Improved Decoding of linear Block Codes using compact Genetic Algorithms with larger tournament size

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Abstract
Soft-decision decoding is a very important NP-hard problem for developers of communication systems. In this work we propose two new dual domain soft decision decoders that use compact Genetic Algorithm (cGA) with larger tournament size: the first algorithm investigates tournament selection with larger size using mutation, and the second employs higher selection pressure with randomly generated individuals. The obtained results are compared to known previous works and show the effectiveness of using larger tournament size in dual domain soft decision decoding problem. Behind performances analysis, a complexity study is done which shows that both proposed decoders are not very complex in comparison with the standard compact Genetic Algorithm based decoder (cGAD).

Keywords: compact Genetic Algorithm, dual domain Soft decision-decoding, higher selection pressure, mutation, optimization, tournament size, updating step size.

1. Introduction
In digital communication, one of the major problems is how to deliver the message of the source to the destination as faithfully as possible. Error correcting codes are a method among others, used to insure reliable transmission, by removing errors due to channel noise. Indeed, error correcting codes are used to improve the reliability of the transmitted data over communication channels susceptible to noise. During data transmission, error-correcting codes are used twice: for coding and decoding. Coding techniques create code words in the emission by adding redundant information to the initial message, and decoding methods try to find in the reception the nearest code word to the received message (Figure 1).

Decoding algorithms are divided in two categories: hard decision and soft decision. Hard decision decoding works with the binary form of the received word. Soft decision one works directly on the received symbols [10]. Soft-decision decoding is an NP-hard problem and has been approached in different ways. Artificial intelligence techniques were recently introduced to solve this problem. Among the different methods used, Genetic Algorithms (GAs) [14] have taken a big interest in decoding problems like many other scientific areas, and was used in many optimization problems. To our knowledge, Maini et al. were the first to introduce GAs in the soft-decision decoding of linear bloc codes [16]. Cardoso et al have also published an influential work on hard-decision decoding of linear bloc codes using GAs [8]. Belkasmi’s team proposed in 2012 [2] a new decoder
based on GAs which deals with the dual code unlike other genetic decoders that use the code itself. To find the optimum solution, the standard GAs has a high processing overhead for generations and manipulations of populations. It also needs to store all the population's individuals in memory, and the lack of simplest way to initialize different operators and stopping criterions make it impractical to solve real-time optimization problems. Recently, a number of algorithms, called Probabilistic Model Building Genetic Algorithms (PMBGAs); or Estimation of Distribution Algorithms (EDAs) have been developed to replace the GAs population and crossover operator with a probabilistic representation and generation method [4] [11] [19] [17] [13]. The EDAs are divided into three categories based on the interdependencies between the variables of solutions [15]: the no dependencies model, the bivariate dependencies model and the multiple dependencies model.

The compact Genetic Algorithm (cGA) [13], is one of the simplest algorithms of the first category, it generates offspring population according to an estimated probabilistic model of parent population instead of using traditional genetic operations. It only needs a small amount of memory which makes it useful for memory-constrained applications. Shakeel first introduced this algorithm for soft-decision decoding using the code itself [18]: his decoder uses the generator matrix of the code; this fact makes the decoding very complicated for high rate codes. We later used this algorithm for dual domain soft-decision decoding of linear codes [3], this decoder is less complex than Shakeel decoder for codes with a high rate (n-k<<n), where n is the code length and k its dimension.

In this work, we propose two new dual domain soft decision decoders that investigates tournament with larger size in the compact Genetic Algorithm using mutation and higher selection pressure, and then, study their performance, compare them with others known decoders and study their complexity. This paper is organized as follows: section 2 gives a description of the standard compact Genetic Algorithm, presents the tournament with larger size and how we can introduce it in the standard compact Genetic Algorithm. Section 3 develops the new Dual Domain Soft-Decision Decoders based on larger tournament size using mutation and higher selection pressure. Performance analysis is presented in section 4, and the complexity one is presented in the section 5. The paper concludes with a summary of the results in Section 6.

2. cGA with larger tournament size

2.1 The compact Genetic Algorithm

Harik et al presented in 1998 [13] the Compact Genetic Algorithm which is one of the simplest EDAs. Instead of dealing with natural operators of GAs, this algorithm uses a probability vector (p). During the processes of optimization, binary solutions are randomly generated from p at each generation, after that, they compete based on their fitness values. p is then updated based on these solutions until reaching the convergence state known when all positions of p became binary (0 or 1). This state of convergence represents the final solution of the optimization problem. The cGA represents the population as a probability vector over a set of solutions and operationally mimics the order-one behavior of the simple GA with the uniform crossover. The cGA has an advantage of using a small amount of memory, and then, may be quite useful in memory-constrained applications. Algorithm 1 describes the pseudo-code of the cGA.

Algorithm 1: Pseudo code of the compact Genetic Algorithm

Require l the length of p;
Require 1/λ: The updating step size.
1. Initialize the probability vector p which length is l

for m := 1 to l do p[m] := 0.5
end for

2. Generate two individuals from p

a := generate(p);
b := generate(p);

3. Let them compete

winner; loser := compete(a; b);

4. Update p towards the better one

for m := 1 to l do
    if winner[m] ≠ loser[m] then
        if winner[m] == 1 then p[m] := p[m] + 1/λ;
        else p[m] := p[m] - 1/λ;
    end if
end if

end for

5. Check if the vector p has converged

for m := 1 to l do
    if p[m] > 0 and p[m] < 1 then return to step 2;
end if

end for

6. The probability vector p represents the final solution.

2.2 Larger tournament size using mutation

2.2.1 Mutation operator

Mutation is an operator referring in nature to an "accident" that changes DNA and adds a new allele which does not exist before. In GAs, its concept is quite simple: a gene is chosen randomly and its allele is changed. In the case of
binary strings, which we are interested on in this work, mutation can be made by complementing the chosen bit. Alone, the mutation does not generally advances the search for a solution, but it insures the evolution of population and avoids the development of a uniform one.

2.2.2 Using mutation to generate individuals

This version of cGA differs from the standard version at the level of competition's vectors generation. Instead of using only two vectors \( a \) and \( b \), this version uses a number of vectors defined according to a parameter \( s \). We generate randomly two binary vectors \( a \) and \( b \) from \( p \), we mutate \( b \) in such way of generating \((s-1)\) other vectors. We select the best according to the objective function between \( b \) and \((s-1)\) resultant vectors of the mutation, then we let it compete with the vector \( a \). This version of the compact Genetic Algorithm is going to replace stages 2, 3, and 4 of the standard version by the stages given in Algorithm 2.

Algorithm 2: Modification of the cGA to implements \( s \) vectors of competition using mutation.

1. Generate two individuals from \( p \)
   
   \( a := generate(p) \)
   
   \( b := generate(p) \)

2. Mutate \( b \) to generate \((s - 1)\) binary vectors
3. Determine the vector with the best value of fitness among \( b \) and \((s - 1)\) other vectors and to name it \( N \);
4. Compete\((a;N)\)
   
   winner; loser := compete\((a;N)\);

5. Update \( p \) towards the winner.

2.3 Larger tournament size using randomly generated vectors with Higher Selection Pressure

The selection operator helps to give more copies to better individuals. But it does not always help to keep the better genes, because the evaluation is done in the context of the individual altogether. Selection pressure controls the selection of individuals from one population to the next. It gives individuals of higher quality, higher probability of being used to create the next generation, and so the algorithm will focus on promising regions in the search space. In other words, higher selection pressure may play the role of memory to store best candidates' solution seen during the search progress. It can be insured with the cGA by introducing elitism [1], or by using tournament selection with increased size [13].

Harik et al introduced the simulation of higher selection pressure by increasing the tournament size in [13], instead of mimicking the pair-wise tournament selection in the standard version of cGA. Tournament with size \( s \) is implemented with the cGA by:

Generating \( s \) individuals from the probability vector and find out the best one;

Let the best individual compete with the others \( s-1 \) individuals, updating the probability vector along the way.

In order to simulate higher selection pressure, steps 2 to 4 of the cGA's pseudo-code (Algorithm 1) have to be replaced by the ones shown in Algorithm 3.

Elitism strategy is also a method to implement higher selection pressure. It is one of the operational characteristics of Genetic Algorithms (GA) that provides a reduction of genetic drift by insuring that the best candidate solution is allowed to contribute to the next generation. That is, the best chromosome is transferred to the next generation without alteration. This one gets the full chance to compete with the other candidates of the next generation. C. A. Wook and R. S. Ramakrishna introduced two elitism-based cGAs: the persistent-elitist cGA and the non persistent-elitist one [1].

Algorithm 3: Modification of the cGA using \( s \) vectors of competition

Require: \( l \) the lenght of \( p \)

Require: \( 1/λ \), The updating step size

1. Generate \( s \) individuals from the probability vector \( p \), and store them in \( S \):

   \( \text{for } i := 1 \text{ to } s \text{ do } \)
   
   \( S[i] := generate(p); \)
   
   \( \text{end for} \)

2. Rearrange \( S \) so that \( S[1] \) is the fittest individual :

3. Let \( S[1] \) compete with the others individuals :

   \( \text{for } j := 1 \text{ to } s \text{ do } \)
   
   \( \text{winner; loser := compete}(S[1]; S[j]); \)
   
   \( \text{update } p \text{ (step 4 of the cGA)} \)
   
   \( \text{end for} \)

3. Decoding algorithms using larger tournament size

The two versions of cGA decoders presented in this work try to solve the dual domain soft-decision decoding of linear error correcting codes. Dual domain Soft-decision decoding is firstly modeled as an optimization problem and then a description of the two decoding algorithms is given.

3.1 Dual domain Soft-decision decoding as an optimization problem

The maximum-likelihood soft-decision decoding problem of linear block codes is an NP-hard problem [7], and can be stated as follows: Given the received vector \( r \) and the parity-check matrix \( H \), let \( S = w^H \) be the syndrome of \( w \),
where \( w \) is the hard decision of \( r \), \( H^T \) is the transpose of matrix \( H \), and \( E(S) \) be the set of all errors patterns whose syndrome is \( S \), find the \( E \in E(S) \) which minimizes correlation discrepancy:

\[
F_r(E) = \sum_{j=1}^{n} (E_j|r_j|) = \sum_{j=1}^{n} \left| E_j(r_j) \right| \tag{1}
\]

Where \( n \) is the code length. The optimization problem (1) has \( n \) error variables, out of which only \((n-k)\) are independent, where \( k \) is the code dimension. Using the algebraic structure of the code, the remaining \( k \) variables can be expressed as a function of these \((n-k)\) variables.

An individual is a set of \( k \) bits. Let \( E' \) be an individual, \( SL \) be a \((n-k)\)-tuple such that \( SL = E' \times A \) where \( A \) is a sub matrix of \( H' \), and \( S2 \) be a \((n-k)\)-tuple such that \( S2 = S + SL \). We form the \( E \) error pattern such that \( E' = (E', E'') \) where \( E' \) is the chosen individual and \( E'' = S2 \). Then \( w + E \) is a code word. The fitness function is the correlation discrepancy between the permuted received word and the estimated error \( E \) such indicated in (1).

Until now, just few numbers of researches treat the soft-decision decoding as an optimization problem because it is identified as a NP-hard problem [7], and also because optimization techniques generally solve a large problems once when the soft-decision decoding occurs repeatedly in communication systems.

### 3.2 Description of decoders

Let \( C \) denotes, binary linear block code of length \( n \), dimension \( k \) and minimum Hamming distance \( d \), defined over the Galois field of order 2 \((GF(2))\). Let \( H \) be its parity check matrix, and \((r_i)\), \( 1 \leq i \leq n \) be the received sequence over a communication channel with noise variance \( \sigma^2 = N_0/2 \), where \( N_0 \) is noise power spectral density. We assume that codewords are modulated by a BPSK modulator and transmitted over an \( AWGN \) channel. Once the message received, the demodulator makes hard-decisions \( w_i \), \( i = 1, ..., n \):

\[
w_i = \begin{cases} 1, & \text{if } r_i \geq 0 \\ 0, & \text{if } r_i < 0 \end{cases}
\]

(2)

The receiver then calculates the syndrome \( w \times H^T \) and accepts \( w \) as the most likely transmitted codeword if the syndrome \( w \times H^T = 0 \). If the syndrome \( w \times H^T \neq 0 \), the soft-decision decoding process begins as described in algorithms 4 and 5: the first one implements higher selection pressure with randomly generated vectors \( cGAD-HSP \), and the second one uses mutation to have tournament with larger size \( cGAD-M \).

#### Algorithm 4: cGAD-HSP

Require: \( k \), The length of \( p \)

Step 1: Sorting the values of sequence \( r \) in decreasing order of reliability \(|r|\). Further, permute the coordinates of \( r \) to ensure that the last \((n-k)\) positions of \( r \) are the least reliable linearly independent positions. Call this vector \( r' \) and let \( \pi \) be the permutation related to this vector \((r' = \pi(r))\). Apply the permutation \( \pi \) to \( H \) to get a new check matrix \( H' = \pi(H) = [A \cdot I_{n-k}] \).

Step 2: Map \( r' \) onto probability vector \( p; p \in R^k \). The probability vector \( p \) defines the starting point for the genetic Search over the \( k \)-dimensional vector space \( F^k \). It is expected that this search would terminate (converge) at a vertex of the \( k \)-dimensional hypercube. An obvious starting point is the center point of the search space, i.e. \( p = (0, 5)^k \). However, the search time and complexity can be greatly reduced if the search is initiated from a point close to the solution vector.

The following steps describe a method that uses soft information of the received vector to determine a starting point close to the optimum solution:

1. \( z \) is the vector formed by the \( k \) first components of \( r' \);
2. Normalize \( z \) using \( |z| \) so that \( 0 \leq z \leq 1 \);
3. Since we are looking for the error pattern, we map the normalized values of \( z \) to the probability vector \( p \) using the transformation:

\[
p_i = 1 - z_i, \quad i = 1, 2, ..., k
\]

(3)

Step 3: Generate \( s \) binary random vectors and store them in \( S; S[j] \in F^k \) following probabilities:

\[
P(S[j] = 1) = p;
\]

\[
P(S[j] = 0) = 1 - p
\]

(4)

An uniform random number generator \( U(0, 1) \) can be used to generate these vectors as follows:

\[
S[i][j] = \begin{cases} 1, & \text{if } U(0, 1) \leq p_i \\ 0, & \text{otherwise} \end{cases}
\]

(5)

Step 4: Rearrange \( S \) so that \( S[1] \) is the fittest individual. This step evaluates the fitness values of each individual \( S[j] \) using the objective function (1), and then arrange \( S \) so that \( S[1] \) will be the fittest one.

Step 5: Let \( S[1] \) compete with the \((s-1)\) others individuals updating the probability vector along the way. Clearly, the best individual wins all the competitions. The update strategy is done following rules:
Step 6: Convergence attained? Here the algorithm checks if all \( p_i \) are equal to 0 or 1. If not, it goes to step 3, but if yes, the converged \( p \) gives the final solution for the objective function (1).

Step 7: Encode \( p \) and apply inverse permutation \( \pi \).

### Algorithm 5: cGAD-M

**Require**: \( k \), The length of \( p \)

**Require**: \( 1/\lambda \), The updating step size

**Step 1**: The same as **Step 1** in algorithm 4

**Step 2**: The same as **Step 2** in algorithm 4

**Step 3**: Produce a pair of binary random vectors \( a \) and \( b \). The pair of vectors is produced with the following probability:

\[
P(a_i; b_i = 1) = p_i;
\]

\[
P(a_i; b_i = 0) = 1 - p_i
\]

(6)

An uniform random number generator \( U(0; 1) \) can be used to generate these vectors as function (5) where \( a_i \) and \( b_i \) will be calculated the same as \( S[i][j] \). Once \( a \) and \( b \) created, we mute \( b \) to create \((s - 1)\) binary random vectors. This group of vectors belongs to \( F^k_2 \).

**Step 4**: Estimate the objective function \( F_r(I) \). This stage estimates the fitness values of the vector \( b \) and \((s - 1)\) generated vectors by using the objective function (1). Once the vector with the best value of fitness is determined, we note it \( N \) and we let it compete with \( a \). The vector with the best fitness (between \( a \) and \( N \)) is identified as the winner whereas the vector with the worst one is identified as the loser.

**Step 5**: Update the probability vector \( p \). The vector of probability is updated towards the winner using the previous rules (step5 of algorithm 4). The size of updating step \( 1/\lambda \) is a parameter defined by the user. It directly affects the performances of the decoder.

**Step 6**: Convergence attained? Here the algorithm checks if all \( p_i \) are equal to 0 or 1. If not, it goes to step 3, but if yes, the converged \( p \) gives the final solution for the objective function (1).

### 4. Performance analysis

In order to show the effectiveness of the new decoders in Dual Domain Soft-Decision Decoding, we carry out intensive simulations. For transmission we used an AWGN channel with a BPSK modulation. The simulations were made with default simulation's parameters outlined in Table 1 and the default cGA parameters in Table 2. The performance is given in terms of bit error rate (BER) as a function of Signal to Noise Ratio (SNR).

<table>
<thead>
<tr>
<th>Table 1: Simulations Parameters</th>
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<tbody>
<tr>
<td><strong>Simulation parameter</strong></td>
</tr>
<tr>
<td>Channel</td>
</tr>
<tr>
<td>Modulation</td>
</tr>
<tr>
<td>Minimum number of transmitted blocks</td>
</tr>
<tr>
<td>Minimum number of residual bit errors</td>
</tr>
<tr>
<td>Default code</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2: cGA Parameters</th>
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</thead>
<tbody>
<tr>
<td><strong>cGA parameter</strong></td>
</tr>
<tr>
<td>( \lambda )</td>
</tr>
<tr>
<td>( s )</td>
</tr>
<tr>
<td>Mutation rate (pm)</td>
</tr>
</tbody>
</table>

4.1 The gain of larger tournament size in Dual Domain Soft-Decision Decoding

In order to show that tournament with larger size improves the decoding performance, in term of performance, we chose binary Golay\(^1\) (23, 12) code and Figure 2 shows the results. From this Figure we can see that the cGAD with higher selection pressure using randomly generated individuals (cGAD-HSP) presents best performances than the one using mutation (cGAD-M). At BER value equal to \( 5 \times 10^-5 \) cGAD-HSP offers a gain of 0.6 dB, when cGAD-M gives the same performance as the standard cGAD presented by the green curve (s=2).

\(^1\)Golay codes: A family of perfect linear error-correcting block codes.
We compare afterwards the decoder based on standard compact Genetic Algorithm that uses only two competition's vectors \((s=2)\), and the proposed decoders. For BCH\(^1\) (63,45,7), As we can see in the Figure 3, the new decoders outperform the standard one (a gain of 0.9 dB at BER value 4.5 \(10^{-5}\) for cGA-M and a gain of 1.2 dB at 2.10\(^{-5}\) for cGA-HSP) which proves the efficiency of the proposed Decoders.

The simulations of the BCH(127,64,21) code show that both decoders cGA-HSP and cGA-M are slightly better than the standard cGAD as illustrated in Figure 4.

We do the same thing for the code QR\(^2\) (71,36) and as expected, both versions improve the decoding performance. The Figure 5 shows that cGA-HSP gives a gain of 1 dB at 4.10\(^{-5}\), where cGA-M gives a gain of about 1.3 dB at 7.10\(^{-5}\). For the binary form of the code RS\(^3\) (15,7,9), the cGA-M has a gain of 0.4 dB in comparison with cGAD, where cGA-HSP outperforms cGAD by a gain of 0.8 dB at BER value of 6.10\(^{-5}\), the listed results are shown in Figure 6.

4.2 The effect of tournament size

After showing the impact of selection with increased size on the improvement of the cGA decoder's performance, we illustrate in Figures 7 and 8 the effect of tournament size \((s)\). As shown in Figure 7, the size of tournament has a considerable effect on the cGA-HSP decoder's performance. Applied to the BCH(63,45,7) code, a tournament of size \(s = 3\) gives approximately the same performance as the standard cGAD. The performances get better along the augmentation of \(s\) value until \(s = 13\): with

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1 BCH codes : Abbreviation of " Bose-Chaudhuri-Hochquenghem " code. A family of cyclic error correcting codes.  
2 QR codes : Abbreviation of " Quadratic Residue" codes. A type of cyclic codes.  
3 RS codes: An Abbreviation of "Reed Solomon" codes. A family of codes that belongs to the class of non-binary cyclic error-correcting codes.
tournament of size 13 the gain is about 1.2 dB at $2 \times 10^{-5}$. Simulating tournament of size equal to $s = 20$ reduces them. The degradation caused by a bigger tournament size can be interpreted by a premature convergence toward local optimum. So we should use the tournament size carefully so as to get the better performances of a given code.

Figure 5 shows that also for the $cGA-M$ decoder; the tournament size has an effect in improving the decoding performance like the $cGA-HSP$. When increasing the tournament size to $s = 3$, we reach approximately a gain of $0.7$ dB at SNR=$4 \times 10^{-5}$. Comparative performance is obtained when tournament size is set to $s = 5$ and $s = 8$. The gain in BER is elevated to 1 dB when $s = 13$.

4.3 The effect of the updating step size

The updating step size $1/\lambda$ is a user defined parameter which is directly related to the performance of the decoder. This part shows the impact of increasing or decreasing this parameter. Figure 9 gives an illustration about the effect of decreasing the $1/\lambda$ parameter applied to BCH(63,45,7) code. We can notice that the performance of our decoder get better when decreasing the size until reaching the value $1/\lambda = 0.002$, after that such a stabilization is remarked even if the size of $1/\lambda$ is decreased. Figure 10 shows also that with $s = 20$, the performances get better when decreasing the updating step size.
Fig. 9 The effect of the updating step size for cGA-HSP on BCH (63,45,7) code for $s = 13$.

Fig. 10 The effect of the updating step size for cGA-HSP on BCH (63,45,7) code for $s = 20$.

The comparison of the updating step size decrease applied for tournament size $s = 13$ and $s = 20$ is made in Figure 11. It shows that for $s = 13$, the decrease of the parameter $1/\lambda$ does not impact the performance after the value $1/\lambda = 0.002$. In contrast, for tournament size $s = 20$ the same value of $1/\lambda$ improves the performances: for SNR = $4.10^{-5}$ a gain of 0.5dB is noticed between $s = 13$ and $s = 20$ for $1/\lambda = 0.0005$. From this result we can say that the updating step size has also a big effect, in addition to tournament size, on the improvement of the decoder performance. When the tournament size get bigger, the $1/\lambda$ parameter should be the smallest possible to have the best performance of the decoder.

Fig. 11 The effect of the updating step size on the performance's improvement to BCH(63,45,7) code for the cGAD-HSP decoder.

4.4 The effect of mutation rate

In this part, we study the effect of the mutation rate into decoding performance of cGA-M, for a fixed tournament selection and updating step size ($s = 13$, $\lambda = 500$). As shown in the Figure 12, when increasing the mutation rate from $p_m = 0.001$ to $p_m = 0.01$, we reach a gain of 0.8 dB at $3.10^{-5}$. After that, even if we increase to $p_m = 0.04$ we don't realize any gain.

Fig. 12 The effect of mutation rate in cGA-M on BCH(63,45,7) code for $s=13$.

4.5 Comparison of the new decoders with other works

After showing the improvement of decoding performance using larger tournament size, and studying the effect of
tournament size, the updating step size and the mutation rate, we compare now our new decoders with previous ones such as Chase-2 algorithm [9], OSD-1 [12], and DDGA [2] decoders in term of performance.

Figure 13 shows the result of the performance's comparison for the code BCH(63,45,7). As we can see there, the standard cGAD presents the worst performance. cGA-M is better than Chase-2 decoder, and the standard cGAD. cGA-HSP is better than Chase-2, and OSD-1 decoders, its performance is equivalent than DDGA decoder. These results show that this new decoders effectively improve the decoding performance and so, the presented algorithms can be efficient for other optimization problems.

![Comparison of cGA-HSP and cGA-M with other decoders for BCH(63,45,7) code.](image)

Fig. 13 Comparison of cGA-HSP and cGA-M with other decoders for BCH(63,45,7) code.

5. Complexity analysis

After showing that the new decoder based on Higher Selection Pressure (cGA-HSP) gives the best performances in comparison with the standard cGA based decoder (cGAD) and the mutation based one (cGA-M), we will compare the three decoders in term of their complexities. For this purpose, we will start by counting the number of iterations required by each decoder to reach the convergence then we will calculate the analytic complexity for them.

5.1 The average number of iterations to reach convergence

Table 3 shows the necessary average number of iterations to reach convergence for the three decoders: cGAD, cGA-HSP and cGA-M. As we can see there, cGAD and cGA-M has an average number of iterations necessary to convergence ($N_i$) around 20000, where cGA-HSP needs just around 2500 iterations. Having into account that $s=13$, if we try to calculate the average number of updates occurred during every decoding process (steps 5 of algorithms 4 and 5), we find that cGAD and cGA-M do around 20000 ones since any of these algorithms do just one update per iteration. cGA-HSP updates the probability vector ($s - 1$) times at each iteration, so the average number of updates for cGA-HSP is (12*2500) around 30000 updates. So we can conclude from this analysis that cGA-HSP is little more complex in term of the number of updates than cGAD and cGA-M.

5.2 The analytic complexity of the new decoders

Table 4 gives the analytic complexity of the three decoders: cGA, cGA-M, and cGA-HSP. It shows that the proposed decoders have the same complexity if we neglect the tournament size $s$, given that $s$ is very small in comparison with the average number of iterations to reach convergence ($N_i$).

<table>
<thead>
<tr>
<th>SNR</th>
<th>Average Number Of Iterations</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>21166</td>
</tr>
<tr>
<td>3</td>
<td>19953</td>
</tr>
<tr>
<td>5</td>
<td>20401</td>
</tr>
</tbody>
</table>

Table 4: The analytic complexity of the new decoders

<table>
<thead>
<tr>
<th>Decoder</th>
<th>Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>cGAD</td>
<td>O($N_i,k(n-k)$)</td>
</tr>
<tr>
<td>cGA-HSP</td>
<td>O($N_i,s,k(n-k)$)</td>
</tr>
<tr>
<td>cGA-M</td>
<td>O($N_i,s,k(n-k)$)</td>
</tr>
</tbody>
</table>

6. Conclusion

In this paper, we presented two new dual domain soft decision decoders that use cGA with larger tournament size: the first algorithm investigates tournament selection with larger size using mutation, and the second employs higher selection pressure with randomly generated individuals. A study of the new decoders has been done and showed that the introduction of mutation and higher selection pressure improves the performance of the decoding. The obtained results are compared to known previous works and show the superiority of the proposed
References


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