An Analytical Comparison between Applying FFT and DWT in WiMAX Systems

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***Abstract*—Discrete Wavelet Transform DWT has advantages over Fast Fourier Transform FFT in analyzing signals containing sharp spikes. DWT processes data at different scales. If we look at the signal with a large window, we would notice big features. Similarly, if we look at the signal with a small window, we would notice small features. The DWT features encourage us to compare them with those of the FFT. In this paper, we analytically compare between the error performance of FFT and DWT in terms of the Bit Error Probability BEP. Then WiMAX system parameters are used to ensure the suitability of application on WiMAX system.**

***Keywords*—Discrete Wavelet Transform (DWT), Fast Fourier Transform (FFT), Probability of Error p(e), Quadrature Mirror Filter (QMF), Worldwide interoperability for Microwave Access (WiMAX).**

1. Introduction

Multicarrier modulation techniques are efficient for wireless communication. They work by dividing the whole bandwidth into N subbands for N subcarriers overlapping with no interference due orthogonality between them [1]. Due to wireless channel fading, orthogonality may be lost leading to InterSymbol Interference ISI and InterCarrier Interference ICI. Fast Fourier Transform-Orthogonal Frequency Division Multiplexing FFT-OFDM is efficient in removing ICI and ISI by using a Cyclic Prefix CP at each transmitted symbol. Using CP results in poor spectral efficiency because it carries no information. Due to the communication growth, we need to transmit more data with high data rate and low Bit Error Rate BER. As a consequence, CP needs to be removed and another way to keep the orthogonality between subcarriers is required. This way is to use the Discrete Wavelet Transform DWT rather than FFT. Wavelet transform is a technique that depends on decomposition and reconstruction to send signal through a communication system. It uses family of wavelets rather than sinusoids. Unlike sinusoids in FFT that are smooth, regular and symmetric, these wavelets can be regular or irregular, symmetric or asymmetric, sharp or smooth [2]. DWT is implemented by dividing the whole bandwidth between subcarriers dyadically so the subcarriers are different in bandwidth [3]. FFT uses a rectangle pulse in time domain which results in large side lobes in frequency domain requiring a good equalization scheme in signal recovery [4]. However, DWT reduces these side lobes due to using different pulse shapes. Additionally, DWT is well localized in frequency domain and time domain so it saves the orthogonality unlike FFT, which is localized in time only [5]. We apply the FFT and DWT model on WiMAX system model and we concluded that using DWT improved the system performance due to reduction in BER compared to FFT. The paper is organized as follows; in sec.II we discuss the WiMAX system model. An analysis of the probability of error $P(e)$ of FFT is presented in sec.III. In sec.IV the probability of error $P(e)$ of DWT model is discussed. Simulation and results are presented in sec.V. And finally the conclusion is presented in sec.VI.

2. WiMAX Model

WiMAX stands for Worldwide Interoperability for Microwave Access. It was designed to provide services of fixed and mobile broadband applications. Fixed wireless broadband supports the service of fixed lines like DSL. Mobile broadband provides portability, nomadicity and mobility. It is an IP architecture network as it allows voice telephony using Voice over IP network VoIP and TV over IP network TVoIP [7]. It provides point-to-point range of 30 miles with throughput or performance of 72 Mpbs [7]. Additionally it supports point to multipoint NLOS 4 miles with 11 Mpbs data rate.

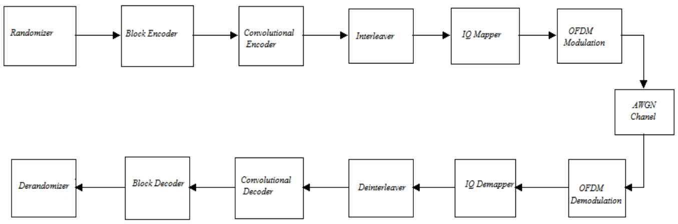


Fig.1 WiMAX system model [6]

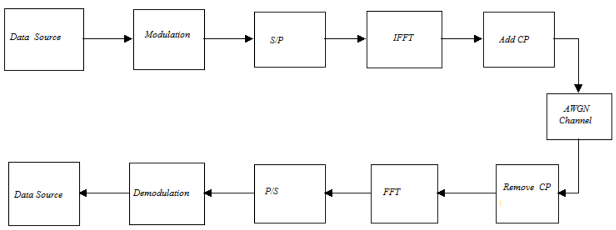


Fig. 2 OFDM modulation model [6]

It uses adaptive channel bandwidth from 1.25 MHz to 20 MHz. As shown in fig.1, WiMAX network uses Forward Error Correction FEC scheme to ensure good quality of information delivery. The signal from data source is fed to modulation block then it passes through serial to parallel converter S/P followed by IFFT block. Then a CP is added. The inverse process takes place at the receiver. Fig.2 shows the construction of OFDM modulation which utilizes the IFFT in transmitter and FFT in the receiver. FFT and IFFT calculate Discrete Fourier Transform DFT and Inverse Discrete Fourier Transform IDFT respectively and both are defined by the following equations [8].

$$\hat{f}[k] = \frac{1}{N}\sum_{n=0}^{N-1} f[n]\, e^{-i2\pi kn/N} \tag{1}$$

$$f[n] = \sum_{n=0}^{N-1} \hat{f}[k]\, e^{i2\pi kn/N} \tag{2}$$

WiMAX was designed to provide maximum performance to maximum distance by using physical layer technologies. It uses different technologies such as OFDM, Time Division Duplex TDD, Frequency Division Duplex FDD and Adaptive Antenna System AAS. WiMAX uses OFDM as an efficient way of transmission in multipath propagation environment. In our analysis, we use the WiMAX simulation model published in [10]. DWT is implemented by using filter bank. It decomposes the signal into low frequency components and high frequency components which results in different frequency and time resolutions. This way in decomposition makes DWT flexible and adaptive in bandwidth allocation to each subcarrier rather than using fixed bandwidth allocation like FFT-OFDM. Using DWT in WiMAX is expected to improve its performance due BER reduction. Additionally, it provides diversity which eliminates ICI because adjacent cells in the network can use the same frequency band without interference [9]. Good localization in time and frequency of DWT will improve BER and increase the spectral efficiency. Mathematically, the DWT is defined by [10].

$$Wf(u,2^j) = \int_{-\infty}^{+\infty} f(t)\frac{1}{\sqrt{2^j}}\,\psi\left(\frac{t-u}{2^j}\right)dt = f * \bar{\psi}_{2j}(u) \tag{3}$$

With

$$\bar{\psi}_{2j}(u) = \psi_{2j}(-t) = \frac{1}{\sqrt{2^j}}\,\psi\left(\frac{-t}{2^j}\right) \tag{4}$$

Eq. (3) represents the inner product between the function and the mother wavelet function ψ that results in wavelet coefficients. These coefficients are scaled and shifted version of the mother wavelet ψ. Additionally, these wavelet basis satisfy orthogonality between subcarriers that is defined by the equation [11].

$$< \psi_{j,k}(t), \psi_{m,n}(t) > = \begin{cases} 1 & j = m, k = n \\ 0 & otherwise \end{cases} \tag{5}$$

3. Analytical Derivation of Probability of Error *P(e)* in FFT-WiMAX

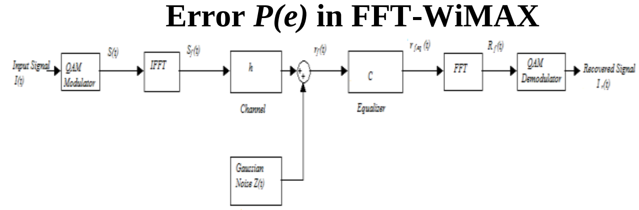


Fig. 3 FFT-WiMAX model

The signal is modulated by I-ary QAM modulator and converted from frequency domain to time domain by IFFT. After propagation through the channel, Gaussian noise $Z(t)$ is added to the signal. An equalizer is used here to remove the channel effect on the signal to go through FFT to convert it from time to frequency domain. Finally it is demodulated to recover the original signal. The signal is composed here from M-carriers corresponding to FFT size and the added noise is Gaussian noise of zero mean and normal distribution over time [$z_i(t) \sim N(0,\delta^2)$][12]. $S_f(t)$ is the signal after FFT. After passing through the channel, its output is ($S_f(t) * h$) and h is the channel impulse response. Before the equalizer, the signal expressed as

$$r_f(t) = S_f(t) * h + Z(t),\ \forall\, t = 0,1,2,\ldots.T \tag{6}$$

Where T is the signaling interval and the operator * denotes linear convolution. The M-carriers here are different in their noises so the channel impulse response is a vector. Consequently, the M-carriers are independent identically distributed i.d.d complex Gaussian noises which are represented by vector $Z(t)$. Each element in this vector corresponds to $z_i(t)$ which is the noise added to the i^{th} carrier ($Z(t) = [\ z_i(t)],\ \ i = 1,2,3,\ldots.M$). The channel equalizer uses zero forcing ZF algorithm which satisfy the following equation

$$\boldsymbol{h} * \boldsymbol{c} = \delta \tag{7}$$

Where δ is the Dirac Delta function. The signal after equalizer is

$$r_{f,eq}(t) = c * r_f(t) \tag{8}$$

$$= c * \left(S_f(t) * h + Z(t)\right)$$

$$= c * S_f(t) * h + c * Z(t)$$

$$= \delta * S_f(t) + c * Z(t)$$

$$= S_f(t) + \hat{Z}(t) \tag{9}$$

The signal $r_{f,eq}(t)$ will be fed to FFT block to get

$$R_f(t) = FFT(r_{f,eq}(t))$$

$$= FFT(S_f(t) + \hat{Z}(t))$$

$$= FFT\left(S_f(t)\right) + FFT\big(\hat{Z}(t)\big)$$

$$= S(t) + Z^f(t) \tag{10}$$

The demodulated signal $R_f(t)$ is influenced by channel noise which is a function of channel equalization coefficients. The analytical expression of Bit Error Probability BEP is a function of ${}^{E_b}/_{N_o}$ where E_b is the energy per bit and N_o is the noise variance of the channel received by demodulator. So we must calculate the variance of $Z^f(t)$.

Since, $Z^f(t) = FFT\big(\hat{Z}(t)\big) = FFT\big(\ c * Z(t)\big)$ and

$$FFT\big(Z(t)\big) = \textstyle\sum_k Z(t) e^{-i\,(2\pi/M)km}$$

Then for each element of $Z^f(t)$ we have

$$z_m^f(t) = \tfrac{1}{\sqrt{M}} \textstyle\sum_{k=0}^{M} \hat{z}_k(t)\ \ e^{-i\,(2\pi/M)km}$$

$$= \tfrac{1}{\sqrt{M}} \textstyle\sum_{k=0}^{M}\big(c * Z(t)\big)\ e^{-i(2\pi/M)km}$$

$$= \tfrac{1}{\sqrt{M}} \textstyle\sum_{k=0}^{M}(\sum_n c_{k-n}\ z_n(t))\ \ e^{-i(2\pi/M)km}$$

$$= \tfrac{1}{\sqrt{M}} \textstyle\sum_n z_n(t)\ \ \sum_{k=0}^{M} c_{k-n}\ e^{-i(2\pi/M)km} \tag{11}$$

Where m stands for the carrier index. To estimate the variance of a function, we must first estimate mean or expectation

$$var\big(x(t)\big) = E\{x(t)^2\} - \ (E\{x(t)\})^2 \tag{12}$$

$$E\{z_m^f(t)\} = E\left(\tfrac{1}{\sqrt{M}} \textstyle\sum_n z_n(t)\ \ \sum_{k=0}^{M} c_{k-n}\ e^{-i(2\pi/M)km}\right)$$

$$= \tfrac{1}{\sqrt{M}} \textstyle\sum_n E(z_n(t))\ \ \sum_{k=0}^{M} E\big(c_{k-n}\ e^{-i(2\pi/M)km}\big)$$

$$= \tfrac{1}{\sqrt{M}} \textstyle\sum_n 0\ \ \sum_{k=0}^{M} c_{k-n}\ e^{-i(2\pi/M)km}$$

$$= 0 \tag{13}$$

The expectation is zero and variance is obtained by

$$var\{z_m^f\} = E\left\{\left(z_m^f(t)\right)^2\right\} - \left(E(z_m^f(t)\right)^2$$

$$\text{So,}\ \ N_m^f = var\{z_m^f(t)\} = \ E\left\{\left(z_m^f(t)\right)^2\right\}$$

$$= E\left\{\left(\tfrac{1}{\sqrt{M}} \textstyle\sum_n z_n(t)\ \ \sum_{k=0}^{M} c_{k-n}\ e^{-i(2\pi/M)km}\ \right)^2\right\}$$

$$= \tfrac{1}{M} \textstyle\sum_n E(z_n(t))^2\ (\sum_{k=0}^{M} E(c_{k-n}\ e^{-i(2\pi/M)km}))^2$$

$$= \tfrac{1}{M}.\ \delta^2\ \textstyle\sum_n(\sum_{k=0}^{M} c_{k-n}\ e^{-i(2\pi/M)km})^2$$

$$= \tfrac{\delta^2}{M}\ \textstyle\sum_n \sum_{k=0}^{M}\big(c_{k-n}\ e^{-i(2\pi/M)km}\big)^2 \tag{14}$$

From Eq. (14), it is obvious that N_m^f is a function of carrier index m, which means that the noises added to carriers are different and not equal.

4. Analytical Derivation of Probability of Error *P(e)* of DWT-WIMAX

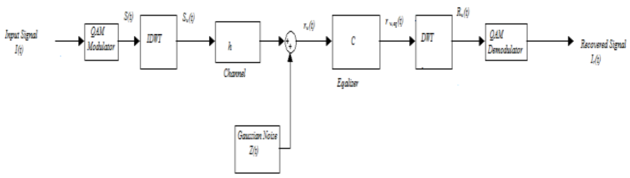


Fig. 4 DWT-WiMAX model

In fig. 4, the IFFT is replaced by the IDWT to perform the same process of converting the signal from frequency to time domain. The DWT conversion at the receiver is done by decomposing the signal into discrete wavelet coefficients using Quadrature Mirror Filter QMF or sometimes it is named filter bank. The filter bank consists of two parts; one is Low Pass Filter LPF of impulse response g_n and the second is High Pass Filter HPF of impulse response h_n. The incoming signal is filtered using these filters to result in half bandwidth of original one. The filters are followed by downsampling to reduce the signal bit rate and keep the signal samples equal to the original signal samples. The output of the LPF is Approximation Coefficient AC and the output of HPF Details Coefficients DC. To perform multilevel DWT, we decompose the AC branch in each level into AC and DC. Fig. 5 shows an example for a filter bank [13]. The transfer functions of these filters are shown in the following [13].

$$r_o[n] = x[n]h_o[n] = \frac{1}{\sqrt{2}}(x[n] + x[n-1]) \qquad (15)$$

After downsampling, alternative samples are dropped to get

$$y_o[n] = r_o[2n] = \frac{1}{\sqrt{2}}(x[2n] + x[2n-1]) \qquad (16)$$

Likewise $y_1[n]$ will be obtained

$$r_1[n] = x[n]h_1[n] = \frac{1}{\sqrt{2}}(x[n] - x[n-1]) \qquad (17)$$

$$y_1[n] = r_1[2n] = \frac{1}{\sqrt{2}}(x[2n] - x[2n-1]) \qquad (18)$$

In reconstructing part, the signal first up sampled by inserting zeros to get the original signal

$$t_o[n] = \begin{cases} y_o[\frac{n}{2}] & n\ even \\ 0 & n\ odd \end{cases} \qquad (19)$$

And

$$t_1[n] = \begin{cases} y_1[\frac{n}{2}] & n\ even \\ 0 & n\ odd \end{cases} \qquad (20)$$

$$\therefore\ v_o[n] = t_o[n]\, f_o[n]$$

$$= \left(\frac{1}{\sqrt{2}}(t_0[n] + t_1[n+1])\right)$$

$$= \begin{cases} \frac{1}{\sqrt{2}}\, y_o\left[\frac{n}{2}\right] & n\ even \\ \frac{1}{\sqrt{2}}\, y_o\left[\frac{n+1}{2}\right] & n\ odd \end{cases} \qquad (21)$$

$$\therefore v_1[n] = t_1[n]\, f_1[n]$$

$$= \left(\frac{1}{\sqrt{2}}(t[n] - t[n+1])\right)$$

$$= \begin{cases} \frac{1}{\sqrt{2}}\, y_1[\frac{n}{2}] & n\ even \\ \frac{-1}{\sqrt{2}}\, y_1[\frac{n+1}{2}] & n\ odd \end{cases} \qquad (22)$$

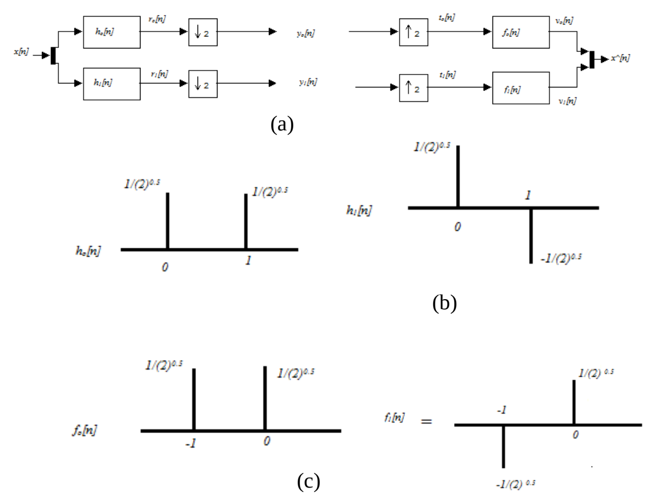


Fig. 5 (a) a Filter bank example [11]

(b) Impulse response of the LPF $h_o[n]$ and HPF $h_1[n]$ in the decomposition part

(c) Impulse response of LPF $f_o[n]$ and HPF $f_1[n]$ in the reconstruction part

The reconstructed signal is

$$\hat{x}[n] = v_o[n] + v_1[n]$$

$$= \begin{cases} \frac{1}{\sqrt{2}}\, y_o\left[\frac{n}{2}\right] + \frac{1}{\sqrt{2}}\, y_1\left[\frac{n}{2}\right] & n\ even \\ \frac{1}{\sqrt{2}}\, y_o\left[\frac{n+1}{2}\right] - \frac{1}{\sqrt{2}}\, y_1\left[\frac{n+1}{2}\right] & n\ odd \end{cases} \quad (23)$$

Fig.6 shows the structure of a three levels filter bank to conduct three levels DWT. The signal expression for each level is given by

(a)First Level Decomposition

$$AC_1 = \sum_{k=-\infty}^{+\infty} x(k)\, g(2n-k) \quad (24)$$

$$DC_1 = \sum_{k=-\infty}^{\infty} x(k)\, h(2n-k) \quad (25)$$

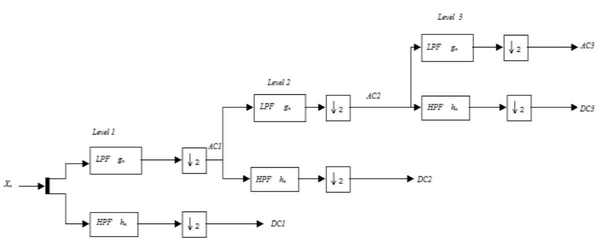
(b) Second Level Decomposing

$$AC_2 = \sum_{k=-\infty}^{\infty} AC_1(k)\ g(2n-k)$$

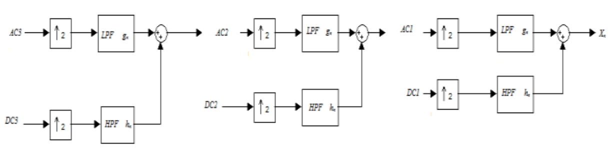
$$= \sum_{k=-\infty}^{\infty} x(k) g(2n-k) \sum_{p=-\infty}^{\infty} g(2n-p) \quad (26)$$

$$DC_2 = \sum_{k=-\infty}^{\infty} AC_1(k) h(2n-k)$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(2n-k) \sum_{p=-\infty}^{\infty} h(2n-p) \quad (27)$$



(a)



(b)

Fig. 6 (a) Three level decomposition filter bank

(b) Three level reconstruction filter bank

(c) Third Level Decomposition

$$AC_3 = \sum_{k=-\infty}^{\infty} AC_2(k)\ g(2n-k)$$

$$= \sum_{k=-\infty}^{\infty} x(k) g(2n-k) \sum_{p=-\infty}^{\infty} g(2n-p) \sum_{m=-\infty}^{\infty} g(2n-m) \quad (28)$$

$$DC_3 = \sum_{k=-\infty}^{\infty} AC_2(k)\, h(2n-k)$$

$$= \sum_{k=-\infty}^{\infty} x(k) h(2n-k) \sum_{p=-\infty}^{\infty} h(2n-p) \sum_{m=-\infty}^{\infty} h(2n-m) \quad (29)$$

The signal before equalizer is expressed as

$$r_w(t) = S_w(t) * h + z(t) \quad (30)$$

Then the equalized signal is

$$r_{eq,w} = r_w(t) * c$$

$$= \left(S_w(t) * h + Z(t)\right) * c$$

$$= S_w(t) * h * c + Z(t) * c$$

$$= S_w(t) * \delta + \hat{Z}(t)$$

$$= S_w(t) + \hat{Z}(t) \quad (31)$$

After Performing DWT Conversion

$$R_w(t) = DWT\{r_{eq,t}(t)\} = DWT\{S_w(t) + \hat{Z}(t)\}$$

$$= DWT\{S_w(t)\} + DWT\{\hat{Z}(t)\}$$

$$= S(t) + Z^w(t) \quad (32)$$

The term $Z^w(t)$ is influencing the demodulated signal at the receiver. So we will obtain the variance of $Z^w(t)$. Since QMF depends on linear convolution as it obvious from Eq. (24) to Eq. (29) and it requires excessive length so we will apply circular convolution. The filter length will be denoted by L.

1- Left Circular Shifting the Input Elements by L/2

Since, $Z^w = DWT\{\hat{Z}(t)\} = DWT\{c * Z(t)\}$, DWT conversion will be done by circular convolution the input by L/2 according to[14].

$$\sigma(i, X) \equiv \left(i + \frac{L}{2}\right) mod\ X \quad (33)$$

For each element $z_m(t)$ of the vector $Z\ (t)$ we have:

$$\bar{z}_m(t) = \hat{z}_{\sigma(m,M)}(t)$$

$$= c_{\sigma(m,M)} * z_m(t)$$

$$= \sum_n c_{\sigma(m,M)-n}\ z_n(t) \quad (34)$$

Since, $\sigma(m, M) = \left(m + \frac{L}{2}\right) mod\ M = m + \frac{L}{2} + M.r$

$$\sigma(m, M) - n = m + \frac{L}{2} + M.r - n,$$

$$-n + \frac{L}{2} + M.r = \left(-n + \frac{L}{2}\right) mod\ M = \sigma(-n, M)$$

So, $\sigma(m,M) - n = m + \sigma(-n,M)$

Then, $\bar{z}_m(t) = \sum_n c_{m+\sigma(-n,M)} . z_n(t)$ (35)

2- Circular Convolution of the Left Circularly Shifted Input Signal of QMF and Down Sampling

The circular convolution of two signals x and y of equal period N is defined by $x_{[n]}^N \otimes y_{[n]}^N = \sum_m x_m \; y_{n-m}^N$ with the operator $\otimes$ denotes circular convolution and it will be done here for the AC part so we will deal with LPF g_n . The first level AC_{1m} circularly shifted is defined by

$$AC_{1m} = \left(\bar{z}_m(t) \otimes g_m\right) \downarrow 2$$

$$= \sum_n (\bar{z}_n(t) \; g_{m-n})^M \downarrow 2$$

$$= \sum_n (\bar{z}_n(t) \; g_{2m-n})^M$$

By putting Eq. (35) into AC_{1m} yielding

$$AC_{1m} = \sum_n \sum_a \left(c_{\sigma(n,M)} * \; z_a(t)\right) g_{2m-n}^M$$

$$= \sum_n \left(\sum_a c_{\sigma(n,M)-a} \; z_a(t)\right) \; g_{2m-n}^M$$

$$= \sum_a z_a(t) \sum_n c_{\sigma(n,M)-a} \; g_{2m-n}^M \quad (36)$$

We inserted a different symbol a other than n in previous equation because we have n circular convolution operations and n of shifting operations in linear convolution. The next step is to estimate the signal variance through the signal expectation.

$$E\{AC_{1m}\} = E\left\{\sum_a z_a(t) \sum_n c_{\sigma(n,M)-a} \; g_{2m-n}^M\right\}$$

$$= \sum_a 0 \; \sum_n c_{\sigma(n,M)-a} \; g_{2m-n}^M$$

$$= 0 \quad (37)$$

$$var\{AC_{1m}\} = E\{(AC_{1m})^2\} - \{E(\; AC_{1m})\}^2$$

$$= E\{(AC_{1m})^2\} - 0$$

$$= E\{(\sum_a z_a(t))^2\} \; E\left\{\left(\sum_n c_{\sigma(n,M)-a} \; g_{2m-n}^M\right)^2\right\}$$

$$= \delta^2 \; \sum_a \left(\sum_n c_{\sigma(n,M)-a} \; g_{2m-n}^M\right)^2 \quad (38)$$

Inverting the shifting direction by inverting the sign of a and n

$$var\{AC_{1m}\} = \delta^2 \; \sum_a \left(\sum_n c_{\sigma(-n,M)+a} \; g_{2m+n}^M\right)^2 \quad (39)$$

Since, $\sigma(-n,M) + a = \left(-n + \frac{L}{2}\right) \; mod \; M + a$

$$= \left(-n + \frac{L}{2}\right) + M.r + a$$

Substituting $\frac{L}{2} + M.r + a = \left(a + \frac{L}{2}\right) \; mod \; M$

$$\sigma(-n,M) + a = -n + \sigma(a,M)$$

So, $var\{AC_{1m}\} = \delta^2 \sum_a \left(\sum_n c_{\sigma(a,M)-n} \; g_{2m-n}\right)^2$

$$= \delta^2 \; \sum_a \left(c_{\sigma(a,M)} \otimes g_n^M\right)^2 \quad (40)$$

Likewise,

$$var\{DC_{1m}\} = \delta^2 \; \sum_a \left(c_{\sigma(a,M)} \otimes h_n^M\right)^2 \quad (41)$$

For the second level, we will obtain DC_{2m} and its variance

$DC_{2m} = \left(AC_{1m\,\sigma(m,M/2)} \otimes h_m^{M/2}\right) \downarrow 2$, this coefficient will be shifted by L/2 a period of *M/2* to result in

$$DC_{2m} = AC_{\sigma(m,M/2)} . h_{2m-b}^{M/2}$$

$$= \sum_b \left(\sum_n \left(\sum_a c_{\sigma(n,M)-a} \; z_a(t)\right) \; g_{2\sigma\left(b,\frac{M}{2}\right)-n}^M\right) \; h_{2m-b}^{M/2}$$

Here g_{2m-n} became $g_{2\sigma\left(b,\frac{M}{2}\right)-n}^M$ because it is twice circularly shifted by L/2.

$$DC_{2m} = \sum_a z_a(t) \sum_n c_{\sigma(n,M)-a} \; \sum_b h_{2m-b}^{M/2} \; g_{2\sigma\left(b,\frac{M}{2}\right)-n}^M$$

$$var\{DC_{2m}\} = E\{(DC_{2m})^2\} =$$

$$\sum_a E\{z_a(t)\}^2 \; \sum_n E\left\{\left(c_{\sigma(n,M)-a}\right)^2\right\} \sum_b E\left\{\left(h_{2m-b}^{M/2} \; g_{2\sigma\left(b,\frac{M}{2}\right)-n}^M\right)^2\right\}$$

The signal after inverting the shifting direction and removing the term *2m* will be

$$var\{DC_{2m}\} =$$

$$\delta^2 \; \sum_a \left(\sum_n c_{\sigma(-n,M)-a} \; \sum_b h_b^{M/2} g_{2\sigma\left(-b,\frac{M}{2}\right)+n}^M\right)^2 \quad (42)$$

By putting $\sum_b \; h_b^{\frac{M}{2}} \; g_{2\sigma\left(-b,\frac{M}{2}\right)+n}^M = S_n$ and

$\sigma(-n, M) - a = a + \sigma(-n, M)$ in Eq. (42) gives

$$var\{DC_{2m}\} = \delta^2 \sum_a \left(\sum_n c_{a+\sigma(-n,M)} S_n\right)^2$$

Since, $a + (-n, M) = a + \left(-n, \frac{L}{2}\right) mod\ M$

$$= a + \left(-n + \frac{L}{2}\right) + M.r = a + \frac{L}{2} + M.r - n$$

$$= \left(a + \frac{L}{2}\right) mod\ M - n = \sigma(a, M) - n$$

Then, $var\{DC_{2m}\} = \delta^2 \sum_a \left(\sum_n c_{\sigma(a,m)-n}\ S_n\right)^2$

$$var\{DC_{2m}\} = \delta^2 \sum_a \left(c_{\sigma(a,M)} * S_n\right)^2 \quad (43)$$

3- Separated convolution

Separated convolution is suitable for any two sequences *x* and *y* where the length of *x* is twice the length of *y*. As a consequence we split the large sequence into odd and even sequences denoted by x^o and x^e respectively. $x^o = [x_1, x_3, x_5, \dots, x_{2N-1}]$ *and* $x^e = [x_0, x_2, x_4, \dots, x_{2N-2}]$. Then by performing circular convolution between *x* and *y* gets we get

$$z_k^o = x_k^o \otimes y_k^N = \sum_n x_{k-n}^o\ y_n^N \quad (44)$$

$$z_k^e = x_k^e \otimes y_k^N = \sum_n x_{k-n}^e\ y_n^N \quad (45)$$

$$z = [z_0^e, z_0^o, z_1^e, z_1^o, \dots, z_{N-1}^e, z_{N-1}^o] \quad (46)$$

Then the general formula of separated convolution is

$$z_a = x_a \odot y_b = \sum_n x_{a-2n}\ y_n^N \quad (47)$$

$$\forall\ a = 0,1,2, \dots, 2N-1 \ and\ b = 0,1,2, \dots, N-1$$

N here is doubled because shifting is done alternatively i.e. we take one sample and drop the following one, *N* is the total number of samples of z_a. Eq. (47) doesn't include the circular shift but it will be performed using Eq. (44) and Eq. (45) and it will be done here by L instead of L/2 because we circularly shift the odd and even samples of *z*.

$$z_{\bar{\sigma}(a,x)} = x_{\bar{\sigma}(a,x)} \odot y_b$$

$$= \sum_n x_{\bar{\sigma}(a,x)-2n}\ y_n^{x/2}\ \sum_n x_{(a+L) mod\ x - 2n}\ \ y_n^{x/2}$$

$$= \sum_n x_{(a+L)+x.r-2n}\ \ y_n^{x/2} = \sum_n x_{a+L-2n+x.r}\ \ y_n^{x/2}$$

$$= \sum_n x_{a+\sigma(-2n,x)}\ \ y_n^{x/2}$$

$$= \sum_n x_{a+2\sigma(-n,\frac{x}{2})}\ \ y_n^{x/2} \quad (48)$$

By applying Eq. (48) on s_n and putting it in Eq. (43) we get

$$S_n = g_{\bar{\sigma}(n,M)} \odot h_k = \sum_b h_b^{M/2}\ \ g^M_{n+2\sigma\left(-b,\frac{M}{2}\right)}$$

$$var\{DC_{2m}\} =$$

$$\delta^2 \sum_a \left(c_{\sigma(a,M)} * \left(g_{\bar{\sigma}(a,M)} \odot h_k^{M/2}\right)\right)^2 \quad (49)$$

Likewise,

$$var\{AC_{2m}\} =$$

$$\delta^2 \sum_a \left(c_{\sigma(a,M)} * \left(g_{\bar{\sigma}(a,M)} \odot g_k^{M/2}\right)\right)^2 \quad (50)$$

Similarly the variance of the third level

$$var\{DC_{3m}\} =$$

$$\delta^2 \sum_a \left(c_{\sigma(a,M)} * \left(g_{\bar{\sigma}(a,M)} \odot \left(g_{\bar{\sigma}(b,M/2)} \odot h_k^{M/4}\right)\right)\right)^2 \quad (51)$$

$$var\{AC_{3m}\} =$$

$$\delta^2 \sum_a \left(c_{\sigma(a,M)} * \left(g_{\bar{\sigma}(a,M)} \odot \left(g_{\bar{\sigma}(b,M/2)} \odot g_k^{M/4}\right)\right)\right)^2 \quad (52)$$

The theoretical Bit Error Probability BEP is calculated from [15]

$$Pb = \frac{\sqrt{I}-1}{\sqrt{I}\ log_2\sqrt{I}}\ erfc\left[\sqrt{\frac{3\ log_2 I\ E_b}{2(I-1)\ N_o}}\right] \quad (53)$$

I is the I-ary QAM modulation

For FFT-OFDM model, *Pb* will be calculated from [15] with Eq. (53)

$$Pb_m^f = \frac{\sqrt{I}-1}{\sqrt{I}\ log_2\sqrt{I}}\ erfc\left[\sqrt{\frac{3\ log_2 I\ E_b}{2(I-1)\ N_m^f}}\right] \quad (54)$$

Similarly for DWT-OFDM model, *Pb* will be calculated from Eq. (40) with Eq. (53)

$$Pb_m^w = \frac{\sqrt{I}-1}{\sqrt{I}\ log_2\sqrt{I}}\ erfc\left[\sqrt{\frac{3\ log_2 I\ E_b}{2(I-1)\ N_m^w}}\right] \quad (55)$$

5. Simulation and Evaluation

We use WiMAX parameters [6], [IEEE 802.16 e WiMAX ETSI HiperMAN]. These parameters are shown in table 1. First we use WiMAX simulation using MATLAB Simulink to get its BER behavior which is shown in fig. 7. First, we apply the FFT analytical model derived in this paper with WiMAX parameters and it gives approximately the same performance as the simulation. Applying the DWT model with the channel impulse response [IEE802.16e WiMAX ETSI HiperMAN] shows the performance enhancement over FFT. Fig. 8 shows this BER behavior of applying DWT. The better performance of DWT than FFT is easily noticed.

parameters	Value
Channel bandwidth *BW*	10 MHz
Number of data subcarriers N_{data}	192
Number of pilot subcarriers N_{pilot}	8
FFT size N_{FFT}	255
Number of used subcarriers N_{used}	200
Modulation	QPSK
Channel	AWGN

Table1: Simulation parameters

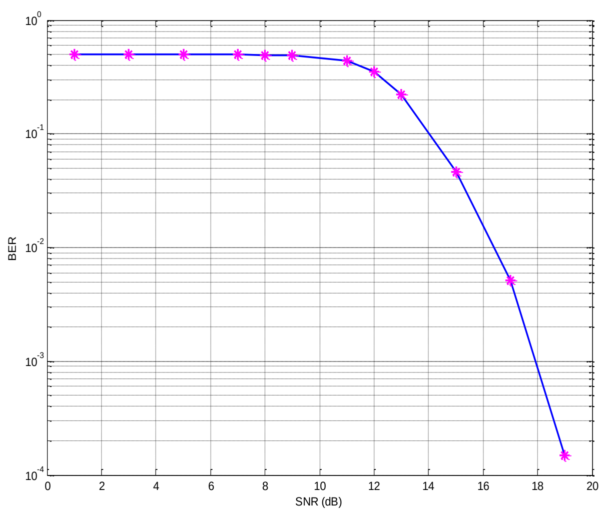


Fig. 7 BER evaluation of simulated FFT-OFDM

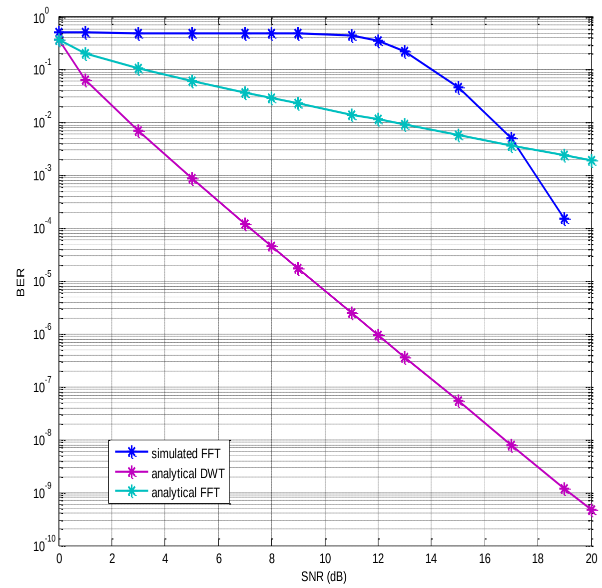


Fig. 8 BER evaluation of analytical FFT and DWT

6. Conclusion

In this paper, we analyzed the DWT and FFT error performances in terms of BEP. Using WIMAX system parameters in our simulation, we proposed the usage of DWT rather than FFT. DWT gives better error performance than FFT. We made a comparison between simulation and analytical results. We show that DWT performs better than FFT with WIMAX system parameters.

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