Towards quantum agents: the superposition state property

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Abstract

The modelling of intelligent complex systems uses the agent paradigm increasingly. However, the problem of decision making with local, incomplete, uncertain, exchanged or observed in asynchronous manner is often present in agent models. Studies on quantum cognition introduce quantum properties such as superposition state and entanglement in the decision process. So how to propose quantum agents models that are capable of implementing both quantum properties of superposition state and entanglement? A case study simulating the "Game of Life" illustrates our proposed quantum agent model.

Keywords: *Quantum Agents, Complex Systems Modelling, Quantum Cognition, Agent-Based Systems, Agent-Based Modelling*

1. Introduction

The modelling of complex systems, particularly when they are intelligent systems, uses the techniques of distributed artificial intelligence and the agent paradigm increasingly. However, some issues and situations are difficult to model when agents must make decisions with uncertain, incomplete or indeterminate knowledge of their context. Thus, studies on quantum cognition are very interesting. They introduce quantum properties such as superposition state and entanglement, in the cognition and therefore in the decision process.

Quantum theories have inspired a wide variety of scientific fields. Thus many quantum-like models have appeared. The potential of using quantum theory to build models of cognition is one example [1]: decision processes, ambiguous perception, probability judgments, memory, cognitive measurements, etc. Indeed, the open, parallel, cooperative and competitive decision processes fully benefit research on quantum probability decision models [2]. The problem of decision making with local, incomplete, uncertain, exchanged or observed in asynchronous manner is often present in agent models [3]. Also, having worked for many years on the agent-based modelling of complex systems, we are interested in modelling quantum agents [4] capable of implementing both quantum properties of superposition state and entanglement.

This paper is organized as follows: in the second section a state of the art in the fields of quantum cognition and agent modelling of complex systems is made; in the third section a quantum agent model is proposed; in the fourth section, a case study on the simulation of the "Game of Life" illustrates our quantum agent model: 1) simulation with classical agents, 2) simulation with quantum agents, and 3) simulation with quantum agents for a continuous and asynchronous version of "Game of Life", are proposed; in the fifth section a general discussion on the use of quantum agents is proposed; finally, conclusions of this research focused on the superposition state property are presented.

2. State of the Art

2.1 Systems and Complex Systems

For minimal definition, a system is a set of elements that interact according to certain principles or rules (*i.e.* the law of system evolution). Such a system is then determined by the nature of its elements (components), all the states of its elements defining its overall state, the interactions between its elements and interactions with their environment [5].

A complex system is a system made up of a large number of components, whose behaviours are both highly variable and highly dependent on the behaviour of other components [6]. From the interactions of the components of a complex system emerge collective behaviour that cannot be derived as a result of the behaviour of each component. The prime examples of complex systems are the human brain and human societies [7].

The many challenges of the science of complex systems include the formal definition of complex systems, the modelling and simulation of these systems with a wide variety of characteristics: many heterogeneous interacting parts, multiple scales, complicated transition laws, unpredicted emergence, sensitive dependence on initial conditions, path-dependent dynamics, ill-defined boundaries, interaction and self-organization of autonomous agents, combinatorial explosion, adaptivity to changing environments, co-evolving subsystems, and multilevel or non-equilibrium dynamics [7].

From this set of complex systems characteristics, we have retained three key properties:

- **Self-organization and complex adaptive systems** [8]. An organization is an integral arrangement according to a distribution of a set of elements in a hierarchical level; a self-organizing system has the ability to create and recreate this structure [9, 10, 11].
- **Nonlinearity**. Behaviour and responses of a complex system are not deterministic and are influenced by the presence of nonlinear relationships and feedback loops. The implicit ability to exhibit linear or non-linear behaviours (order and/or chaos), is based on a response of the system to these stimuli [7].
- **Emerging behaviours**. From nonlinear interactions, self-organized or chaotic result emergent properties and complex behaviours [12, 13, 14, 15].

2.2 Quantum Cognition

When a human being thinks, reasons and makes a decision, he does not use the rules of classical logic, but those of quantum mechanics, where things are not well defined but intricate, ubiquitous, oscillating and superposed [16, 17, 18]. Like photons and electrons, thoughts are superposed, interfere, and are entangled in our brain.

Using limited cognitive resources, the quantum cognition gives humans the opportunity to answer an unlimited number of questions with limited rationality [16]. Quantum theory allows the reactions of human beings to the questions put to them or to situations in which they are placed to be evaluated [17, 19]. Each time reasoning is applied to a decision process, human decisions are typically quantum, because opinions are not always determined [20, 21].

Wang et al. [2] present five reasons, which become five challenges, to use quantum theory to build models of human cognition:

1) The challenge of formalizing psychological concepts of conflict, ambiguity, and uncertainty – quantum modelling allows us to formalize the state of a cognitive system moving across time in its state space until a decision is reached, at which time the state collapses to a definite value (*i.e.* indefinite state, called a superposition state at each moment in time) [3].

2) The challenge of formalizing the cognitive system's sensitivity to measurements – the quantum principles are also consistent with the idea that a choice can alter preferences (*i.e.* an intermediate choice affects the final decision) [22].

3) The challenge of formalizing order effects of cognitive measurements.

4) The challenge of understanding violations of classical probability laws in cognitive and decision studies.

5) And the last challenge of understanding non-decomposability of cognition.

As noted above, the quantum properties are numerous: indeterminacy, wave interference, ubiquity, oscillation, entanglement between the states, superposition state principle. From this set of properties, we have retained as first studies two essential properties:

- **The superposition state**. Quantum thinking is to do massively parallel calculations, reasoning operating on mental representations consisting of a superposition of states. When an observation is made, the superposition state reduces to a single classical and definite state. The superposition state provides a very good representation of conflict, ambiguity or uncertainty that we feel when we doubt.
- **Quantum entanglement**. Entanglement is the propensity that can have two (or multiple) objects, two ideas, or two arguments to form only one.

Quantum formalisms have already been proposed to develop quantum cognition models [23], such as SCOP model [20]. The SCOP model is defined by five elements Eq. (1):

$$(\Sigma, M, \Lambda, \mu, \nu) \qquad (1)$$

where Σ is the set of states, M is the set of contexts, Λ is the set of properties, μ is a probability function that describes how a state changes to another state under the influence of a context, ν is a function that describes a weight for a specific state.

In a SCOP model each concept is represented by defined sets of states, contexts and properties. The concepts are continuously changing under the influence of contexts. These changes are described by state changes [20].

2.3 Agent-Based Complex Systems Modelling

The concept of software agents is a response to the desire to design and develop intelligent systems composed of entities which are themselves intelligent. This opens new perspectives in the research field of Distributed Artificial Intelligence [24]. The intelligence of these artificial entities is collective or individual. This allows modelling, simulating, or developing a wide variety of complex systems [25]. At least an agent is an autonomous entity that can adapt to, react to, or interact with its environment [26].

The main properties of these entities are then: autonomous, interactive and adaptive. Agents may also have excessively cognitive properties as in the BDI model (Belief, Desire, and Intention). This model is built around three concepts inspired by human behaviour models: (1) beliefs, based on agent knowledge, (2) desires, corresponding to the knowledge that agents would express, and (3) intentions, or actions, that agents decide to do [27].

There are many definitions of the agent paradigm [28, 29, 30, 31] and new types of agents continue to emerge [32, 33, 34]. So, it is difficult to establish a consensual definition. However, through these definitions we observe that three functions characterize agent activity: perceive, decide, and act. An agent has its own knowledge. It acts in autonomy to reason and decides according to its objectives, its interactions with other agents in the system, and its environment perception. By extension, considering cognitive agents, experts of this domain generally agree on the following characteristics: intentionality, rationality, commitment, adaptability, and intelligence. Agent-based systems are systems that allow distributing agents, communicating, autonomous, reactive, skilful, and finalized entities. They form intelligent solver networks, weakly bound, working together to solve problems beyond their individual capabilities and knowledge [35]. Agents may possess many other properties, like: sociable, mobile, proxy, intelligent, rational, temporally continuous, credible, transparent and accountable, coordinative, cooperative, competitive, rugged, or trustworthy [26].

Agent-based modelling and simulation is a relatively new approach to modelling complex systems made up of interactive and autonomous entities – the agents. Multi-agents modelling is a way of modelling the dynamics of complex systems and complex adaptive systems [6]. These systems are often self-organized and can create emergent orders. The behaviours of agents are described by simple rules. They interact with other agents and their environment. These interactions in turn influence their own behaviour. Thus, at the system level, structures and behaviours emerge that were not explicitly programmed into the initial model, but appear through interaction of agents. This emergence in multi-agent systems is also subject to formalization proposal [36]. The focus on the modelling of heterogeneous agents in a population, the emergence of self-organization, and the self-adaptation of multi-agent systems are three distinctive features of agent-based simulation [37, 38, 39].

3. A Quantum Agent Model

A software agent, according to the model of Newell and Simon [40], is an independent information processing system, which means it is made up of: (1) receiving and transmitting modules to exchange messages with its environment or other software agents, (2) its own execution capacity, and (3) a memory (*i.e.* a knowledge base). Thus, an agent-based system is a society of autonomous agents that work together to achieve a common goal through interaction, communication or transaction. Autonomy is the main differentiation of agent paradigm compared to the object paradigm. This autonomy can be achieved by: (1) an independent computer process, (2) an individual memory, (3) the ability to interact (perception/reception, communication/action) [25], and (4) for a quantum agent the ability to control the superposition states.

The quantum agent-based formal approach we follow to model and design complex systems is to define the modular architecture of quantum agents (qAgents), to define their model of interaction, communication and knowledge, and to respect a rigorous methodology for acquiring expertise. Thus, a quantum agent-based system M is described by a 5-tuple Eq. (2):

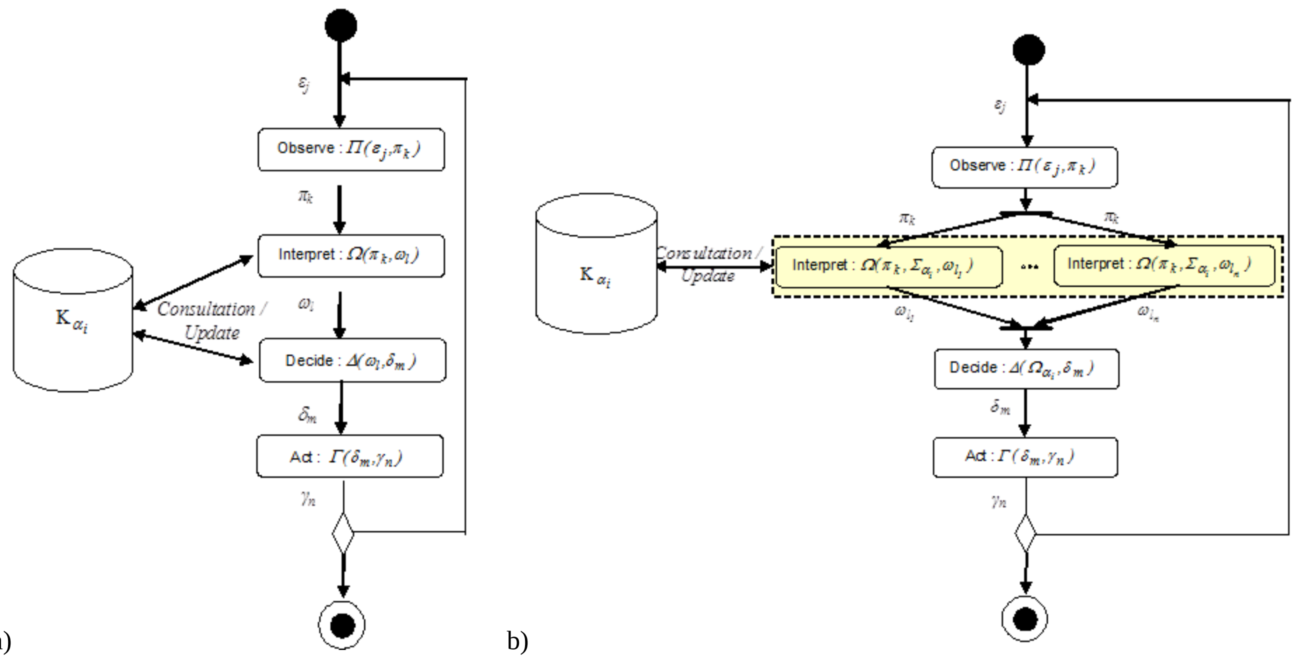
$$\mathrm{M} = \langle \mathrm{A}, \mathrm{I}, \mathrm{P}, \mathrm{O}, \Sigma \rangle \tag{2}$$

where A is a set of qAgents ($\alpha_i \in A$) that can superpose several states included in the set Σ; I is the set of interactions defined for the qAgents of A ($\iota_i \in I$); P is the set of roles to be played by the qAgents of A ($\rho_i \in P$), and O is the set of organizations of the qAgents of A into communities ($o_i \in O$).

Many basic agent behaviours are inspired by the cycle: perceive or observe, then decide, finally act [25]. The behaviour of quantum agents is similar. They continually perform four functions: observation, interpretation according to their possible states, decision, and eventually action (Figure 1). Thus, a qAgent $\alpha_i \in \mathrm{A}$ is described by the following tuples Eq. (3):

$$\alpha_i =< \Pi(\varepsilon_j, \pi_k), \Omega^*(\pi_k, \Sigma_{\alpha_i}, \Omega_{\alpha_i}), \Delta(\Omega_{\alpha_i}, \delta_m), \Gamma(\delta_m, \gamma_n), K_{\alpha_i} > \tag{3}$$

where $\Pi(\varepsilon_j, \pi_k)$ is the function of observations of the qAgent α_i (ε_j is an event and π_k is its observation); $\Omega^*(\pi_k, \Sigma_{\alpha_i}, \Omega_{\alpha_i})$ is the multi-function of interpretations of the qAgent α_i (Σ_{α_i} is the finite set of states of qAgent α_i, moreover, at a given time the state vector of the qAgent α_i is noted $|\psi_{\alpha_i}\rangle$), and Ω_{α_i} is the finite set of interpretations of observations made by qAgent α_i); $\Delta(\Omega_{\alpha_i}, \delta_m)$ is the function of decisions of the qAgent α_i (δ_m is a decision); $\Gamma(\delta_m, \gamma_n)$ is the function of actions of the qAgent α_i (γ_n is an action); K_{α_i} is the finite set of knowledge of the qAgent α_i (decision rules, values of the domain, acquaintances and/or networks of affinities between quantum agents, observed events, internal states, etc.).



a) b)

Fig. 1 a) Basic behaviour of a classical agent, b) Basic behaviour of a quantum agent

The quantum agents communicate and influence each other (quantum property), they also have quantum states. When these states take only two values, such as cellular automata that we present below in Section 4, we can compare them to the qbit (or qubit) concept. A qbit (contraction of quantum and bit) represents the storage unit of quantum information [41]. A qbit is composed of a superposition of two basic states, written |0> and |1>. A qbit state consists of a linear superposition of these two states. If a classical bit is always either in the |0> state, or in the |1> state, a qbit is in a superposition of these two states. This can be described by a linear combination of two states Eq. (4):

$$|\Psi\rangle = \alpha \bullet |0\rangle + \beta \bullet |1\rangle \quad (4)$$

where the coefficients α and β are two complex numbers satisfying the normalized relation $|\alpha|^2 + |\beta|^2 = 1$.

4. Application to the Game of Life Simulation

To calculate is to observe, remember and act; thus performs a finite-state automaton. Consider a set of identical finite-state automata, with a limited number of states, placed on the squares of a chessboard (these kinds of automata are called cellular automata, or CA). Any one of these automata observes the automata around, remembers the state it is in and changes state respecting the invariable rules that characterize it (rules similar to a program). This change relates all squares of the chessboard and determines a new generation of states of the squares. By applying this process again, a new generation is obtained. The most famous cellular automaton is the Conway automaton, known as “Game of Life” [42].

The Conway automaton has two states, 0 and 1, also called “dead state” and “living state” respectively. From one generation to another, a Conway automaton looks at its eight nearest neighbouring squares: if it is dead and if exactly three neighbours are alive, then it passes into the living state (birth) in the next generation; if living and if exactly two or three neighbours are alive, it is alive in the next generation. In all other cases, the automaton is found in the death state (death by isolation or choking) the next generation. An illustration of a simulation based on classical agent’s model is given in Figure 3 (Each cell is modelled by an agent; the initial configuration had 200 agents). Three steps of the evolution of a simple configuration of “Game of Life” are shown in the figure (3.a$_1$, 3.a$_2$ and 3.a$_3$).

4.1 Simulation with Quantum Agents

A quantum system can be described by the sum of different superposed states Eq. (5):

$$|\Psi\rangle = \alpha_1|\psi_1\rangle + \alpha_2|\psi_2\rangle + \ldots + \alpha_n|\psi_n\rangle \qquad (5)$$

where α_i is a coefficient called "probability amplitude".

In the case of a cell of a cellular automaton, which can be living or dead, without interference from its environment, the sum of different superposed states becomes Eq. (6):

$$|cell\rangle = \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle \qquad (6)$$

where 0 represents the death state and 1 represents the living state (*i.e.*, a kind of bipolar quantum cellular automata [43]).

In the case of the "Game of Life", probability of a cell to be dead or alive, coefficients α or β respectively, will depend on the states of the 8 neighbouring cells Eq. (7):

$$|cell\rangle = \alpha|0\rangle + \beta|1\rangle,\ where\ \alpha + \beta = 1 \qquad (7)$$

By applying the SCOP model to the "Game of Life", where the concept is the cell (C is the set of cells and $c_i \in C$ is a given cell), we get:

- Σ is a set of 2 states: {0, 1}, dead or alive;
- M is the set of cells contexts; the context e of a cell c_i is defined by the states of the 8 neighbouring cells of c_i;
- Λ is the set of cell properties: position, size, colour, ...;
- μ is the function of state change, *i.e.* the probability that a cell in a state p under the influence of context e changes in state q (knowing that there are 2^8=256 possible contexts for a given cell c_i) Eq. (8):

$$\mu : \Sigma \times M \times \Lambda \rightarrow [0,1]; (q,e,p) \rightarrow \mu(q,e,p) \qquad (8)$$

- ν is the weight of a cell property a in a state p Eq. (9):

$$\nu : \Sigma \times \Lambda \rightarrow [0,1]; (p,a) \rightarrow \nu(p,a) \qquad (9)$$

4.2 From the Discrete Mode to the Continuous Mode

Cellular automata are synchronous massively parallel computers. Each cell is a finite state transducer which takes its inputs from neighbouring cells, and determines its own output state. At every tick of the clock, cells determine their states (change of states or not). Each cell observes states of neighbouring cells and, taking its own state into account, applies a defined transition rule, to decide its state at the next tick of the clock. All cells are changed at the same time. From the physical point of view, a cellular automaton belongs to the field of digital classical theories, in which space, time and states are discrete [6].

Now we propose another quantum agents model of the "Game of Life", in a continuous and asynchronous mode. Quantum agents are temporally autonomous (*i.e.* each qAgents manages its evolution cycle time), they observe the state of their neighbours (qAgents can also transmit their state by exchanging messages) and apply the same defined transition rules as the previous discrete model. The asynchrony of this version of the "Game of Life" leads to uncertainty for agents about the updating or indeterminacy of the state of neighbours. Indeed, when a cell becomes aware of (observes) the status of a neighbouring cell, the latter may be planning to change its state. Also, each cell will determine its future state by calculating the probability of death or life, α or β (8).

Figure 2 illustrates a possible evolution for a set of 20 cells (4x5), whereas the evolution of these cells is rather synchronous. Consider the cell $c_{2,1}$ (coloured red in the figure), its probability of death or life are defined by Eq. (10):

$$\alpha_{2,1}|0\rangle + \beta_{2,1}|1\rangle \Rightarrow \frac{5}{16}|0\rangle + \frac{11}{16}|1\rangle \qquad (10)$$

Figure 3 illustrates three steps of evolution of a configuration of a "Game of Life" using: a) classical agents, b) quantum agents in a discrete model, and c) quantum agents in a continuous model. In this last case, the colour code for the cells depends on the value of probability of life state β (black *if* $\beta = 1$, blue *if* $\beta = 0$, red *if* $2/3 \leq \beta < 1$, orange *if* $1/3 \leq \beta < 2/3$, and yellow *if* $0 \leq \beta < 1/3$).

$$\begin{pmatrix} 0&0&0&0 \\ 0&0&1&0 \\ 0&?&1&0 \\ 0&0&1&0 \\ 0&0&0&0 \end{pmatrix} \rightarrow \begin{pmatrix} \alpha_{0,0}|0\rangle + \beta_{0,0}|1\rangle & \alpha_{0,1}|0\rangle + \beta_{0,1}|1\rangle & \alpha_{0,2}|0\rangle + \beta_{0,2}|1\rangle & \alpha_{0,3}|0\rangle + \beta_{0,3}|1\rangle \\ \alpha_{1,0}|0\rangle + \beta_{1,0}|1\rangle & \alpha_{1,1}|0\rangle + \beta_{1,1}|1\rangle & \alpha_{1,2}|0\rangle + \beta_{1,2}|1\rangle & \alpha_{1,3}|0\rangle + \beta_{1,3}|1\rangle \\ \alpha_{2,0}|0\rangle + \beta_{2,0}|1\rangle & \alpha_{2,1}|0\rangle + \beta_{2,1}|1\rangle & \alpha_{2,2}|0\rangle + \beta_{2,2}|1\rangle & \alpha_{2,3}|0\rangle + \beta_{2,3}|1\rangle \\ \alpha_{3,0}|0\rangle + \beta_{3,0}|1\rangle & \alpha_{3,1}|0\rangle + \beta_{3,1}|1\rangle & \alpha_{3,2}|0\rangle + \beta_{3,2}|1\rangle & \alpha_{3,3}|0\rangle + \beta_{3,3}|1\rangle \\ \alpha_{4,0}|0\rangle + \beta_{4,0}|1\rangle & \alpha_{4,1}|0\rangle + \beta_{4,1}|1\rangle & \alpha_{4,2}|0\rangle + \beta_{4,2}|1\rangle & \alpha_{4,3}|0\rangle + \beta_{4,3}|1\rangle \end{pmatrix} \rightarrow \begin{pmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&1&1&1 \\ 0&0&0&0 \\ 0&0&0&0 \end{pmatrix}$$

Fig. 2 Illustration of the quantum evolution of cell states in the "Game of Life"

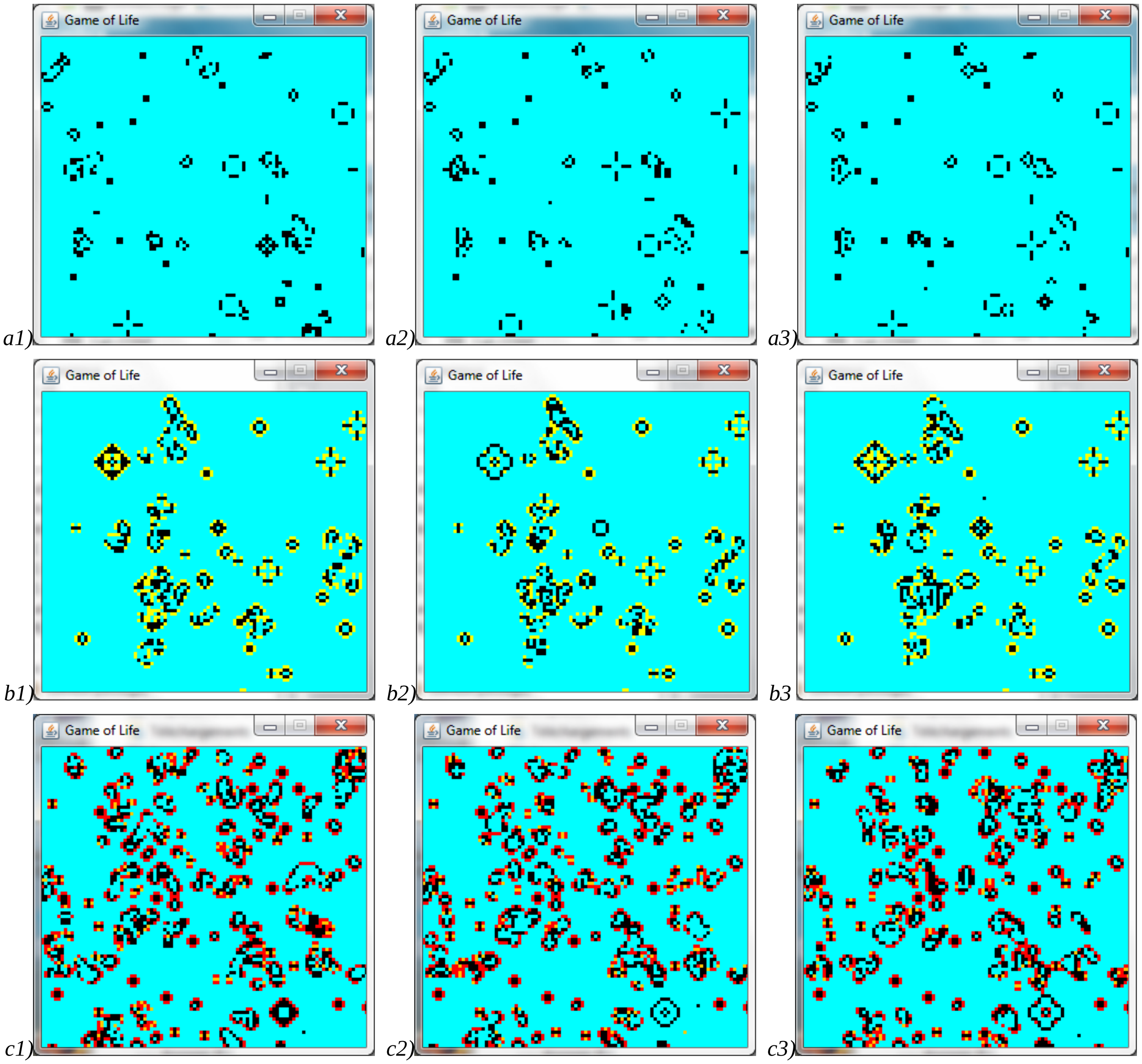


Fig. 3 Three steps of evolution in a simulation of "Game of Life": *a1→a3*) with classical agents; *b1→b3*) with quantum agents in a discrete model; *c1→c3*) with quantum agents in a continuous model

5. Discussion

When an observation is made by a quantum agent, the superposition state is reduced to only one definite state. Conversely, between two situations observed, quantum agents superpose their states, allowing them not to make an assumption on an opinion/decision but to consider all possibilities at once and so represent any uncertainty of the situation including the state of its environment and the states of other agents. Thus, quantum thinking is a more powerful thought.

In return, the design of quantum agents is complex. It is for the designer/programmer to identify, distinguish and evaluate in terms of probabilities the different quantum states that an agent can have. These states will be determined based on events that may occur in the environment of the agent and that he can observe continuously and not necessarily synchronously. So, the programmer's training should include the study of logic as much as that of probabilities.

At the algorithmic level, the superposition of states induced a program structure with the logical operator *AND* (*FOR* $state_1$ *DO {actions}; AND …; AND FOR* $state_n$ *DO {actions'}*), rather than alternative or conditional structures of the type *IF THEN ELSE* (*IF {conditions on* $state_i$*} THEN {actions} ELSE {actions'}*). To translate this logical structure *AND*, two ways are possible:

- From the perspective of sequential program, the iterative structure is most appropriate: *FOR EACH possible state DO {actions}*;
- From the perspective of distributed program, a structure of parallel processing where each process considers one possible state is, at this time, the most appropriate.

Moreover, the distributed vision of a system, a fortiori a complex system, helps to think quantum (without meaning, it's only a hypothesis, either a quantum vision). A system includes both multiple states, the states of different distributed components, and a single coherent and real state of the overall system.

In the case study of "Game of Life", the determined state of a cell (living or dead) depends on the moment of its observation of neighbouring cells. This means two probable states before determining a state during observation or measurement (display in the environment). This presents a complex timing problem for computers. Consider a cell c_i, if two adjacent cells are alive at the time of observation t_1, they are perhaps not alive when the cell c_i determines its new living state (observation at time $t_1 + \delta t$). That's why we discussed the "Game of Life" in discrete or continuous mode, in the previous section, but other factors are involved and thus remain to be studied, such as correlations of states between cells. Thus our next experiments will focus on the entanglement and oscillation of states, reflecting the hesitations of an agent in the process of decision making.

6. Conclusion

The modelling of intelligent complex systems uses the agent paradigm increasingly. However, the problem of decision making with local, incomplete, uncertain, exchanged or observed in asynchronous manner is often present in the agent model. Studies on quantum cognition introduce quantum properties such as superposition state and entanglement in the decision process. In this context, we have assumed that agent modelling can be inspired by quantum-like models. In this paper, a quantum agent's model that is capable of implementing both quantum properties of superposition state and entanglement is proposed.

A simple case study of simulation of the "Game of Life" illustrates our proposed quantum agent model. We have begun by presenting a simulation of the "Game of Life" with classical agents, before presenting a simulation with quantum agents. Then we have proposed a second simulation based on quantum agents for presenting the potentiality of a quantum approach for modelling and simulating a continuous and asynchronous version of "Game of Life". In this last simulation we have shown the interest of the superposition state property for the decision process of quantum agents.

We are continuing to work on better integration of quantum properties in models based on quantum agents. After the property of superposition state, we are now interested in the quantum properties of entanglement and interference between states. Another perspective for our work is the comparison of the relevance of the approach based on fuzzy agents [44] and the approach based on quantum agents for the treatment of uncertainty in the process of decision making by intelligent agents.

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