Image Compression Using Wavelets

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Abstract

Image compression is a fast paced and dynamically changing field with many different varieties of compression methods available. Wavelet transform techniques currently provide a promising approach to image compression and concentrate on achieving higher compression ratio without sacrificing the quality of the image. In image compression, the choice of wavelet depends on the image content. The quantitative measures Compression Ratio(CR), Bit-Per-Pixel(BPP) and perceptual quality measures Mean Square Error(MSE) and Peak Signal To Noise Ratio(PSNR) are calculated for different images with different wavelets and comparative study has been made.

Keywords: *Discrete Wavelet Transform (DWT), Bit-Per-Pixel (BPP), Mean Square Error (MSE), Peak Signal To Noise Ratio (PSNR), Compression ratio (CR)*

1.Introduction

Image compression is very important for efficient transmission and storage of images. It has many applications in information theory[1],applied harmonic analysis[2] and in many other fields. The objective of image compression is to minimize the size of an image by exploiting the redundancy within the data without degrading the quality of the image. The reduction in file size allows more images to be stored in a given amount of disk or memory space. The common redundancies are spatial redundancy, temporal redundancy, inter-pixel redundancy, psycho-visual redundancy and statistical redundancy[3,4]. Some common image compression methods are JPEG, Discrete Cosine Transform, Wavelet Algorithm Transform, Fractal based and several other techniques like NTT and Neural Network. Discrete Wavelet Transform (DWT) can be efficiently used in image coding applications because of their data reduction capabilities. Basis of DWT can be composed of any function (wavelet) that satisfy the requirements of multiresolution analysis [5]. V. Elamaran et al.[6] have described the basic idea of compression and attempted to reduce the average number of bits per pixel to adequately represent an image. Fourier based transforms (e.g. DCT and DFT) are efficient in exploiting the low frequency nature of an image. The high frequency coefficients are coarsely quantized, and hence reconstructed image have poor quality at edges. M. Gupta et al. [7] have developed some simple functions to compute the DCT and to compress images. Image Compression is studied using 2-D discrete Cosine Transform. The original image is transformed in 8-by-8 blocks then inverse transform in 8-by-8 blocks to reconstruct image and the error image (the difference between the original and reconstructed image) has been displayed. M. Chowdhury et al. [8] have described a new image compression scheme with pruning proposal based on discrete wavelet transformation. A new image compression scheme based on discrete wavelet transform is proposed which provides sufficient high compression ratios with no appreciable degradation of image quality.

In this paper, we have studied the behavior of different type of wavelet functions with different types of images and suggested the appropriate wavelet function that can perform optimum compression for a given type of images. The effects of different wavelet functions and compression ratios are assessed. This investigation is carried out by calculating the Compression Ratio(CR), Mean Square Error(MSE), Bit Per Pixel(BPP) and Peak Signal to Noise ratio(PSNR) for different wavelets.

This paper is organized into four sections : Section 2 presents the theory behind wavelets and Discrete Wavelet Transform. Section 3 describes image compression using wavelets and how the wavelet transform is implemented and used in image compression systems. In section 4, we present some results of different wavelets used for different images and the conclusion.

2. Basics of Wavelet Transform

Wavelets basis functions are generated by scaling and translation of function $\psi(t) \in L^2(R)$ and $\psi(t)$ is called mother wavelet. Mother wavelet is scaled by a factor of a and translation by a factor of b to give

$$\psi_{a,b}(t) = |a|^{\frac{-1}{2}} \psi\left(\frac{t-b}{a}\right),$$

where $a \neq 0$ and $b \in R$
where a and b are two arbitrary real numbers representing the dilation and translation parameters in the time axis. The parameter 'a' contracts $\psi(t)$ in the time axis when $a < 1$ and expands or stretches when $a > 1$.

Wavelets are special kind of mathematical functions that exhibits oscillatory behaviour for short period of time and then die out. Wavelet function $\psi(t) \in L^2(R)$ has zero average value and also satisfy admissibility condition, i.e.

$$\int_{-\infty}^{\infty} \psi(x)dx = 0$$

and

$$c_\psi = 2\pi \int_R \frac{|\,\widehat{\psi}(w)|^2}{|w|} dw < \infty$$

2.1 Multiresolution Analysis and scaling function

A multiresolution analysis on R is a sequence of subspaces V_j of functions $L^2(R)$ which approximate space $L^2(R)$ and satisfies the following properties.
(i) $... \subset V_{-1} \subset V_0 \subset V_1 \subset ...$
(ii) span $\bigcup_{j \epsilon Z} V_j = L^2(R)$
(iii) $\bigcap_{j \epsilon Z} V_j = 0$
(iv) $f(x) \in V_j$ if and only if $f(2^{-j}x) \in V_0$
(vi) there exist a function $\phi(t) \in L^2(R)$ called scaling function such that the system $\{T_n\phi(t) = \phi(t-n)\}$ is an orthonormal system of translates and

$$V_0 = \overline{span}\{T_n\phi(x)\}$$

The subspaces $\{V_j\}$ is generated by basis functions

$$\phi_{j,k}(t) = 2^{\frac{j}{2}}\phi(2^j t - k), \; j, k \in Z$$

where $2^{j/2}$ denotes scale of scaling function$\{\phi_{j,k}(t)\}$ and k denotes translation in time. Since $V_0 \subset V_1$, any function from subspace V_0 can be represented with basis function from V_1.

$$\phi(t) = \sqrt{2}\sum_k h_k \phi(2t-k)$$

Using the fact that $\{\phi_{j,k}(t)\}_{k \in Z}$ are orthonormal basis for V_j, the coefficients h_k can be obtained by computing the inner product:

$$h_k = \sqrt{2}\int_{-\infty}^{\infty} \phi(t)\phi(2t-k)dt$$

$\{h_k\}$ are called low pass filter coefficients.
By defining W_j as an orthogonal complement of V_j in V_{j+1},

$$V_{j+1} = V_j \oplus W_j,$$

the space $L^2(\mathbf{R})$ is represented as a direct sum of W_j's as

$$L^2(\mathbf{R}) = \oplus_{j\in Z} W_j$$

The entire space of square integrable functions $L^2(R)$ can be decomposed into orthogonal subspaces W_j each containing information about details at given resolution. Detail space W_j has orthonormal basis $\psi_{j,k}(t)$, where

$$\psi_{j,k}(t) = 2^{\frac{j}{2}}\psi(2^j t - k)$$

So $L^2(R)$ has an orthonormal basis $\psi_{j,k}(t)$ called wavelet basis. Since $\psi(t-k)$ is in $W_0 \subset V_1$, $\psi(t)$ can be represented as superposition of basis function for V_1.

$$\psi(t) = \sqrt{2}\sum_k g_k \phi(2t-k)$$

Using the fact that $\{\phi_{j,k}(t)\}_{k\in Z}$ are orthonormal for V_j, the coefficients g_k can be obtained by computing the inner product

$$g_k = \sqrt{2}\int_{-\infty}^{\infty} \psi(t)\phi(2t-k)dt$$

$\{g_k\}$ are called high pass filter coefficients. These can also be calculated from coefficients of low pass filter using

$$g_k = (-1)^k h_{1-k}$$

2.2 Discrete Wavelet Transform

Supposing that input function $f(t)$ only known to the certain level j and that details on the scales smaller than 2^j is ignored, the approximation of $f(t)$ on level j is determined by equation

$$f_j(t) = \sum_k a_k^j \phi_{j,k}(t)$$

where $\phi_{j,k}(t)$ is a scaling function on resolution level j and translated by integer k and a_k are coefficients given by

$$a_k^j = \int_R f_j(t)\phi_{j,k}(t)dt$$

The function $f_j(t)$ can be uniquely represented by the coefficients a_k. The function $f_j(t)$ can be decomposed into a smooth part $f_{j-1}(t)$ on the next coarser level $j-1$ and detail d_{j-1}

$$f_j(t) = f_{j-1}(t) + d_{j-1}(t)$$

$$= \sum_k a_k^{j-1}\phi_{j-1,k}(t) + \sum_k d_k^{j-1}\psi_{j-1.k}(t)$$

where $\phi_{j,k}(t)$ is a wavelet function on resolution level j and translated by integer k and $d_k^j$are the coefficients given by

$$d_k^j = \int_R f_j(t)\psi_{j,k}(t)dt$$

Smooth part of $f_j(t)$ can be further decomposed to smooth part and detail part on resolution level $j-2$. Decomposition of input function can be repeated untill the coarsest level j_0 is reached.

2.3 2-D Discrete Wavelet Transform

Discrete Wavelet Transform for two dimensional signals or images can be derived from one-dimensional DWT. Easiest way for obtaining scaling and wavelet function for two-dimensions is by multiplying two one-dimensional functions.
Scaling function for 2-D DWT can be obtained by tensor product of two 1-D scaling functions. Generally different scaling functions can be used for each direction but in practice those functions are in most cases the same.

$$\phi(x,y) = \phi(x)\phi(y)$$

Wavelet functions for 2-D DWT can be obtained by multiplying two wavelet functions or wavelet and scaling function for one-dimensional analysis. For 2-D case there exist three wavelet functions that analysis details in horizontal, vertical and diagonal direction respectively.

$$\psi^{(I)}(x,y) = \psi(x)\phi(y)$$

$$\psi^{(II)}(x,y) = \phi(x)\psi(y)$$

$$\psi^{(III)}(x,y) = \psi(x)\psi(y)$$

3. Image Compression Based On DWT

A typical black and white image is an $M * M$ array of integers chosen from some specified range say 0 through $L - 1$.Each element of this array is referred to as picture element or pixel and the value of each pixel is grayscale value and represents the shades of the gray of the given pixel. If pixel value 0 then color is black and if $L - 1$ color is white. If $M = 256$ and $N = 256$ then the storage required for an image would be $256 * 256 * 8 = 524288$ bits. The goal of image compression is to take advantage of hidden structure in the image to reduce these storage requirements.

In the discrete wavelet transform, an image can be analyzed by passing it through an analysis filter bank followed by a decimation operation. This analysis filter bank consisting of a low pass filter and a high pass filter at each decomposition stage, is commonly used in image compression. When a signal passes through these filters, it splits into two bands. The low pass filter, which corresponds to an averaging operation, extracts the coarse information. The high pass filter, which corresponds to a differencing operation, extracts the detail information.

A two dimensional transform can be done by performing two separate one-dimensional transforms.First, the image is filtered along the x-dimension using low pass filter and high pass filter and decimated by two. Low pass filtered coefficients are stored on the left part of the matrix and high pass filtered are stored on the right part. Because of decimation the total size of the image is same as the original image. Then it followed by filtering the sub-image along the y-direction and decimated by two. We have split the image into four bands denoted by LL, HL, LH, HH after one level decomposition.

Images contain a large amount of information that requires large transmission bandwidths, much storage space and long transmission times. Therefore it is crucial to compress the image by storing only the essential information needed to reconstruct the image. An image can be thought of as a matrix of pixel values. In order to compress the image, redundancies must be exploited, for example, areas where there is little or no change between pixel values. Therefore large redundancies occur in the images having large areas of uniform color, and conversely images that have frequent and large change in color will be less redundant and harder to compress.

In general, there are three essential stages in a wavelet transform image compression system:
(1) The Transform step
(2) The Quantization step
(3) The Coding step

(1)The Transform step: In this step, the image data is acted upon by some invertible transform whose purpose is to decorrelate the data as much as possible. This means to remove the redundancy or hidden structure in the image. Such a transform usually amounts to computing the coefficients of the image in some orthonormal or non-orthogonal basis.

(2) The Quantization step: The coefficients calculated in the transform step will in general be real numbers, or at least high precision floating point numbers even if the original data consists of integer values. As such the number of bits required to store each coefficients can be quite high. Quantization is the process of replacing these real numbers with approximations that require fewer bits to store.

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(3) The Coding step: Most of the coefficients calculated in the transform step will be close to zero, and in the quantization step will actually be set to zero. Hence the step (1) and (2) will be a sequence of bits containing long stretches of zeros It is known that bit sequences with that kind of structure can be very efficiently compressed. The idea behind coding is to exploit redundancy in order to reduce the number of bits required to store bit sequence.

Huffman Coding : Huffman coding is statistical compression technique developed by David Huffman. It uses the probability of occurrence of symbols to determine the codeword representing the symbols. The length of the codeword is variable which means that individual symbols which make a message are represented with bit sequences that have distinct length[10]. This helps to reduce the redundancy in data. Symbols with higher probability of occurrence have shorter codeword length while symbols with lower probability of occurrence will have longer codeword lengths. As a result, the average codeword length per symbol reduces and this leads to a smaller output data size. In order to use Huffman encoding to encode the quantize data, we have to obtain the frequencies of occurrence of the symbols from the input.

4.Results

Results have been obtained by calculating few parameters obtained by comparing the original image and uncompressed image.

(i)Mean Square Error: It represents the mean square error between the original and compressed image.It is defined as:

$$MSE = \frac{1}{mn}\sum_{i=0}^{m}\sum_{j=0}^{n}|X(i,j) - X_c(i,j)|^2$$

The lower the value of MSE ,the lower the error.

(ii) Peak Signal To Noise Ratio: It is most commonly used as a measure of quality of reconstruction of lossy compression. It represents the measure of peak error and is expressed in decibels. It is defined as :

$$PSNR = 10\log_{10}\left(\frac{255^2}{MSE}\right)$$

(iii)Bit Per Pixel: It gives the number of bits required to store one pixel of the image. Thus for the purpose of compression BPP should be less to reduce storage on the memory.

(iv)Compression Ratio: It is defined as the ratio of the size of the original image over the size of the compressed image.

$$C_R = \frac{n_1}{n_2}$$

where C_R is the compression ratio,n_1 and n_2 is the number of information carrying units in the original and encoded images respectively.CR is expressed in percentage.

The results over the images have been obtained using Biorthogonal4.4, Haar,Symlets8 and db4 wavelets.Results are analyzed in tabular form.

The first test image is Brain.The best results is obtained with Biorthogonal4.4 wavelet.The MSE is as low as 34.69 and PSNR is 32.73dB at the BPP 1.837. The compression ratio at this point 7.6%. The results of various parameters of the Brain images are recorded in Table-I over different wavelets.(See figure (1) and (2))

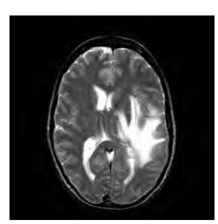

Fig. 1: Original Image

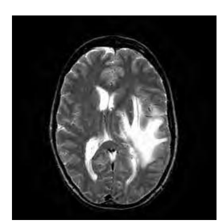

Fig. 2: Compressed Image

Table 1: Values of Brain image Parameters

image	wavelet used	BPP	MSE	PSNR(dB)	CR(%)
Brain	Bio 4.4	1.0355	37.83	32.25	4.31
		1.8378	34.69	32.73	7.66
	Haar	1.0327	77.21	29.25	4.30
		1.8339	43.76	31.72	7.64
	sym 8	1.0347	40.17	32.09	4.31
		1.837	36.75	32.48	7.66
	db 4	1.0273	44.67	31.64	4.28
		1.8467	39.00	32.22	7.69

In the second test image that of Laure, the best results are obtained with Biorthogonal4.4 at BPP 1.0306 with the resulting MSE 26.76dB. The compression ratio and PSNR are 4.29% and 33.86 respectively. The result for laure iamge has been enlisted in Table-II.(See figure (3) and (4))

Table 2: Values of Laure image Parameters

image	wavelet used	BPP	MSE	PSNR(dB)	CR(%)
Laure	Bio 4.4	1.0306	26.76	33.86	4.29
		1.8361	43.62	31.73	7.65
	Haar	1.0392	43.16	31.78	4.33
		1.8447	44.18	31.68	7.69
	sym 8	1.024	28.02	33.66	4.27
		1.894	43.55	31.74	7.66
	db 4	1.0348	29.9	33.37	4.31
		1.8413	47.31	31.38	7.67

Fig. 3: Original Image

Fig. 4: Compressed Image

In the third image that of baby, the best results are obtained with Biorthogonal4.4 at BPP 1.0303 with resulting MSE 30.44. The PSNR and compression ratio are 33.3 dB and 4.29 %respectively. The various results for baby image are tabulated in Table- III.(See figure (5) and (6))

Table 3: Values of Baby image Parameters

image	wavelet used	BPP	MSE	PSNR(dB)	CR(%)
Baby	Bio 4.4	1.0303	30.44	33.3	4.29
		1.8369	33.72	32.85	7.65
	Haar	1.0289	54.95	30.73	4.29
		1.8417	37.87	32.35	7.65
	sym 8	1.0331	34.01	32.81	4.30
		1.837	38.56	32.27	7.64
	db 4	1.0314	34.75	32.72	4.30
		1.8369	38.27	32.3	7.65

Fig. 5: Original Image

Fig. 6: Compressed Image

4.Conclusion: We have used different wavelets for compression of different images. Peak signal to Noise Ratio is a qualitative measure based on Mean Square Error of the reconstructed image. Typical Value of PSNR range between 20 dB to 40 dB. The actual value is not meaningful but the comparison between the two values for different reconstructed image gives a measure of image quality. Compression of images using Biorthogonal4.4 can maintain the quality of image, produces high compression performance and minimize the amount of the data so that it can be transmitted effectively.

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