Numerical modeling and algorithms of creation of finite element model of multicoherent area

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Abstract

There is the technique of creation of finite element model of designs with a difficult configuration is described in article. Constructed the algorithms of creation of a finite element grid for designs with a simple configuration. Through their association the finite element model set designs be formed. Developed the algorithms of a choice of the initial front for minimization of width of a tape of nonzero coefficients. Carried out the streamlining of numbers of knots of finite element model on the basis of a frontal method. Examples of the solution of specific objectives are given.

Keywords: *finite element, sewing together, discrete model, grid, edge, knot.*

1. Introduction

There is the technique of creation of finite element model of designs with a difficult configuration is described in article. Constructed the algorithms of creation of a finite element grid for designs with a simple configuration. Through their association the finite element model set designs be formed. Developed the algorithms of a choice of the initial front for minimization of width of a tape of nonzero coefficients. Carried out the streamlining of numbers of knots of finite element model on the basis of a frontal method. Examples of the solution of specific objectives are given.

At the solution of applied tasks by finite elements method (FEM) the main problem connected with formation and the decision of the allowing system of the linear algebraic equations of a high order arises. Coefficients of a system`s matrix have simmetriny and tape structure. In this case it is enough to store in memory of the computer only diagonal and not zero elements which are below the main diagonal of a matrix, limited by width of not zero coefficients tape.It is known that tape width directly depends on a way of numbering of knots in finite element model. This dependence is described by the relation (1):

$$l = \max_{1 \le i \le N} (K_i^{\max} - K_i^{\min} + 1) * V \qquad (1)$$

where - $K_i^{\max}$ and $K_i^{\min}$ the maximum and minimum numbers of knots in i-th finite element, V – dimension of a task, N – total number of knots of a grid.

When carrying out computing experiments on the basis of FEM it is necessary to automate process of creation of finite element model of a real object. If the area of a body has difficult configuration, creation of finite element grid will be the labor-intensive process demanding big ability and bunch of time. In this regard the technique of creation of finite element model of multicoherent area is offered. This method based on formation of finite element model of object by means of "sewing together" (association) of elementary subareas. The elementary area is meant as area with simple configuration for which there is an algorithm of creation of finite element grid. Quadrangles and quadrangular prisms are used as finite elements.

2. Technology of the solution

The topology of finite element model of object is represented simple hierarchy of volumes, surfaces, lines and points [1]:

1) three-dimensional area in the form of system of the volume elements connected among themselves limited to the surfaces which are crossed in nodal points; boundary surfaces and lines, as well as each volume element, can have some number of internal knots; surfaces can be crossed only along boundary lines;

2) two-dimensional area with system of the surfaces adjoining lengthways the boundary lines connected among themselves which are crossed in nodal points; thus boundary lines can also include some number of intermediate knots; surfaces can be limited to several lines; two lines have to connect so that one of them crossed another in a trailer point.

The finite element model of area is described by the following discrete set (2):

$$\Omega = \{N, M, MK, MN\}, \qquad (2)$$

where

N – number of knots of a finite element grid;

M – quantity of finite elements;

MK – the massif of coordinates of knots;

MN – an array of numbers of knots on elements.
"Sewing together" of two subareas has an appearance (3):

$$\Omega = \Omega' + \Omega'', \qquad (3)$$

where

$$\Omega' = \{N_1, M_1, MK', MN'\},$$
$$\Omega'' = \{N_2, M_2, MK'', MN''\},$$

discrete models of the subareas which are subject to association. Generalizing the above, it is possible to conclude that if Ω – resultant area, and – the corresponding elementary subareas (4):

$$\Omega = \sum_{i=1}^{p} \Omega^{i} \qquad (4)$$

where
p – number of the subareas which are subject to association. Thus, formation of a finite element grid of multicoherent area is carried out by means of consecutive "sewing together" of elementary subareas.

3. Solution Method

For descriptive reasons we will consider process of formation of finite element model of two-dimensional area of a difficult configuration (figure 1).

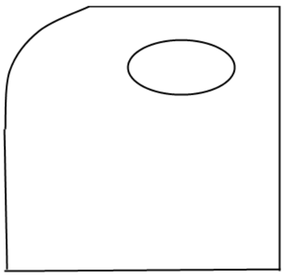

Fig. 1. Initial area.

The following subareas are used as elementary areas (figure 2):
1) any quadrangle;
2) a rectangle with elliptic cut in top;
3) 1⁄4 part torah;
4) 1⁄4 part of an ellipse.
Basic data for creation of a finite element grid of these areas are:
1) any quadrangle – coordinates of tops, number of splittings on axes of OX and OY;
2) a rectangle with elliptic cut in top – coordinates of the center and radiuses of an ellipse, the sizes of the parties of a rectangle, number of radial splittings and number of divisions on an axis OX;
3) 1⁄4 part a torah - coordinates of the center and radiuses of an ellipse, number of radial splittings and number of divisions on an axis OX;
4) 1⁄4 part of an ellipse – coordinates of the center and radiuses of an ellipse, number of splittings on axes of coordinates.

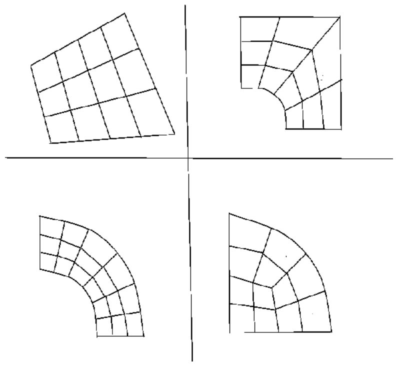

Fig. 2. Forms of elementary subareas.

Taking into account a configuration the studied area breaks into a set of elementary areas (figure 3).

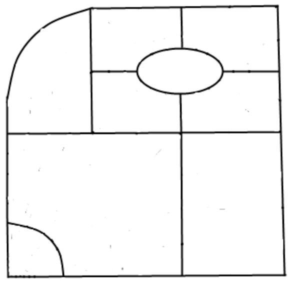

Fig. 3. The studied area breaks into a set of elementary areas.

Formation of area of a difficult configuration through of using the library of elementary areas allows simplifying process of creation of finite element grid due to reduction of volume of the entered basic data. The algorithm of construction is reduced to a consecutive task of parameters of elementary subareas, formation of a finite element grid and their association (figure 4).
The described technique of creation of finite element model of area of a difficult configuration allows, without increasing quantity of finite elements and number of knots, to consider all geometrical features of a configuration of area.

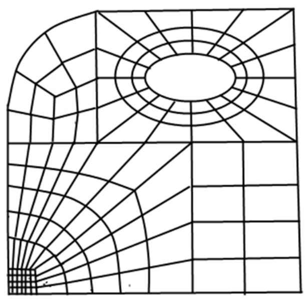

Fig. 4. Formation of discrete model of a difficult configuration.

4. Algorithm for three-dimensional finite element model

Three-dimensional finite elements can be received by such operations as expression or leaving of a trace at rotation applied to the surfaces covered with a grid. We will consider as a design example in the form of a rectangular parallelepiped with ellipsoidal dredging in top (figure 5). It represents 1/8 part of a parallelepiped with an ellipsoidal cavity in the center.

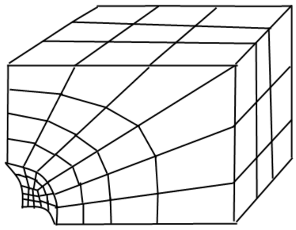

Fig. 5. A parallelepiped with ellipsoidal dredging in top.

The beginning of system of coordinates we will arrange in the center of a cavity, and we will send to an axis of coordinates along edges. Rectangle sides with elliptic cut in a corner, break as follows. Ellipse points of intersection connect to axes of coordinates a straight line. The piece shares on n of equal parts. From the beginning of coordinates through points of splitting radial straight lines before crossing with an ellipse contour are drawn. Coordinates of points of intersection are defined from the following system of the equations:

$$\begin{cases} \dfrac{x}{x_i} = \dfrac{y}{y_i} \\ \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \end{cases},$$

where (x_i, y_i) - coordinates of points on a straight line $(i = \overline{1,n};)$,

n- plural number,

a, b – ellipse half shafts.

Further, the average knot, received on an ellipse, connects to an opposite corner of a rectangle. The parties not adjacent to cut break into pieces in the same relation. The knots constructed on an ellipse connect to the knots received on the parties of a rectangle. These straight lines break into pieces on the basis of a proportion (5):

$$\frac{a_i}{a} = \frac{l_i}{l} \quad i=1,\dots,m, \qquad (5)$$

where

a_i – the set piece i length on the party of a rectangle, adjacent to cut,

l – length of the straight line connecting knot on an ellipse to the party of a rectangle,

l_i – piece i-go length on this straight line,

m – quantity of pieces on the party of a rectangle, adjacent to an ellipse.

Length of a piece of a straight line is calculated on the following formula (6):

$$l=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}, \qquad (6)$$

where (x_1, y_1)– knot coordinates on an ellipse, (x_2, y_2)– knot coordinates on the party of a rectangle.

Coordinates of internal knots of a side are determined by ratios (7):

$$\begin{cases} x=\dfrac{x_1+\lambda_i x_2}{1+\lambda_i} \\ y=\dfrac{y_1+\lambda_i y_2}{1+\lambda_i} \end{cases} \qquad (7)$$

where

$$\lambda_i = \frac{\sum_{j=1}^{i} l_j}{\sum_{j=i+1}^{m} l_j}$$

Creation of discrete model of a surface of ellipsoidal dredging is connected with formation of a spatial triangle (figure 6) which angular knots have coordinates $(a;0;0),(0;b;0),(0;0;c)$ where a,b,c – ellipsoid half shafts.

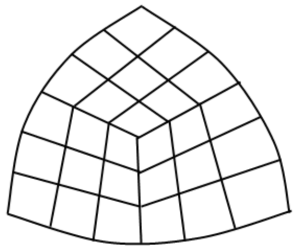

Fig. 6. Finite element representation of an ellipsoidal triangle.

The parties of this triangle share on n of equal parts. Then the geometrical center of a triangle connects to median knots of the parties of a triangle and the received pieces break as equals' parts. In each of the received quadrangles we draw the straight lines connecting the splitting knots located on the opposite sides of quadrangles. Coordinates of knots on the parties of quadrangles are defined from the ratios similar (7), and coordinates of internal knots - from the decision of system of the equations:

$$\begin{cases} \dfrac{x-x_1}{x_2-x_1}=\dfrac{y-y_1}{y_2-y_1}=\dfrac{z-z_1}{z_2-z_1} \\ \dfrac{x-x_3}{x_4-x_3}=\dfrac{y-y_3}{y_4-y_3}=\dfrac{z-z_3}{z_4-z_3} \end{cases},$$

where $(x_1,y_1,z_1),(x_2,y_2,z_2),(x_3,y_3,z_3),(x_4,y_4,z_4)$– coordinates of the knots lying on the opposite sides of quadrangles. Coordinates of the knots located on a surface of ellipsoidal dredging decide from a condition of crossing of the radial straight lines passing through the knots lying on a triangle on an ellipsoid surface as follows:

$$\begin{cases} \dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}= 1 \\ \dfrac{x}{x_i}=\dfrac{y}{y_i}=\dfrac{z}{z_i} \end{cases}$$

where

(x_i, y_i, z_i) – i-th coordinates – that knot, (i = 1,…, N),
N – number of knots on a triangle, $N=(n+1)(n/2+1)+(n/2)^2$.

The final stage in creation of a finite element grid of a design is splitting other three sides which aren't adjoining dredging. Each side shares $n^2/4$ on rectangles. Then the knots located on the middle of the parties of dredging and also in its tops connect to parallelepiped tops. To the central knot there has to correspond the point of intersection of three sides not adjacent to dredging. Splitting points on these straight lines are defined on the basis of proportions (5).

Thus, it is possible to hurt an initial body into hexagons.Numbering of knots of a body with ellipsoidal dredging in top is carried out by the frontal method described above. For what at first dredging knots are consistently numbered. Then, without interrupting a numbering order, knots of the subsequent layers are numbered. The number of knots on each layer will be identical. It should be noted that all layers in a form will be similar to dredging, except the last which is formed by three crossed parallelepiped sides.To form a finite element grid of a parallelepiped with a through cylindrical cavity in the middle, it is necessary to use finite element a rectangle grid with an elliptic cavity in top with addition of the coordinates set on OZ axis.

Coordinates of nodal points of each layer of a parallelepiped on axes of OX and OY coincide with coordinates of knots of a rectangle.

The algorithm of splitting a rectangle with elliptic dredging in top can be used for creation of finite element representation of the hollow cylinder. For this purpose it is necessary to replace the equations of the parties of a rectangle with the equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

and to add the corresponding values on OZ axis.

5. Algorithm of construction

The algorithm of construction includes the following stages:
1) formation of library of finite element models of elementary subareas;
2) procedure of association of subareas;
3) procedure of definition of the initial front of knots;
4) procedure of streamlining of numbers of knots of finite element model.

Procedure of association of two subareas includes the following stages:
1) on the basis of comparison of coordinates of the knots located in the MK′ and MK″ are formed:
kp - number of the coinciding knots located on border of association of subareas;
the M_1 and M_2 including the corresponding numbers of knots located on border of association of subareas;
2) $N = N_1 + N_2 – kp$;
3) $M = M_1 + M_2$;
4) initial M_1 to lines of the MN array appropriates the corresponding values of the MN′ sets Ω′;
5) the subsequent values of elements of lines of the MN array are defined by means of procedure of replacement of local numbers of knots of a set Ω″, located in the MN″ array, on global numbers. If value of the current i-th numbers of the knot located in the MN″ is present at the MN″, the corresponding local number of knot from the M1 array is assigned to it. Otherwise, its value is calculated on the basis of a ratio: $i + N_1 – z$, where value of a variable z is defined as number of knots of the M2 array which number there is less than value i;
6) for formation of initial N_1 of lines of the MK array values of the MK′ array are used;
7) the next lines of the MK array are formed of MK″, with the withdrawn lines which numbers are specified in the M2.

The final stage of algorithm of creation of finite element model is streamlining of numbers of knots that is connected with reduction of width of a tape of system of the allowing FEM equations. The essence of streamlining consists in renumbering of knots on the basis of a frontal method [2]. In the real work the frontal method is modified taking into account that the initial front gets out as sequence of numbers of the knots located on border of the considered area [3]. For streamlining of numbers of knots three fronts are used: in the first numbers of knots of initial or current fronts, in the second – numbers of knots previous settle down, and in the third – the new front is formed.

The algorithm of a method consists of the following stages:
1) as the initial front boundary knots get out;
2) finite elements which contain knots with the same numbers, as well as numbers of knots of the front are defined;
3) are excluded from (2) numbers of knots chosen on a step participating in current previous and in formed fronts;
4) the grid knots having identical numbers, as numbers of knots in the current front according to the following rule are renumbered: everyone the following numbered knot gets on unit a bigger number, than previous, and the initial renumbered knot has number one;
5) contents of the current front (now it becomes previous) remain, contents of the new created front in flowing with its subsequent clarification are copied, i.e. the front "moves" on a konechnoelementny grid;
6) all actions described in points (2)-(5) until the created front becomes empty repeat.

On the example of two-dimensional area the best results turn out at a choice as the initial front of the ordered set of the knots located on edges and tops of a finite element grid of multicoherent area. In this regard we will enter the corresponding definitions.

In finite element representation of multicoherent area the knot which is found in the only finite element is called as top. The set of the knots located on border of area or on the border concluded between two tops is called as an edge.

Process of search of an edge is carried out as follows:
1) in a random way the knot which is found in two finite elements gets out and is added to the front;
2) search of the knots which are found only once in two finite elements described in point (1) is carried out. If those aren't present, search of the knots which are found two times if those aren't present is run in them, formation of the initial front comes to an end. Then it is cleared and carried out transition to point (1). Otherwise, number of this knot is added to the front;
3) if in the course of repeated use of the described actions the top is found in the second step two times, performance of search of an edge comes to an end and streamlining on the front is carried out;

4) all knots which are found one or two times, are added to the front and the actions described in points (2) - are carried out (3);
5) steps (1) - repeat (4), all edges concluded between two tops won't be found yet.
To prevent it is necessary "to mark" and not to use the cycling of this process, all knots which are found in two final elements further. Other edges can be found replacement of a condition in the third point, i.e. the exit will come from a cycle if the "marked" knot is found.

6. Computing experiment

On the basis of this algorithm the software is developed and streamlining of numbers of knots of the two-dimensional multicoherent area (figure 7) consisting of association of triangular and quadrangular areas [4] is carried out.

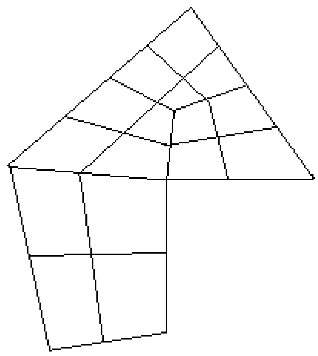

Fig. 7. Initial finite element grid.

The parties of a triangle are divided into k, and the parties of a quadrangle on k/2 of pieces, respectively. At k=4 reduction of a difference between maximum and minimum numbers of knots with 25 to 7 is observed. At k=46 – with 2209 to 49 (i.e. approximately by 45 times).

7. Conclusion

The developed way of formation of a finite element grid allows breaking into smaller elements of different vicinity of inclusions that fully captures the physical essence of process of deformation of the bodies subject to external influences.The constructive elements considered above having cavities or inclusions can be components of the areas representing constructional composite materials, conglomerates of disperse particles in material, deposits of rocks, etc.

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