

# Development of Wavelength Division Multiplexing Model with Mathematical Equations

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## Abstract

The rate of bandwidth consumption for transferring data during internet services and telephony has generated a lot of problems to the users in terms of the speed of the network, the price rate and its overall efficiency, especially in Africa. Wavelength Division Multiplexing (WDM) system is a new technique that provides solutions to this common problem, the main subject of this paper is based on the understanding of Wavelength Division Multiplexing (WDM) subsystem, theoretical modelling of (WDM) subsystem using mathematical equations, critical evaluation of the performance of each of the components and overall subsystem. Thorough critique of the subsystem, description and analysis from the results obtained and how the system can be implemented were established.

**Keywords:** Wavelength, Optical Fiber, Bandwidth, Optical receiver, Amplifiers, Multiplexers, and Demultiplexers.

## 1. Introduction

The large bandwidth of fiber can be made full use of by transmitting several channels simultaneously on a single mode fibre using multiplexing techniques. The multichannel light wave systems that result can provide savings. One of the major techniques used for optical signal multiplexing is called Wavelength Division Multiplexing (WDM). This technique can also be used in multiple access environments such as for broadcast in a LAN, the network that employs WDM is referred to as wavelength Division Multiple Access (WDMA) network. The bandwidth of an optical fibre spans quite a range when considering the low loss windows. If a single wavelength signal was transmitted over this then much of the bandwidth will go unused, Wavelength Division Multiplexing exploits the large potential bandwidth of the fibre by combining several wavelengths [1].

WDM system can also be classified from the architectural view point into three categories; 1. Point -to-point Links, 2. Broadcast-and-select networks and 3. Local area networks (LANs). But in this research, only WDM point to point would be considered.

## 2. WDM Waveform

In WDM, the optical transmission spectrum of a fibre is divided into a number of non-overlapping wavelength bands, with each wavelength supporting a single communication channel see figure1 (WDM waveform).

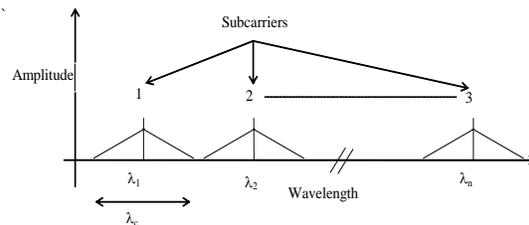


Fig 1. WDM waveform

Each signal is allocated a bandwidth of  $\lambda_c$  and the sub carrier wavelengths are designated  $\lambda_1 \dots \lambda_n$ . The composite signal containing all channels is then placed onto a fibre and transmitted.

The channel spacing needs careful consideration in order that crosstalk (the presence of energy of one wavelength channel in an adjacent channel) does not occur e.g. The extra frequencies (wavelength) caused by FWM [2].

## 3. Typical WDM System

As a point to point link WDM is acting as a physical layer device and basically carries data from one node to the next. A typical WDM link is shown in the figure 2 for the details.

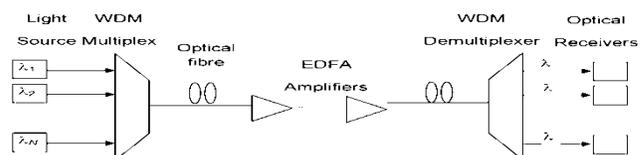


Fig. 2 WDM Point to Point Link

The signal sources composed of lasers with different wavelengths are modulated as can be seen from the diagram above. WDM multiplexing combines the wavelength into single fiber as illustrated in fig 2. This can be achieved by passing the multiple wavelength channels into directional couplers which eventually summed them together. Here the insertion loss can be specified as a total loss, take for instance, if N wavelengths are input and each coupler loss Zdb, then the total loss =  $\log_2(N) \times Z$ . The optical fiber is the transmission media, fiber amplifiers are commonly used to amplify optical signals over a long transmission span. Effectively, the more wavelengths that can be transmitted the aggregate bandwidth of the transmission.

Demultiplexing WDM, obviously at the receiving end of the fibre link, the different wavelength is separated by a demultiplexer using a tunable optical filter. And finally at the optical receiver, individual detection of the channels is executed by changing the optical signal back to the original signal that was applied to regulate the light source. Point to Point WDM transmission offers the following advantages:

Several signals can be carried by one fibre, thereby increasing system capacity with little complexity. System capacity can be easily upgraded [3-4].

### 3.1 Aim and Objective of the Study

The main aim of this paper is to develop a WDM model system using mathematical equations for effective utilization of bandwidth. Objectives of this Study are as follows:

- To design and Implement a WDM model that would increase the efficiency of fibre bandwidth.
- To critically evaluate and analyze the model's performance using mathematical equations.

The development of WDM model would no doubt increase the efficiency of bandwidth, i.e. With a very small bandwidth; we would be able to achieve more in WDM comparable to that of CDMA. It also increases the speed of the network and therefore reduces the monthly cost of bandwidth.

## 4. Methodology

The approach that will incorporate into the development of this model is basically by generating mathematical equations for each of the components of WDM, the losses are also represented through

mathematical equations and the performance of the overall system was determined by critical analysis and evaluation of the model equations.

### 4.1 Description of WDM Subsystem

Directional Couplers: Exchanging of power between guided modes of adjacent waveguides is known as directional coupling. Waveguide directional couplers perform a number of useful functions in thin-film devices, including power division, modulation, switching, frequency selection, and polarization selection. Directional couplers rely on a concept known as an evanescent field for its operation which explicitly explained the transfer of signal energy between closely placed waveguides.

## 5. Evanescent Field in A Waveguide

Evanescent field is a field that propagates outside of the waveguide, with the wave, in the direction of propagation. The actual field profile as it propagates along the waveguide is typically shown in figure 3 below.

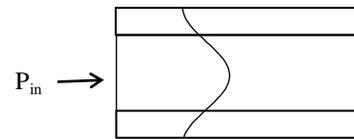


Fig. 3 Evanescent Field in Waveguide

Mode Coupling: This is effected by placing in close proximity a section of the waveguide, please see fig 4 (mode coupling) for the details.

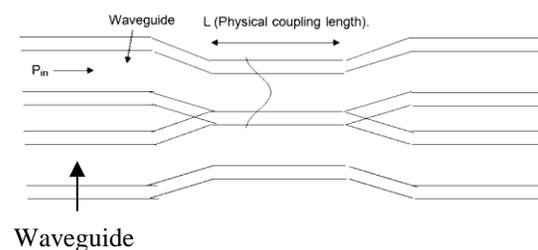


Fig. 4 Mode coupling

From this diagram, (i) as the signal reaches the end of the coupling section it leaves via the same waveguide. (ii) The signal is shared by some ratio between the waveguides. (iii) The signal leaves via the other fibre and this effect is known as "mode coupling". The parameters of interest in this model for analysis are. The power coupling of the field to the other guide is total if the length of the physical coupling length (L) equals a parameter

called the coupling length  $L_c$ . In between this the power coupling is zero or partial and is given by the formula;

$$\frac{P_{coupled}}{P_{in}} = \cos^2\left(\frac{\pi L}{L_c}\right) \quad (1)$$

Where  $\pi L/L_c$  is in radians, this expression is also known as the power coupling ratio. Where  $L$  is the actual coupling length of the fibre and  $L_c$  the coupling length, this describes the operating principle of a device called the directional coupler (figure 5).

## 6. Critical Evaluation on Mode Coupling

In [1-1] above, if  $L$  is less than or greater than  $L_c$ , the power ratio will fall and the maximum power ratio can only be achieved when  $L = L_c$ . This can be proved further by assigning different values for  $L$  and  $L_c$  to determine the corresponding values of the power ratio. When  $L$  is greater than  $L_c$ , the power ratio remains the same as when  $L$  equals  $L_c$ . Please see in Figure Power ratio below, as long as  $L$  is greater than or equal to  $L_c$  the power ratio remains constant, but when the values of  $L_c$  increases relatively to  $L$  i.e.  $L_c = 2L, 3L, 4L$  e.t.c then the value of power ratio decreases.

Power Ratio = 0.54 when  $L=L_c, L=2L_c, L= 3L_c, L=4L_c$ .  
 Power Ratio when  $L < L_c$  i.e  $L=1/2L_c, L = 1/3L_c$  and  $L=1/4L_c$  have been calculated to be 1.0, 0.87, 0.76 respectively. That is to say, as  $L$  decreases, the power ratio also decreases while the increase in the value of  $L$  does not change the value of the Power Ratio.

We can rewrite equation [1-1] as a power transfer function of 3dB couplers as  $P_{out} = P_{in} \cos^2(D\theta)$ , where  $D\theta$  is the relative phase difference between the signals which can take a value 0 or  $\pi$ .

Now let's assume the input power applied to the directional couplers is 5mw then, we can calculate the output power of the system by  $P_{out} = P_{in} \cos^2(\pi)$ , Output power =  $5 \times \cos^2(\pi)$  where  $\pi$  is in radians. Note that this is like equation [1-1], if  $L=L_c$ . Therefore the output power =  $5 \times 0.54 = 2.70mw$

Let's plot the output power against coupling length  $L_c$ , assuming the actual coupling length of the fiber  $L$  is 1m then from our equation we derive the following table.

TABLE1: Power versus Coupling Length

Output Power (mw)	Coupling length $L_c$ (m)
5.0	2
4.3	3
3.8	4
3.4	5
	6

3.2

Fig 5 illustrates a four port directional coupler, the direction of allowed power flow are indicated by the arrows in the figure. For instance, let's assume power  $P_1$  is incident on port1 of the coupler.

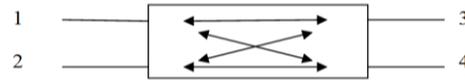


Fig. 5 Four port directional coupler

This power will be divided between ports 2 and 3 according to the desired splitting ratio. We can assume that the power emerging from port2 ( $P_2$ ) is equal to, or greater than the power emerging from port 3 ( $P_3$ ), ideally no power will reach port 4, the isolated port.

The following characteristics of the coupler losses (in db) can be defined as follows.

1. Throughput loss

$$L_{THP} = -10 \log P_2/P_1 \quad (2)$$

This specifies the amount of transmission loss between the input port and the favoured port (port2).

2. Tap loss:

$$L_{TAP} = -10 \log P_3/P_1 \quad (3)$$

This specifies the transmission loss between the input port and the tap (port 3).

3. Directionality

$$L_D = -10 \log P_4/P_1 \quad (4)$$

This specifies the loss between the input port and the port we wish to isolate (port 4).

4. Excess Loss

$$L_E = -10 \log (P_2 + P_3) / P_1 \quad (5)$$

This specifies the power lost within the coupler, this includes radiation scattering, absorption and coupling to the isolated port. Ideally, no power reaches port 4 ( $L_D = \infty$ ). The total emerging power from ports 2 and 3 equals the input power ( $P_2 + P_3 = P_1$ ), making the excess loss zero. The splitting ratio is  $P_2/P_3$ , the ratio of the powers at the two output ports.

For the lossless coupler,  $P_2 = P_1 - P_3$  so the throughput loss [Eq. (1-2)] can be written as

$$L_{THP} = -10 \log (1 - 10^{-L_{TAP}/10}) \quad (6)$$

TABLE 2: Four-Port Directional Couplers

2
3
4
5

Coupler	L <sub>TAP</sub> (db)	L <sub>THP</sub> (db)	Splitting Ratio
Description			
3dB	3	3	1:1
6dB	6	1.25	3:1
10dB	10	0.46	9:1
12	12	0.28	15:1

This result provides the relationship between the tap loss and the throughput loss. Additionally, from fig 5, assuming an input at port 1, the coupling to ports 2 and 3 are given by

$$P_2/P_1 = \cos^2(\Delta\beta L), P_3/P_1 = \sin^2(\Delta\beta L) \quad (7)$$

Where  $\Delta\beta$  is the coupling coefficient (given in radians per meter) between the wave guide and L is the length of fiber over which interaction exists. From these equations, the input power divides between the two output ports with no loss, as we can see from preceding equations; all power appears at port 3 when the length of the interaction region is

$$L_c = \pi/2\Delta\beta \quad (8)$$

The resulting length is called the coupling length. Figure 6 is a plot of the coupled power as a function of the length of the interaction region.

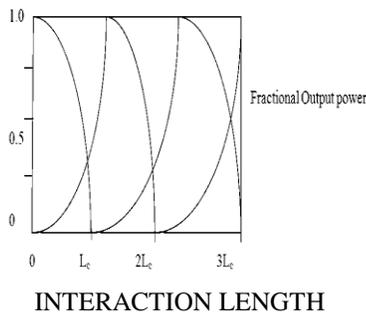


Fig 6. Fractional coupled power as a function of the coupling region.

Note also that a coupler can be constructed to form a device that splits the input signal equally in terms of power, at the output, this can be demonstrated in fig 7: 3dB Coupler.

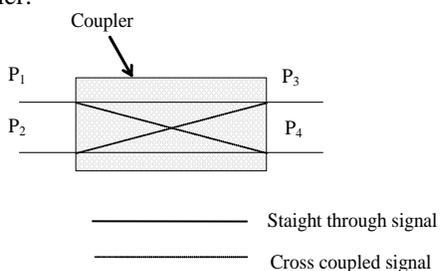


Fig. 7 3dB coupler

The signal entering at port 1 splits into two signals at ports 3 and 4. The coupler can be described by the power/intensity transfer function given by:

$$\begin{bmatrix} P_3 \\ P_4 \end{bmatrix} = 1 - \gamma \begin{bmatrix} \eta_{(1,3)} & \eta_{(2,3)} \\ \eta_{(1,4)} & \eta_{(2,4)} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \quad (9)$$

Where  $\eta_{(m,n)}$  represents the power coupling coefficient between ports m and n, and  $\gamma$  the proportion of power lost through the coupler (insertion loss). This equation shows that coupling occurs also with a signal input to port 2. We try to evaluate this further in the future [8-10].

## 7. Optical Filters Etalons

It is often a requirement that an optical signal be filtered out of an aggregation of different wavelength signals and optical filters that do this are one of the main components in a WDM de-multiplexer. A number of devices possess wavelength selectivity one being the Fabry Perot etalon. The Etalon is a device that has a certain thickness and reflectivity, a typical etalon transmission function is:

$$\frac{P_o}{P_i} = T = \frac{1}{1 + \frac{4R}{(1-R)^2} \sin^2\left(\frac{2\pi l}{\lambda}\right)} \quad (10)$$

Where R is the reflectance of the device, l the thickness,  $\lambda$  the input wavelength. A typical response with respect to signal wavelength is shown in the figure 8 Transmission of etalon v wavelength for three different values of relectivity note l = 500x1.55e-6 m.

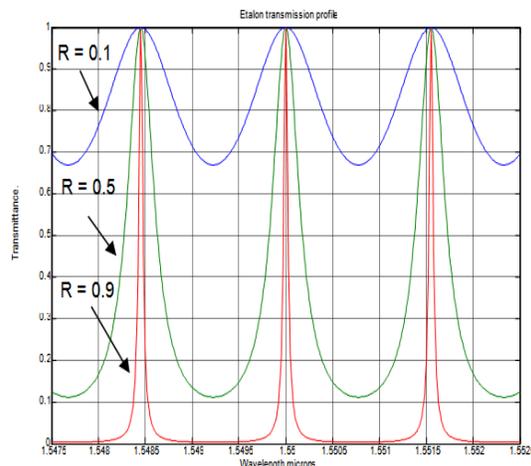


Fig. 8 Transmission of etalon v wavelength for three different values of relectivity.

### 8. Theory Fabry-Perot Etalons

The plane-parallel etalon acts as a frequency filter, or interferometer, through the interaction of multiple reflections from the partially reflecting dielectric interfaces of the etalon. The etalon performs a simple transfer function of changing optical frequency into transmitted intensity. As an example, consider an uncoated solid etalon made of fused silica in an ambient medium of air. Figure 9 shows the multiple reflections that the input beam undergoes as it traverses the etalon. In this example, light enters an etalon of length  $l$  from the left with incident angle  $\theta_0$ .

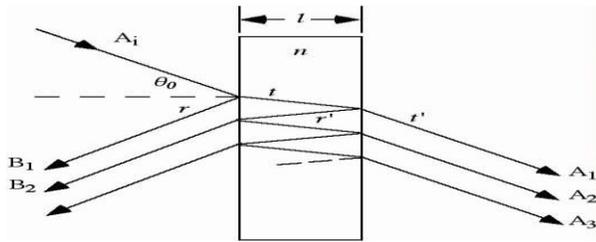


Fig. 9 Fabry-Perot Etalon: Multiple Reflection Model

Starting with the amplitude of the incident electric field,  $A_i$ , the reflected amplitude from the first interface is given by  $B_1$ , while the partially transmitted amplitude from the second interface is given by  $A_1$ . The coefficients of amplitude reflection,  $r$ , and transmission,  $t$ , denote light travelling from air to silica while the coefficients,  $r'$  and  $t'$ , denote light travelling from silica to air. The multiple output beams differ in phase due to the different path lengths traversed by each of the beams. The optical phase acquired by the light on one round trip through the etalon is given by:

$$\delta = 4\pi n l \cos\theta / \lambda$$

Where the  $n$  = the index of refraction,

$l$  = thickness of the etalon

$\lambda$  = wavelength of the laser (11)

The amplitude of each of the transmitted waves can thus be written:

$$A_1 = tt' A_i, A_2 = tt' r'^2 e^{i\delta} A_i, A_3 = tt' r'^4 e^{2i\delta} A_i \dots (12)$$

The sum of transmitted wave amplitudes,  $A_t$ , is:

$$A_t = A_i tt'(1 + r'^2 e^{i\delta} + r'^4 e^{2i\delta} + \dots) = tt' / (1 - rr' e^{i\delta}) A_i (13)$$

The fractional output intensity, or power transmission,  $T = I_t / I_i$ , from the ideal etalon is given by:

$$T = I_t / I_i = A_t A_t' (14)$$

In a lossless system, and with  $r = r'$  for identical etalon surfaces, this equation simplifies to  $T = 1 / (1 + F \sin^2(\delta/2))$  with  $F = 4R / (1 - R)^2$ , Where we have introduced the power reflectivity  $R = r^2$  with lossless interface  $r^2 + r'^2 = 1$ . This function is known as the Airy function showing in fig 2.1. And this is also equivalent to etalon transmission equation quoted from (10)

### 9. Evaluation of output power against Thickness of Etalon

Firstly, according to the model, the output power of the directional coupler would be applied as an input power to calculate the power output of Etalon by using equation 2-0 above. Remember that the output power calculated from Directional coupler is 2.7mw.

From (11), 
$$\frac{P_o}{P_i} = T = \frac{1}{1 + \frac{4R}{(1-R)^2} \sin^2\left(\frac{2\pi l}{\lambda}\right)}$$

With  $l = 500 \times 1.55e-6$ , Reflectance = 0.5, input wavelength  $\lambda = 1.55$ , therefore

Power output =  $2.7 / (1 + 4 \times 0.5) / (1 - 0.5)^2 \sin^2(2 \times 3.142 \times 500 \times 1.55e-6) / 1.55$

Power output =  $2.7 / 1.015276 = 2.6595mw$

Therefore the output power of the Etalon filter is 2.65mw.

Power output (MW)	Etalon Thickness (m)
1.5495	1.9
2.6352	2.0
2.6353	2.5
2.6228	3.0
2.6106	1.5
2.5980	4.0

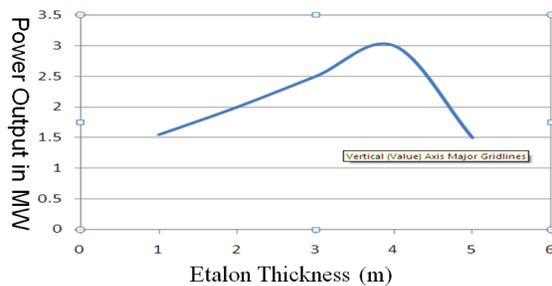


Fig. 10 Output Power versus Etalon Thickness.

## 10. Non Linear Optical Loop Mirror

This is commonly used in many applications such as optical switching and demultiplexing, mode locking, pedestal suppression, pulse shaping or regeneration of ultra fast data streams. Most NOLM designs rely on self-phase modulation, which causes a differential nonlinear phase shift to accumulate only if a power imbalance exists between the beams propagating in the loop. An architecture that puts the phase modulation produced by the Kerr effect in to good use is known as Non-linear optical loop mirror. This loop is formed by fusing together the two loose ends of piece of fibre which forms a directional coupler. Fig 11 describes the operation of NOLM.

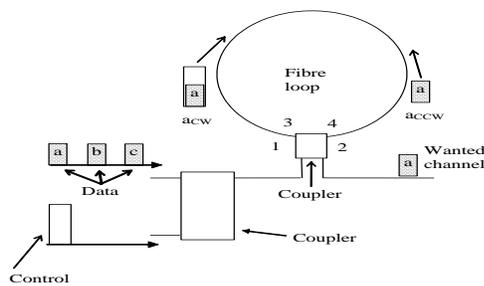


Fig. 11 Non linear optical loop mirror.

This loop has as its input the high frequency data stream plus a control pulse. The data split at the coupler equally and propagates around the loop in contra directions (clockwise aCW counter clockwise aCCW) and recombines back at the coupler. The control pulse also enters the loop after the coupler and is timed to propagate with one of the data.

The effect in terms of switching is if no control pulse is applied then the data pulses emerges back through the input. If the control pulse causes the phase shift to be  $\pi$  via the Kerr effect then the data pulse it propagates with emerges from the output port 2. The procedure to determine the phase shifts follows closely that of the Mach Zehnder.

## 11. Loop transmission against phase shift

The description given was for the two extreme cases with the phase shifts equal to 0 and  $\pi$ . However phase shifts that are not at these values give an output power defined by the transmission equation:

$$T = 1 - \cos^2\left(\frac{\Delta\phi}{2}\right) \text{ And this relates the power out from port 2 to the power in to port 1 by } P_{\text{out}} = P_{\text{in}}T. \quad (15)$$

Now the pulse may not have a square shape e.g. when a Gaussian control pulse is used for the control the phase shift is time dependent meaning the actual phase shift produced by the control varies with time now it can be written as:

$$\Delta\phi_D(t) = \frac{2\pi L \Delta n(t)}{\lambda_D} = 2 \frac{2\pi}{\lambda_D} n_2 L I_C(t) \quad (16)$$

This leads to a time dependent power transmission and the equation to define the transmission of the NOLM is now:

$$T(t) = 1 - \cos^2\left(\frac{\Delta\phi_D(t)}{2}\right)$$

Using a Gaussian control means that the phase shift is determined by the intensity at a particular time due to the Gaussian profile the transmission is the phase shift for a Gaussian shape is substituted for  $I_{CG}(t)$ :

$$T(t) = 1 - \cos^2\left(2 \frac{2\pi n_2}{2\lambda_D} L I_{CG}(t)\right) = 1 - \cos^2\left(\frac{2\pi n_2}{\lambda_D} L I_{CG}(t)\right) \quad (17)$$

Application of the equation is usually confined to substituting in values and calculating the phase shift over a suitable time range. If the peak intensity of the control pulse  $I_C$  is calculated to give a  $\pi$  phase shift then the transmission profile (switching window) has a typical profile as depicted in the figure 12.

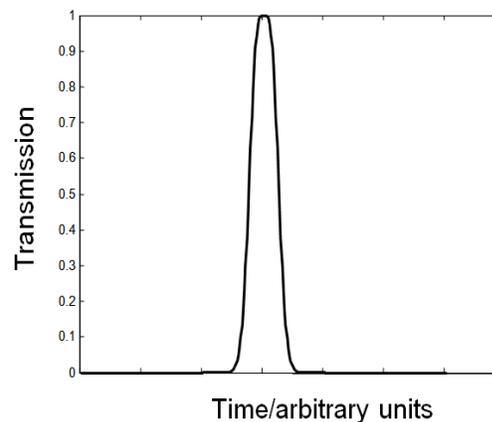


Fig. 12 Transmission profile using Gaussian control pulse exhibiting  $\pi$  phase shift at peak intensity.

## 12. Discussion and Results

The input power here would be the output power from Etalon filter which is 2.65mw, from (2-6), we have

$$T = 1 - \cos^2\left(\frac{\Delta\phi}{2}\right), \text{ where } P_{out}=P_{in} T. \text{ note the values}$$

of phase shift for this transmission equation should not be 0 or  $\pi$ . For the sake of this analysis, let phase shift =  $\pi/4$ . Therefore  $T = 1 - \cos^2(\pi/8) = 0.397$

Output power = input power x transmission, therefore output power = 2.65mw x 0.397 = **1.05mw**. Now lets try to input different values of phase shift to get different values of transmission, which will be used to compute the corresponding values of output powers using fromular  $P_{out} = P_{in} T$ . Please see the results in table below

TABLE 4: Transmission versus Power Output

Transmission	output power
0.636	0.2
0.927	0.35
1.084	0.41
1.143	0.43
2.208	.833
2.324	575
2.385	.305
2.4398	.12

We will plot also the graph of output power against transmission.

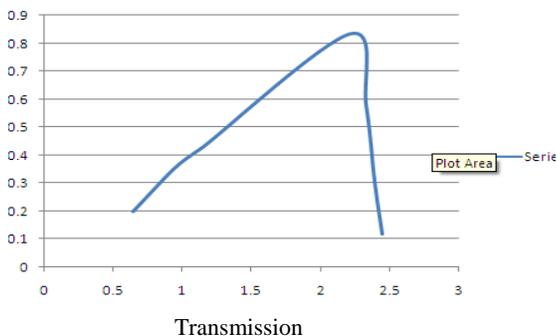


Fig. 13 Output Power against Transmission.

### 13. Optical Receiver

The receiver is constituted of a multi-channel WDM receiver array capable of receiving signals; Monolithic receivers integrate a photodiode array with a demultiplexer on the same chip. The receiver fabrication is divided into two units, the optical chip which includes the fiber pigtailed AWG chip integrated with photodiode (PD) array, and the Radio Frequency (RF) electronics which includes the pre-amplification and switching functions as well as control DC signals. The WDM

receiver was designed to be programmable such as each WDM channel can be selected for the right output using very fast PIN switches, with switching speed faster than 100ns. The WDM receiver consists of an AWG, InGaAs PD array, and RF electronics, including built in test (BIT) functions. Generally speaking, receiver comprised of a laser pre amplifier, an optical filter and a photodiode followed by a low noise electrical amplifier.

Optical amplifiers are commonly used as preamplifiers before detection to enhance the system sensitivity. Hence it is of importance to assess the performance of optically pre amplified receiver in the presence of crosstalk. We will consider an *amplitude shift keying/direct detection (ASK/DD) system*, optically preamplifier receiver whose schematic diagram is depicted in **Figure 14**. The incoming optical signal  $Y(t)$ . (information-carrying signal and crosstalk), after traversing one or several optical cross-connections, is amplified and subsequently filtered in order to reduce the effect of the ASE noise. The photo-detector output passes through a post-detection filter of type integrate-and-dump filter and sampled to form the decision variable  $Z$ . The decision device derives an estimate of a transmitted binary symbol by comparing the value of the decision variable with a preselected detection threshold. We are interested in assessing the error performance of the system, to accomplish this goal we use a statistical method for evaluating the error probabilities. The method is called saddle point approximation, which makes use of the moment generating function (MGF) for the receiver decision variable. The photocurrent intensity  $I(t)$ , in a normalized form can be written as:

$$\lambda(t) = 1/2 B(t)^2 \tag{18}$$

In which  $B(t)$  represents the optical field in equivalent baseband form, falling upon the photo-detector.

### 14. Optical Pre-Amplification

Consider an optical signal  $Y(t)$  at the input of the erbium-doped fiber amplifier (EDFA) preamplifier which is modelled as an optical field amplifier with power gain  $G$ , an additive noise source  $X(t)$ , representing the ASE noise and an optical filter with complex equivalent baseband impulse response  $r(t)$ . See Fig. 2. The optical field at the output of the amplifier is:

$$B(t)=\sqrt{G} Y(t) + X(t) \tag{19}$$

The density of  $X(t)$  expressed in photons per second is given by:

$N_0 = n_{sp} (G-1)$ , in which  $n_{sp}$  represents the spontaneous emission parameter. For the further analysis, we assume that  $Y(t)$  is confined in the bit interval and that the impulse response  $r(t)$  is limited to the same time interval.

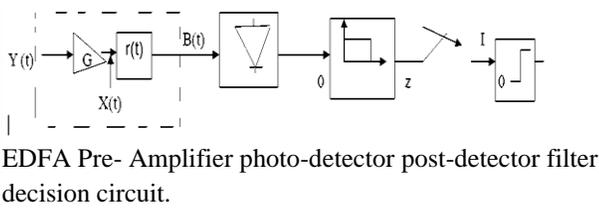


Fig. 14 Optical Pre-amplified ASK/DD receiver

We can therefore expand  $B(t)$  in Fourier series. Consequently, the optical field  $B(t)$  can be written as:

$$B(t) = \sum_{k=-L}^L (\sqrt{G} Y_k + X_k) e^{j\pi kt/T} \quad (20)$$

Where  $\beta = 2L+1$  (the number of temporal modes) equals the ratio of the bandwidth  $B_0$  of the optical filter and the data rate  $B = 1/T$ ;  $\beta = B_0/B$

The power transfer function for receiver is

$$P_o = P_i [SRL]/2 \quad (21)$$

Where  $S$  is the slope of the laser diode characteristic curve,  $R$  is the responsivity of the photodiode and  $L$  is the loss in fibre.

### 15. Critique Evaluation of optical Receiver

When the optical power level into a receiver is increased there will be a point at which peak clipping or overload occurs and the performance degrades. Similarly, if the signal is reduced sufficiently an input level is reached at which the performance is again degraded. Broadly speaking, dynamic range of a receiver can be considered as the difference between these two input levels for a specified error rate.

Dynamic Range of an optical receiver can be defined in several different ways, one of which is given below:

$$D = 10 \log P_{MAX}/P_{MIN},$$

Where  $P_{MAX}$  is the maximum allowable optical input at a given bit error rate (ber) and  $P_{MIN}$  is the receiver sensitivity at the same bit error rate.

Please see fig 15 for the profiles of sensitivity and overload versus frequency for a typical receiver.

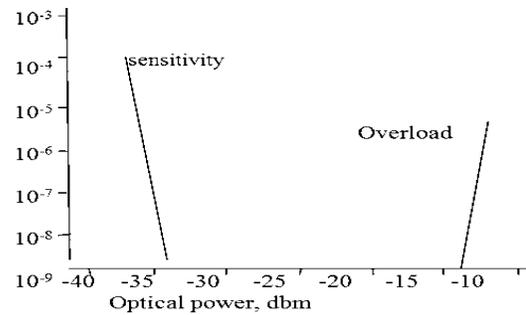


Fig. 15. Sensitivity and Overload vs. Frequency.

Note that the input power for this system will now be the output power from the NOLM which is 1.05mw. Assuming we have the slope of the laser diode to be 0.1 with the responsivity of 10.0 and loss of 1mw.

Then the output power of the receiver can be calculated as:  
 Output power =  $1.05 \times (0.1 \times 1 \times 10) / 2$  mw = 0.525mw  
 We can change the responsivity of the photodiode and calculate the corresponding output powers for each. Then output power can be plotted against the responsivity. Please see Figure 16.

TABLE 5: Responsivity versus Power Output

Responsivity	Output Power (mw)
0.0	0.30
7.0	0.25
8.0	0.20
9.0	0.12
10.0	0.05

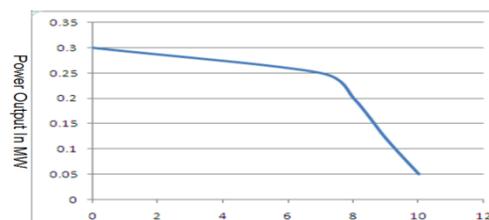


Fig. 16 Output Power against Responsivity

Convert all input and output values from Mw to dB using  $10 \log_{10} P_i (MW) = DB$

TABLE 6: Input Power, Values and Components

Input Power (MW)	Components	Input Values (DB)
5.0	Directional coupler	6.8

2.7	Etalon	4.3
2.65	NOLM	4.2
1.05	Receiver	0.2

Total input power to the system in dB = 15.5dB.

TABLE 7: Output Power, Values and Components

Output Power	Components	Output Values (dB)
2.7	Directional Coupler	4.3
2.65	Etalon	4.2
1.05	NOLM	0.2
0.525	Receivers	-2.7

Total output power of the system =6.7Db

In summary of this analysis, it can be observed that NOLM as a component has the greatest loss in power as we can see that the gain to the system is 4.2dB but the output is 0.2dB and this reflects its contribution to the performance of the WDM as a whole, removing this particular component will increase the performance of the system. On the other hand, Etalon Filter has got the minimum power loss, the least the power loss in the system the higher the strength of the signal generated; therefore, the contribution of Etalon to this model is higher than any other components in this model. Addition of Etalon to this model would yield the best performance of the model while removing this component would definitely reduce the general performance of the WDM model.

## Conclusion

In this paper, we have successfully developed mathematical models with relevant equations for each of the WDM components and critical analysis of the relationships of parameters has also been established. This model has been thoroughly linked together by analysing the performance of each WDM component in terms of the power inputs/outputs, plotting relevant graphs to further explain the relationships between the parameters and by looking at the effects of one of the sub-components relative to the performance of the entire system.

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