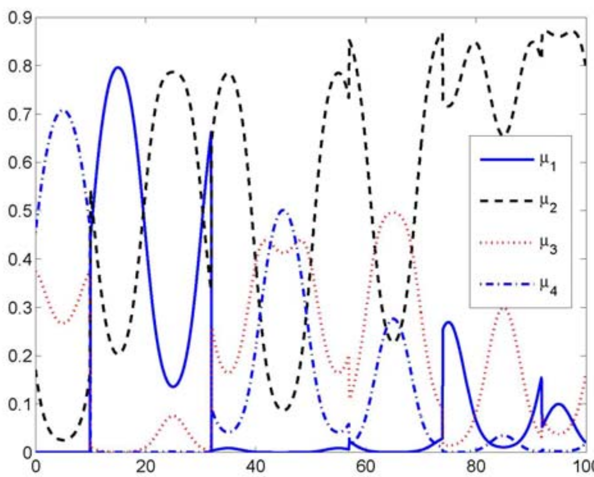

Fig. 2. Weighting functions

Choosing, $\bar{A} = 10 \times I$ the $\lambda, K_1, K_2, K_3, K_4, L_1, L_2, L_3, L_4$ and G computation gives: $\lambda = 1.2936$,

$$L_1 = \begin{bmatrix} 37.57 & 11.81 & -18.60 \\ 26.93 & 8.46 & 23.66 \\ -12.26 & 27.19 & -31.1 \\ -3.78 & 39.31 & 20.545 \end{bmatrix} \quad L_2 = \begin{bmatrix} 37.16 & 11.1 & -20.35 \\ 27.73 & 8.56 & 23.34 \\ -21.75 & 53.53 & -49.31 \\ -11.23 & 74.74 & 33.12 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} 37.98 & 10.65 & -22.71 \\ 30.44 & 8.71 & -24.84 \\ -32.98 & 81.68 & -71.08 \\ -18.38 & 113.29 & 47.59 \end{bmatrix} \quad L_4 = \begin{bmatrix} 36.87 & 10.1 & -22.74 \\ 30.96 & 8.56 & 24.05 \\ -43.14 & 106.16 & -92.72 \\ -24.36 & 148.41 & 62.18 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} -5.24 & -1.17 & -14 \\ 15.32 & 17.34 & 39.18 \\ -7.40 & -1.30 & -9.80 \\ 0.47 & 5.87 & 3.80 \\ -0.08 & 8.87 & 2.12 \\ 4.93 & 3.29 & 14.11 \end{bmatrix} \quad K_2 = \begin{bmatrix} -4.67 & 1.20 & -12.76 \\ 13.85 & 20.79 & 40.87 \\ -9.07 & -1.29 & -8.01 \\ -3.88 & 6.37 & 5.61 \\ 2.36 & 4.68 & 4.49 \\ 5.99 & 3.977 & 11.88 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} -3.91 & 2.67 & -13.4 \\ 10.22 & 25.26 & 45.13 \\ -10.14 & -1.40 & -7.17 \\ -9.05 & -5.74 & -6 \\ 4.56 & -0.03 & 4.28 \\ 7 & 5.03 & 9.81 \end{bmatrix} \quad K_4 = \begin{bmatrix} -3.79 & 6.85 & -12.68 \\ 9.01 & 26.6 & 36.03 \\ -11.86 & -1.24 & -5.94 \\ -13.37 & 6.66 & 7.90 \\ 6.86 & -3.94 & 5.54 \\ 8.47 & 3.65 & 5.40 \end{bmatrix}$$

$$G = \begin{bmatrix} -2.44 & -2.99 & 1.96 & -4.35 & 3.17 & 7.58 \\ -0.53 & 4.53 & 7.53 & -2.52 & 5.43 & -1.02 \end{bmatrix}$$

The obtained results are shown in figures (3) to (7).

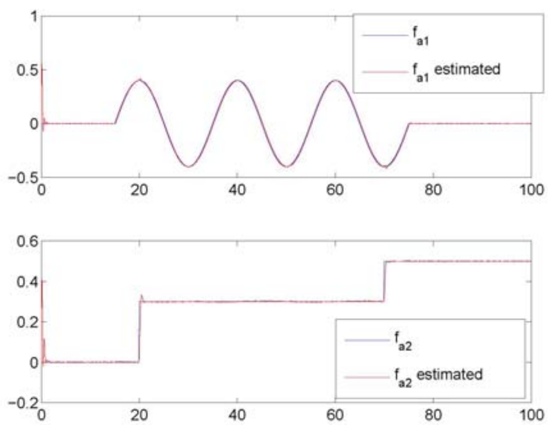

Fig. 3. Actuator faults and their estimation

Figure (3) visualizes the two actuator faults ($f_{a1}(t)$ and $f_{a2}(t)$) and their estimations, the two sensor faults ($f_{s1}(t)$ and $f_{s2}(t)$) and their estimations are represented in figure (4). In figure (5), the state error estimation is visualized.

These three figures show that the proposed observer permits to estimate simultaneously the sensor and actuator faults and the system state. The application of the proposed method to the three tanks system shows its robustness. Simulation results show that the fault is estimated well and the effect of the measurement noise is minimized. This method allows estimating well the sensor and actuator faults even in the case of time-varying faults.

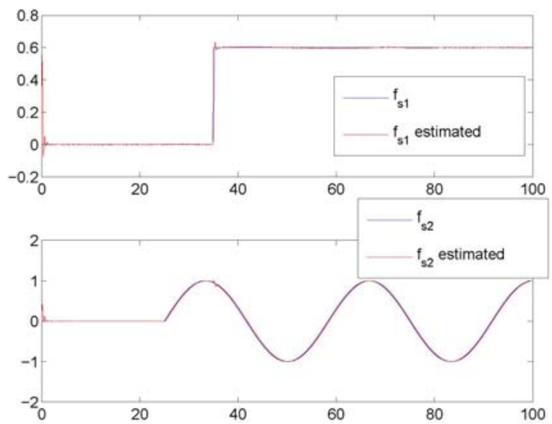

Fig. 4. Sensor faults and their estimation

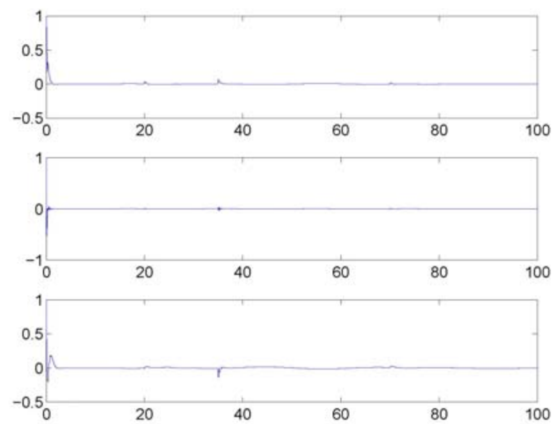

Fig. 5. state error estimation

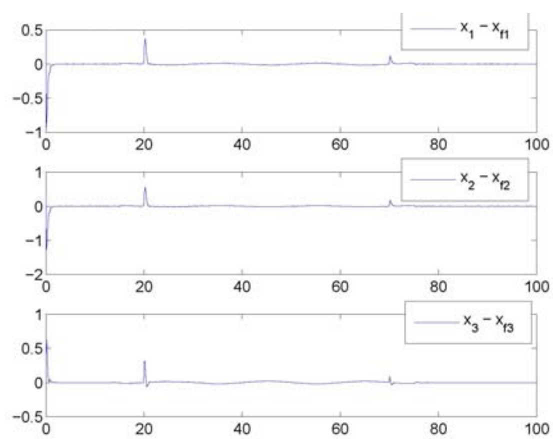

Fig.6. error between x and x_f

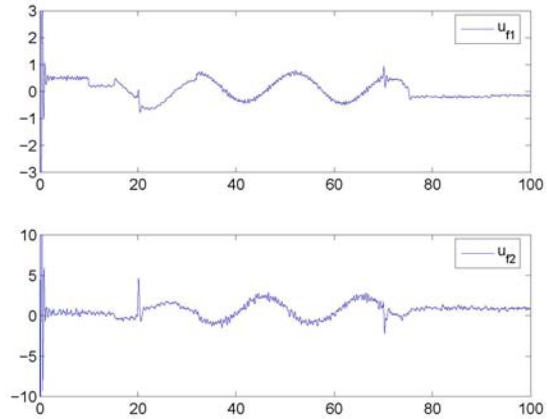

Fig. 7. Fault tolerant control u_f

Figure (6) shows the time-evolution of the error $\tilde{X}(t)$ between the reference state $x(t)$ and the faulty state $x_f(t)$. This error converges toward zero. So the application of the conceived fault tolerant control law $u_f(t)$ to the faulty system let the behavior of the system affected by the sensor and the actuator fault similar to the reference system behavior. The action of the proposed fault tolerant control is quick.

Fault and state estimation is very important because the fault and state estimated are used to conceive the fault tolerant control strategy. This control is shown in the figure (7)

5. Conclusion

This work proposes a direct application of the use of proportional integral observer to the fault tolerant control design. This control was conceived for systems described by Takagi-Sugeno fuzzy models with weighting function depending on the FTC. The proposed method is based on the estimation of the state and faults affecting the system.

To make faults estimation, a mathematical transformation was used to conceive an augmented system in which the sensor fault affecting the initial system appears as an actuator fault. Then an adaptive proportional integral observer is used to estimate simultaneously actuator and sensor faults and the system state. The main contribution in this work is that the considered systems have the weighting functions depending on the fault tolerant control which is a very important case and is the subject of few works and in the use of the mathematical transformation and the proportion integral observer to estimate time-varying sensor and actuator faults. The FTC controller is implemented as a state feedback controller. This controller is designed such that it can stabilize the faulty plant using Lyapunov theory and LMIs.

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