Inventory Lot-Sizing Problem with Supplier Selection under Storage Space and Budget Constraints

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Abstract

In this paper, we consider a multi-period inventory lot-sizing problem with supplier selection under storage space and budget constraints. The objective of this research is to calculate the optimal inventory lot-sizing for each supplier and minimize the total inventory cost which includes joint purchase cost of the products, transaction cost for the suppliers, and holding cost for remaining inventory. It is assumed that demand of multiple products is known over a planning horizon. The problem is formulated as a mixed integer linear programming and is solved with optimization package like LINGO12. Finally, numerical example is provided to illustrate the solution procedure. The results determine what products to order in what quantities with which suppliers in which periods, in order to satisfy overall demand.

Keywords: *Inventory lot-sizing, Supplier selection, Storage space.*

1. Introduction

Lot-sizing problems are production planning problems with the objective of determining the periods where production should take place and the quantities to be produced in order to satisfy demand while minimizing production and inventory costs [1]. Since lot-sizing decisions are critical to the efficiency of production and inventory systems, it is very important to determine the right lot-sizes in order to minimize the overall cost.

Lot-sizing problems have attracted the attention of researchers. The multi-period inventory lot-sizing scenario with a single product was introduced by Wagner and Whitin [2], where a dynamic programming solution algorithm was proposed to obtain feasible solutions to the problem.

Soon afterwards, Basnet and Leung [3] developed the multi-period inventory lot-sizing scenario which involves multiple products and multiple suppliers. The model used in these former research works is formed by a single-level unconstrained resources indicating the type, amount, suppliers and purchasing time of the product. This model is not able to consider the capacity limitations. One of the important modifications we consider in this paper is that of introducing storage capacity and budget constraints.

This paper is built upon Basnet and Leung [3] model. We formulate the multi-product and multi-period inventory lot-sizing problem with supplier selection under storage space. The problem is formulated as a mixed integer linear programming. The objective of this research is to calculate the optimal inventory lot-sizing for each supplier and minimize the total inventory cost (including holding, transaction and, purchasing cost). The results determine what products to order in what quantities with which suppliers in which periods, in order to satisfy overall demand.

This paper is organized as follows: Section 2 provides a literature review on the current inventory lot-sizing. Section 3 we describe our model. In Section 4 presents a numerical example of the model. Finally, computation results and conclusions are presented in Section 5 and 6.

2. Literature review

Inventory lot-sizing has been one of the most studied problems in production and inventory management literature. Bahl et al [4] proposed four categories for classifying works in this area: (1) single-level unconstrained resources, (2) single-level constrained resources, (3) multiple-level constrained resources, and (4) multiple-level unconstrained resources. Levels refer to the different levels in a bill of material structure where dependency of requirements exists, and constrained resources refer to production capacity limitations.

The scenario discussed in this paper belongs to the second category. The multi-period inventory lot-sizing which involves with multiple products and multiple suppliers under storage space and budget constraints. The study lot-sizing began with Wagner and Whitin [2], provided a dynamic programming algorithm for a single product case. This problem is known as the uncapacitated single item single level lot-sizing problem.

With the advent of supply chain management, much attention is now devoted to supplier selection. Rosenthal et al [5] studied a purchasing problem where one needs to select among suppliers who offer discounts selling a ‘‘bundle’’ of multiply products. Then a mixed integer programming formulation was presented. Chaudhry et al [6] considered vendor selection under quality, delivery and capacity constraints and price-break regimes. Ganeshan [7] presented a model to determine lot sizes that involve multiple suppliers including multiple retailers, and consequent demand on a warehouse. Kasilingam and Lee [8] incorporated the fixed cost of establishing a vendor in a single-period model that includes demand uncertainties and quality considerations in the selection of vendors. Also vein, Jayaraman et al [9] proposed a supplier selection model that considers quality (in terms of proportion of defectives supplied by a supplier), production capacity (constraining the order placed on a supplier), leadtime, and storage capacity limits. This is also a single period model that attaches a fixed cost to deal with a supplier.

Included in the stream of researches integrating supplier selection and procurement lot-sizing are works by Oliver [10], Rule [11], Chappell [12], Williams and Redwood [13], Anthony and Buffa [14], Buffa and Jackson [15], Bender et al [16], Pan [17], Tempelmeier [18], and Basnet and Leung [3]. They consider a multi-period planning horizon and define variables to determine the quantity purchased in each elementary period. Buffa and Jackson [15] presented a schedule purchase for a single product over a defined planning horizon via a goal programming model considering price, quality and delivery criteria. Bender et al [16] studied a purchasing problem faced by IBM involving multiple products, multiple time periods, and quantity discounts. The authors described, but not developed, a mixed integer optimization model, to minimize the sum of purchasing, transportation and inventory costs over the planning horizon, without exceeding vendor production capacities and various policy constraints. Tempelmeier [18] proposed a planning model for supporting short-term selection and order sizing under time varying parameters.

Basnet and Leung [3] presented a multi-period inventory lot-sizing scenario where there are multiple products and multiple suppliers. They considered a situation where the demand of multiple discrete products is known over a planning horizon. The model determines the type, amount, supplier and purchasing time of products. Their model is one of the most useful ones for supply selection in a single stage category. They proposed an uncapacitated mixed integer programming that minimizes the aggregate purchasing, ordering and holding costs subject to demand satisfaction. The authors proposed an enumerative search algorithm and a heuristic algorithm to solve the problem.

3. Formulation

We also make the following assumptions and mathematical for the model:

3.1 Assumptions

- Demand of products in period is known over a planning horizon.
- All requirements must be fulfilled in the period in which they occur: shortage or backordering is not allowed.
- Transaction cost is supplier dependent, but does not depend on the variety and quantity of products involved.
- Holding cost of product per period is product-dependent.
- Initial inventory of the first period and the inventory at the end of the last period are assumed to be zero.
- Product needs a storage space and available total storage space is limited.
- Order lead-time is deterministic and is the same for each period are assumed to be zero.

Base on the above assumption of model, Fig. 1 shows the behavior of the model considering the scenario of multi-period inventory lot-sizing problem with supplier selection under storage space and budget constraints.

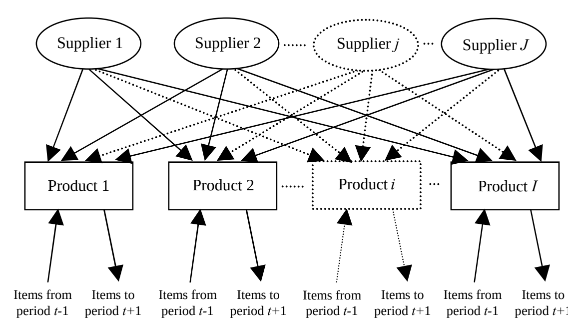

Fig. 1 Behavior of the model in period t .

3.2 Mathematical modeling

This paper is built upon Basnet and Leung [3] model. We formulate the multi-product and multi-period inventory lot-sizing problem with supplier selection under storage space and budget constraints using the following notation:

Indices:

i = $1,\ldots,I$ index of products
j = $1,\ldots,J$ index of suppliers
t = $1,\ldots,T$ index of time periods

Parameters:

D_{it} = demand of product i in period t
P_{ij} = purchase price of product i from supplier j
H_i = holding cost of product i per period
O_j = transaction cost for supplier j
w_i = storage space product i
S = total storage space
B_t = purchasing budget in period t

Decision variables:

X_{ijt} = number of product i ordered from supplier j in period t
Y_{jt} = 1 if an order is placed on supplier j in time period t, 0 otherwise

Intermediate variable:

R_{it} = Inventory of product i, carried over from period t to period $t+1$

Regarding the above notation, the mixed integer programming is formulated as follows:

$$Minimize\ (TC) = \sum_i\sum_j\sum_t P_{ij}X_{ijt} + \sum_j\sum_t O_jY_{jt} + \sum_i\sum_t H_i\left(\sum_{k=1}^{t}\sum_j X_{ijk} - \sum_{k=1}^{t} D_{ik}\right) \quad (1)$$

Subject to:

$$R_{it} = \sum_{k=1}^{t}\sum_j X_{ijk} - \sum_{k=1}^{t} D_{ik} \geq 0 \quad \text{for all } i \text{ and } t, \quad (2)$$

$$\left(\sum_{k=t}^{T} D_{ik}\right) Y_{jt} - X_{ijt} \geq 0 \quad \text{for all } i,\ j, \text{ and } t, \quad (3)$$

$$\sum_i w_i\left(\sum_{k=1}^{t}\sum_j X_{ijk} - \sum_{k=1}^{t} D_{ik}\right) \leq S \quad \text{for all } t, \quad (4)$$

$$\sum_i\sum_j X_{ijt}P_{ij} \leq B_t \quad \text{for all } t, \quad (5)$$

$$Y_{jt} = 0 \text{ or } 1 \quad \text{for all } j \text{ and } t, \quad (6)$$

$$X_{ijt} \geq 0 \quad \text{for all } i,\ j, \text{ and } t, \quad (7)$$

The objective function as shown in Eq. (1) consists of three parts: 1) purchase cost of the products, 2) transaction cost for the suppliers, and 3) holding cost for remaining inventory in each period in $t+1$.
Constraint in Eq. (2) all requirements must be filled in the period in which they occur: shortage or backordering is not allowed. Constraint in Eq. (3) there is not an order without charging an appropriate transaction cost. Constraint in Eq. (4) each products have limited capacity. Constraint in Eq. (5) the total purchasing payment for each item cannot exceed the budget in period. Constraint in Eq. (6) is binary variable 0 or 1 and Constraint in Eq. (7) is non-negativity restrictions on the decision variable.

4. A numerical example

In this section we solved a numerical example of the model using the LINGO12. We consider a scenario with three products over a planning horizon of five periods whose requirements are as follows: demands of three products over a planning horizon of five periods and purchasing budget are show in Table 1.
There are three suppliers and their prices and transaction cost, holding cost and storage space are show in Table 2 and Table 3, respectively.

Table 1: Demands of three products over a planning horizon of five periods (D_{it}) and budget of them (B_t).

	Planning Horizon (Five Periods)				
Products	1	2	3	4	5
A	12	15	17	20	13
B	20	21	22	23	24
C	20	19	18	17	16
Budget	1,820	2,000	3,500	3,000	3,500

Table 2: Price of three products by each of three suppliers X, Y, Z (P_{ij}) and transaction cost of them (O_j).

Table 3: Holding cost of three products A, B, C (H_i) and

	Price		
Products	X	Y	Z
A	30	33	32
B	32	35	30
C	45	43	45
Transaction Cost	110	80	102

storage space of them (w_i).

The total storage space (S) is equal to 200.

The results of applying the proposed method are shown in

	Products		
	A	B	C
Holding Cost	1	2	3
Storage Space	10	40	50

Table 4. The solution of this problem (I = 3, J = 3, and T = 5) is to place the following orders.
All other $X_{ijt} = 0$:

Table 4: Order of three products over a planning horizon of five periods (X_{ijt}).

	Planning Horizon (Five Periods)				
Products	1	2	3	4	5
A	$X_{111}=12$	$X_{132}=15$	$X_{113}=37$	-	$X_{135}=13$
B	$X_{231}=20$	$X_{232}=21$	$X_{213}=22$	$X_{234}=23$	$X_{235}=24$
C	$X_{321}=20$	$X_{332}=19$	$X_{313}=18$	$X_{334}=17$	$X_{335}=16$

Cost calculation for this solution:
Purchase cost for product 1 from supplier 1, 3
= (12+37) × 30 + (15+13) × 32 = 2,366.
Purchase cost for product 2 from supplier 1, 3
= (22×32) + (20+21+23+24) × 30 = 3,344.
Purchase cost for product 3 from supplier 1, 2, 3
= (18×45) + (20×43) + (19+17+16) × 45 = 4,010.
Transaction cost from supplier 1, 2, and 3
= (2×110) + (1×80) + (4×102) = 708.
Holding cost for product 1

$R_{13} = X_{113} - D_{13} = 37 - 17 = 20.$

$= H_1 \sum R_{1t} = 1\times (0 + 0 + 20 + 0 + 0) = 20.$

Thus, the total cost for this solution
= 2,366 + 3,344 + 4,010 + 708 + 20
= 10,448.

5. Computation results

In this section, we are using a commercially available optimization package like LINGO12. Experiments are conducted on a personal computer equipped with an Intel Core 2 duo 2.00 GHz, CPU speeds, and 1 GB of RAM. The transaction costs are generated from *int* [50; 200], a uniform integer distribution including 50 and 200. The prices are from *int* [20; 50], the holding costs from *int* [1; 5], the storage space from *int* [10; 50], and the demands are from *int* [10; 200]. Computational results using the problem size are documented in Table 5. A problem size of $I; J; T$ indicates number of suppliers = I, number of products = J, and number of periods = T. Computation time limit is set at 120 minutes [3]. The optimal objective value for this model is shown in Fig. 2. The solution time of LINGO12 to optimal is a short time as the small problem size (with the problem sizes 3 x 3 x 5; 3 x 3 x 10; 3 x 3 x 15; and 4 x 4 x 10).
For large problems sizes LINGO12 cannot obtain optimal solutions within limit time due to as the larger problem size (with the problem sizes 4 x 4 x 15; 5 x 5 x 20; 10 x 10 x 50; 10 x 10 x 80; and 15 x 15 x 50). Next, we study % error in the problem sizes solutions. The results are show in Fig. 3 LINGO12 used a maximum % error from the optimal solutions is found to be 4.41% (at the problem size 10 x 10 x 80). For comparison, the percentage error is calculated by Eq. (8).

Percentage error of LINGO12

$$= \left[\frac{\text{Upper bound - Lower bound}}{\text{Upper bound}}\right] \times 100 \qquad (8)$$

As show in Fig. 4 a plot of the problem size versus solution time. LINGO12 uses longer computation time (with the problem sizes 4 x 4 x 15; 5 x 5 x 20; 10 x 10 x 50; 10 x 10 x 80; and 15 x 15 x 50). Additionally, the computation time when using LINGO12 is also short, making it a very practical means for solving the multiple products and multi-period inventory lot-sizing problem with supplier selection under storage space and budget constraints.

Table 5: Computational results

Problem size	Total cost	Solution time (minutes)	% Error
3 x 3 x 5	10,448	0.01	0
3 x 3 x 10	20,644	0.14	0
3 x 3 x 15	30,966	14.35	0
4 x 4 x 10	25,436	6.34	0
4 x 4 x 15	38,154[a] , 37,828[b]	120	0.85
5 x 5 x 20	60,218[a] , 59,527[b]	120	1.14
10 x 10 x 50	285,344[a] , 274,758[b]	120	3.70
10 x 10 x 80	456,494[a] , 436,317[b]	120	4.41
15 x 15 x 50	417,800[a] , 405,155[b]	120	2.66

[a]LINGO12 = Upper bound, [b]LINGO12 = Lower bound.

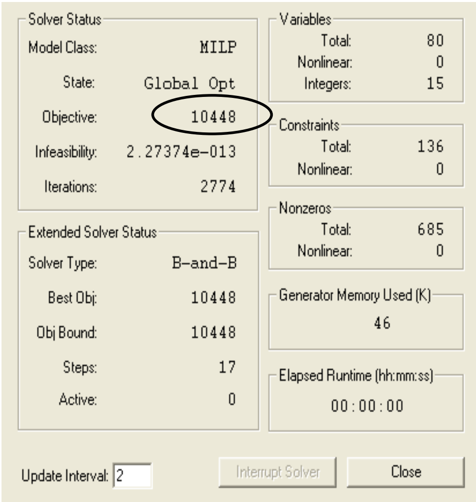

Fig. 2 The optimal objective value for this model

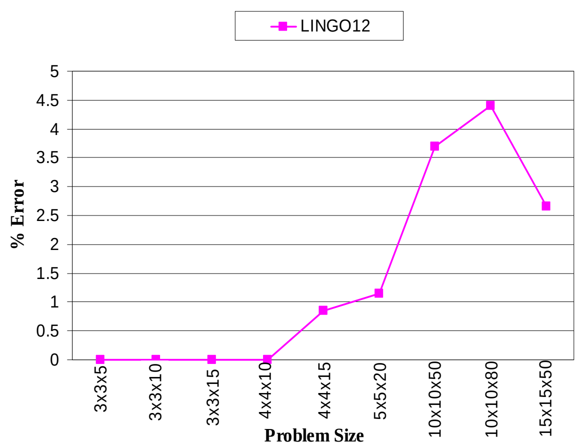

Fig. 3 Plot of the problem size vs. % error

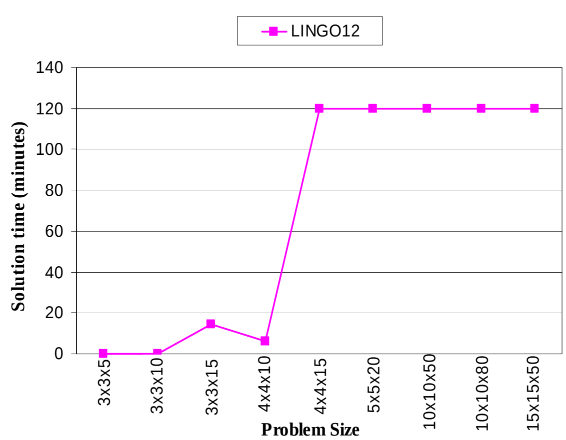

Fig. 4 Plot of the problem size vs. solution time

6. Conclusions

In this paper, we consider a multi-period inventory lot-sizing problem with supplier selection under storage space and budget constraints. The results determine what products to order in what quantities with which suppliers in which periods, in order to satisfy overall demand. The mathematical model is give and the use of the model is illustrated though a numerical example. The problem is formulated as a mixed integer programming and is solved with LINGO12. Future works for this research includes providing computational tests results, comparing the test results with other solution methodologies.

Appendix

A coding schema of Lingo12 (at the problem size 3 x 3 x 5)

```
ITEMS / 1..3 / ;
SUPPLIER / 1..3 / ;
PERIODS / 1..5 / ;
PLINK(ITEMS, SUPPLIER) : P ;
LINKS(ITEMS, SUPPLIER, PERIODS) : X ;
ODERING(SUPPLIER): O ;
YLINKS(SUPPLIER, PERIODS) : Y ;
HOLDING(ITEMS) : H ;
DLINKS(ITEMS, PERIODS) : D, temp1;
ITEM(ITEMS): temp 2,W;
PER(PERIODS): temp3, BT;
ENDSETS

DATA :

    H    =    1     2     3;
    O    =    110   80    102;
    P    =    30    33    32
              32    35    30
              45    43    45;
    D    =    12  15  17  20  13
              20  21  22  23  24
              20  19  18  17  16;
    W      =  10   40    50;
    S    =    200;
    BT   =    1820  2000  3500  3000  3500;

ENDDATA

 !Objective functions minimize cost. ;
MIN     =@SUM(LINKS(I,J,T) : P(I,J)*X(I,J,T)) +
          @SUM(YLINKS(J,T) : O(J)*Y(J,T))+
          @SUM (DLINKS(I,T): H(I) * temp1(I,T));
          @FOR(DLINKs(I,T):temp1(I,T)=
          @SUM(YLINKS(J,k)  |k  #LE#  T:  X(I,J,k))-
          @SUM(PER(k)|k #LE# T:D(I,k)));
```

```
 !Constraint 1;
@FOR(DLINKS(I,T):@SUM(YLINKS(J,k)|k#LE#
T:X(I,J,k))-@SUM(PER(k)|k #LE# T:D(I,k)) >=(0;
 !Constraint. 2;
@FOR)ITEM(I):temp)2I)=@SUM)PER(T):D(I,T)));
@FOR)LINKS(I,J,T) : temp)2I)* Y(J,T)- X(I,J,T) >=(0 ;
 !Constraint. 3;
@FOR)PER(T):@SUM)LINKS(I,J,k)|k#LE#T:
X(I,J,k)*W(I))-
@SUM)DLINKs(I,k)|k#LE#T:D(I,k)*W(I))<=S);
 !Constraint. 4;
@FOR)PER(T):temp3)T)=
@SUM)PLINK(I,J):P(I,J)*X(I,J,T)));@FOR)PER(T)
:temp3)T) <=  BT(T));
 !Constraint. 5;
@FOR)YLINKS(J,T):@BIN)Y(J,T)));
 !Constraint. 6;
@FOR)LINKS(I,J,T) :  (X(I,J,T)) >= (0;
END
```

Acknowledgments

The authors gratefully acknowledge the King Mongkut's University of Technology North Bangkok (KMUTNB). Out thanks go to Dr.Krisada Asawarungsaengkul help in programming and running some of the computational experiments.

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