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Monte Carlo simulation in the case of a single risk factor and evaluation of a European option

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Abstract

This article presents the Monte Carlo method in the context of stochastic simulation models. It is used to calculate a numerical value using random processes. Indeed, it is to isolate a number of variables and their effect a probability distribution. Our research aims to make practical use of the main operative techniques[1] of Monte Carlo simulation applied to finance. In this article, we describe how to develop Monte Carlo simulations in the presence of a single risk factor Y.

Keywords: Monte Carlo, Finance, Risk Factor, Simulation, Stochastic.

1 Introduction

This article aims to introduce probabilistic techniques to understand the most current financial models. Indeed, in recent years, financial experts describe various phenomena and develop computational methods through mathematical tools that are becoming increasingly sophisticated. In fact, the intervention of probability in financial modeling dates back to the early 20th century when Bachelier[2] introduced the use of finance " Brownian motion" to achieve his " theory of speculation"İn addition, Black-Scholes[3] and Merton[4] have used the so-called theory in terms of pricing and hedging options. Therefore, the options markets are experiencing a development using the methods of Black and Merton who have advanced in terms of generality, clarity and mathematical rigor. Thus, our work focuses on the problem of options considered as the most salient example of the methods of stochastic calculus in finance for relevance. Black and Scholes were the first to propose a model leading to an explicit formula for the price of a European call on an action that does not provide dividends and to a management strategy in the context of the model which allows the seller of the option to hedge perfectly. The price of the call is, in the Black-Scholes model, the amount of money that must initially have to be able to follow the hedging strategy and produce exactly wealth to maturity. In addition, the resulting formula depends only on one parameter which is not directly observable in the market and it's called " volatility" by particiens. In the context of stochastic simulation models, our work focuses on the Monte Carlo method[5]. Indeed, it is known that the simulation ensures that a system is studied and experimented. This system contains complex interactions that may undergo changes whose effects on the system in question are measured using the so-called simulation. Moreover, in a simulation, it is possible that element intervenes at random: it is a random simulation. That said, the faithful representation of the phenomena is fraught with difficulties whose cause is not explicit calculations. Thus, simulation techniques permit approaching numerically these calculations. In this sense, the Monte Carlo methods[6] are intended for the use of repeated experiments to assess the quantity and solve a deterministic system. These methods[7] are used to calculate the integral to solve partial differential equations, linear systems and optimization problems .

2 Problematic

we consider a wallet or security whose value as t is denoted by $V(t,\overrightarrow{Y}(t))$ because it depends on the time and m factors $\overrightarrow{Y}(t) = (Y_1(t), Y_2(t), ..., Y_m(t))$. It may be, for example, an option whose value depends on two random factors, the price of the underlying $S(t)$, and the interest rate $r(t)$ (in this case $m = 2$, $Y_1(t) = S(t)$ and $Y_2(t) = r(t)$). We seek to understand, at least empirically, the probability distribution of $V(t,\overrightarrow{Y}(t))$ and some moments of this distribution. Depending on the situation, it is important to know $V(t,\overrightarrow{Y}(t))$ in the whole time interval $(0, T)$ (we are interested therefore in the trajectories of V) or simply the value $V(t,\overrightarrow{Y}(t))$ in T only horizon. Monte Carlo simulation is a probabilistic method when, unable to analytically determine the law (or the first moments) of the distribution of $V(t,\overrightarrow{Y}(t))$, we simulate an empirical distribution. At first, we generate a large sample from the law of $\overrightarrow{Y}_t$ and each element of the sample is associated to the corresponding value $V(t,\overrightarrow{Y}(t))$ to construct an empirical distribution. According to the problem addressed, the Monte Carlo simulation will involve the generation of a large number M of possible trajectories $\overrightarrow{Y}_t]_{t=t_1,.....,t_N}$ from which we calculate the corresponding trajectories $V(t,\overrightarrow{Y}(t))]_{t=t_1,....,t_N}$ or we will simply calculate M value of $(\overrightarrow{Y}_t, V(t,\overrightarrow{Y}(t)))$ in a single date T. We note that $\overrightarrow{Y}$ denotes a particle realization of the random variable $\overrightarrow{Y}(t)$. Indeed, this difference in notation allows to distinguish a toss $\overrightarrow{Y}_t$ of a random variable of the variable $\overrightarrow{Y}(t)$ itself.

3 Monte Carlo simulation in the case of a single risk factor

Let us deal with the case of a single risk factor Y. We distinguish two situations. In the first, the nature of the problem requires the simulation of different trajectories of $V(t,\overrightarrow{Y}(t))$ between 0 and T

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(dynamic simulation). In the second, the simulation of different values $V(t, \overrightarrow{Y}(t))$ for one date T is sufficient to treat the problem (static simulation).

1. Dynamic simulations of trajectories of $\overrightarrow{Y}(t)$ and $V(t, \overrightarrow{Y}(t))$ in the interval $[0, T]$

a. Theory

It starts with the simulation of $\overrightarrow{Y}(t)$. This simulation is based on the stochastic equation that is supposed to govern the evolution of $\overrightarrow{Y}(t)$ in time. This evolution is written in discrete time or continuous time. In the case of continuous time, it is such a diffusion process:

$$dY = \mu(t, Y(t))dt + \sigma(t, Y(t))dw \quad (1)$$

Where dw is the increment of a standard Brownian motion and $\mu()$ and $\sigma()$ are two known functions representing the drift and the coefficient of the process respectively. For the discretization of (1) we decompose the interval $[0, T]$ in N periods of the same duration equal to $\Delta t = T/N$ (N is large). We define $t_i \equiv i\Delta t$. So the discretization of (1) will be written as following:

$$Y(t_j) - Y(t_{j-1}) = \mu(t_{j-1}, Y(t_{j-1}))\Delta t$$
$$+ \sigma(t_{j-1}, Y(t_{j-1}, Y(t_{j-1}))\sqrt{\Delta t}U_j \quad (2)$$

Where u_j a particular embodiment of $U(j)$ (for $j = 1, ...N$). The $U(j)$ are normal variables, centered, reduced and independently distributed. A trajectory $Y(t_j)$ is calculated from series of N independent tosses $U_j]_{j=1,....,N}$ from a standardized Gaussian law and the equation (2).

b. Example 1

Consider an action s with price S and with a position composed of the action as well as derivative products on s such as options. We will be in the universe of Black-Scholes (BS) in which the value $V(t, S(t))$ of this position is affected by time and one random factor $Y(t) \equiv S(t)$, the function $V(t, S)$ is presumed known (option prices are for example given by the formula BS). In accordance with BS we suppose that $S(t)$ follows the geometric Brownian motion: $(3.a)\frac{dS}{S} = \mu dt + \sigma dw \iff$
$(3.b)S(t) = S(0)\exp((\mu - 0,5\sigma^2)t + \sigma w(t))$

Where $w(t)$ is a standard Brownian μ and σ are known constants annualized, and time is measured in years. We will develop simulations with:

- Step time is weekly.
- The parameters of rate of return and weekly volatility are: $\frac{\mu}{52}$ and $\frac{\sigma}{\sqrt{52}}$
- $S(0) = 100$
- Rate of return and volatility are annualized $\mu =$ 12% and $\sigma = 36\%$
- Rate of return and weekly volatility are: $\frac{\mu}{52} =$ 0.0023 and $\frac{\sigma}{\sqrt{52}} = 0.0505$

S (n) $= 100\exp(0.00105n + 0.05(U_1 + U_2 + ... + U_n))$ (4)

$U_1 + U_2 + ... + U_n$ follows the normal centered law and variance equal to n. A trajectory $S_n]_{n=1,....,N}$ is obtained by using (4) by a sequence of N toss poster $U_n]_{n=1,....,N}$. The Table (figure 1) below contains the first 15 trajectories obtained during a simulation of 2000 trajectories of 10 weeks.

	1	2	3	4	5	6	7	8	9	10
1	=RACINE(-2*	1.16935614	0.00797447	-0.63868025	0.65221792	0.43946116	0.38354978	1.62939474	0.3163534	-0.37783237
2	0.96109376	-1.16935614	-0.00797447	0.63868025	-0.65221792	-0.43946116	-0.38354978	-1.62939474	-0.3163534	0.37783237
3	0.64151199	0.1067405	-0.43627948	1.66902319	-0.21465999	-0.48156016	0.5609588	-0.40087607	1.02956766	-0.74497298
4	-0.64151199	-0.1067405	0.43627948	-1.66902319	0.21465999	0.48156016	-0.5609588	0.40087607	-1.02956766	0.74497298
5	-0.42605819	0.89538095	-0.80084841	0.34743103	-1.0287803	0.05235845	0.12824501	-1.73190036	0.69201536	0.10296358
6	0.42605819	-0.89538095	0.80084841	-0.34743103	1.0287803	-0.05235845	-0.12824501	1.73190036	-0.69201536	-0.10296358
7	-0.14720724	-0.15556938	-0.48391493	-0.04098442	-0.48036327	0.92396722	-0.14253661	-0.00234461	0.38786724	-1.01373476
8	0.14720724	0.15556938	0.48391493	0.04098442	0.48036327	-0.92396722	0.14253661	0.00234461	-0.38786724	1.01373476
9	-0.19312503	0.84761601	-0.36272841	-0.66479126	-0.01748296	-1.11794087	0.28352076	-0.29749568	0.09086957	0.81133458
10	0.19312503	-0.84761601	0.36272841	0.66479126	0.01748296	1.11794087	-0.28352076	0.29749568	-0.09086957	-0.81133458
11	0.80236664	-0.91460273	-1.01086526	0.78828341	0.37895631	-1.05391096	-0.56242354	0.30289239	1.90016833	0.81686092
12	-0.80236664	0.91460273	1.01086526	-0.78828341	-0.37895631	1.05391096	0.56242354	-0.30289239	-1.90016833	-0.81686092
13	-0.38246331	-0.4901476	0.80541456	-0.23098981	-0.46105031	-1.12636736	-1.26638553	0.23116796	-0.88102271	-0.22144835
14	0.38246331	0.4901476	-0.80541456	0.23098981	0.46105031	1.12636736	1.26638553	-0.23116796	0.88102271	0.22144835
15	-1.14131307	0.23490942	0.01583536	-0.33446486	-0.24174356	-1.50853691	-0.36263468	0.7426962	-0.44496306	-0.43669339

Figure 1: $S_n^i]_{n=1,....,N}$ for $i = 1$ to 2000 (2000 trajectories of 10 weeks)

2. Static simulation trajectories of $\overrightarrow{Y}(t)$ and of $V(t, \overrightarrow{Y}(t))$ at the time T

In the above we showed how we simulate i trajectories $(Y_{t_j}, V(t, Y_{t_j}))_{j=1,...,N}$ In many cases, especially when it comes to evaluating European options[8] maturity T or enjoying a VaR to horizon T, only the knowledge of the empirical distribution of $V(T, Y(T))$ is useful.

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a. Example 2

Use the previous example in which the only risk factor $Y(t)$ is the current $S(t)$ which follows the geometric Brownian parameters (annualized) $\mu = 12\%$ and $sigma = 36\%$. Unlike the case of this example, we are interested here solely in terminal values $S(T)$ and $V(T, S(T))$ $S(T) = 100\exp((\mu - 0.5\sigma^2)t + \sigma\sqrt{T}U)$
U: Standard Gaussian variable
Generate variables U_i and these antitheticals $-U_i$

=RACINE(-2*LOG(ALEA()))*COS(2*PI()*ALEA())

	Ui	- Ui
1	=RACINE(-2*	-1.03894723
2	0.40905412	-0.40905412
3	-0.93175159	0.93175159
4	-0.83880652	0.83880652
5	-1.07224152	1.07224152
6	0.10919138	-0.10919138
7	-0.66456251	0.66456251
8	0.68726781	-0.68726781
9	1.11444778	-1.11444778
10	-0.98974561	0.98974561
11	-0.446135	0.446135
12	-1.19616081	1.19616081
13	0.6428049	-0.6428049
14	-1.21116956	1.21116956
15	0.33851053	-0.33851053

Figure 2: $U_i \rightsquigarrow N(0, 1)$ and $-U_i$

Table (figure 3) below shows the first 15 tosses obtained from 2000 tosses of U, $U_i]_{i=1,....,2000}$ to which are coupled antitheticals 4000 to obtain simulations of $S(T)$.

			Call europ		Put europ	
	Si(T)	Si'(T)	Vi(T,Si(T))	Vi'(T,Si'(T))	Vi(T,Si(T))	Vi'(T,Si'(T))
1	112,13501	91,091704	10,1350057	0	0	10,908296
2	111,56346	91,558376	9,56345508	0	0	10,441624
3	108,71088	93,960869	6,71087981	0	0	8,0391306
4	97,098169	105,19837	0	3,198366987	4,9018313	0
5	115,49659	88,440435	13,4965915	0	0	13,559565
6	117,55568	86,891324	15,5556806	0	0	15,108676
7	92,619432	110,28538	0	8,285375427	9,3805685	0
8	97,973312	104,25869	0	2,258686398	4,0266883	0
9	102,88706	99,279433	0,88705833	0	0	2,7205673
10	92,823751	110,04262	0	8,042619774	9,1762486	0
11	115,1606	88,698464	13,1606045	0	0	13,301536
12	103,47585	98,714518	1,47585162	0	0	3,2854824
13	124,8521	81,813349	22,8521046	0	0	20,186651
14	102,00083	100,14201	0,00083379	0	0	1,8579874
15	110,39017	92,531505	8,39017204	0	0	9,4684948

Figure 3: The values of $S_i(T)$ and $V(t, S_i(T))$

Example 2 is applied in the calculation of VaR and the Expected Shortfall (ES). Indeed, the calculation of the VaR of the portfolio whose value at time T is $V(T, S(T))$ is made from simulated values of $V(T, S(T))$ and loss $V(0, S(0)) - .V(T, S(T))$. In this example, the VaR $(10days, 5\%)$ is assessed using the 200^e the worst result among the 4000 simulated values. In addition, the arithmetic average of the 200 highest losses gives the ES $(10days, 5\%)$

b. Example 3: Evaluation of a European option

Evaluate the price $O(0)$ a European option of maturity T whose payoff is $V(T, S(T))$. Monte Carlo simulations are used to evaluate this option simply by updating safely to rate the empirical average of payoff resulting from simulations developed using a risk-neutral dynamics. The procedure is as follows: We simulate $M(M > 1000)$ values $S_i]_{i=1,...,M}$ of $S(T)$, deduced from a risk-neutral dynamics, and from M Gaussian tosses u_i for example using the equation: $S_i = 100\exp((\mu - 0,5\sigma^2)t + \sigma\sqrt{T}U)$. Remember that in the risk-neutral universe the expectation of growth rate of price $S(t)$ is equal to the interest rate r (different from μ)
*Algorithm
- We calculate M the value $V(T; S_i)]_{i=1,...,M}$ corresponding to payoff
- We calculate the arithmetic average of these M payoffs and we update the result over a period T of rate r to obtain the value $O(0)$ of the option in current date 0 : $O(0) = \exp(-rT)\frac{1}{M}\sum_{i=1}^{M} V(T, S_i))$

*Statement

Retake the data of Examples 1 and 2 and suppose the r continuous rate equal to 4% and constant. This is to assess the premium O of a European option maturity $T = 10$ weeks, written an a action price $S(t)$ and volatility $\sigma = 36\%$ the payoff of the option is $V(T, S(T))$. Simulations of $S(T)$ are operating here from the formula:

S(T)= 100 $\exp((\mu - 0,5\sigma^2)T + \sigma\sqrt{T}U)$
$= 100exp(-0,004769 + 0,158U)$

Table (figure 4) below shows the first 21 tosses obtained from 2000 tosses of U, $U_i]_{i=1,....,2000}$ to which are coupled antitheticals to obtain 4000 simulations of $S(T)$.

Table (figure 5) below contains the premium European option "Call and Put" and Parity check. The option value is estimated at

O(0)=$\exp(-0,00769)\frac{1}{4000}\sum_{i=1}^{400} V(T, S_i)$

			Call europ		Put europ	
	Si(T)	Si'(T)	Vi(T,Si(T))	Vi'(T,Si'(T))	Vi(T,Si(T))	Vi'(T,Si'(T))
1	104,14643	95,107172	3,14643195	0	0	5,8928279
2	91,895894	107,7858	0	6,785802414	9,1041064	0
3	88,562923	111,8422	0	10,84220491	12,437077	0
4	74,140111	133,59938	0	32,59937761	26,859889	0
5	114,04907	86,849222	13,0490653	0	0	14,150778
6	98,075491	100,99437	0	0	2,9245092	0,0056277
7	89,666356	110,46588	0	9,465876973	11,333644	0
8	109,63749	90,343849	8,63748815	0	0	10,656151
9	107,95268	91,753841	6,95267548	0	0	9,2461589
10	104,80602	94,508619	3,80602406	0	0	6,4913807
11	99,260219	99,788946	0	0	1,7397811	1,2110539
12	104,44128	94,838674	3,44128177	0	0	6,1613264
13	91,419628	108,34733	0	7,34732984	9,5803716	0
14	79,381464	124,77816	0	23,77815551	21,618536	0
15	77,905949	127,14142	0	26,14141534	23,094051	0
16	86,116899	115,01892	0	14,0189193	14,883101	0
17	92,421103	107,17328	0	6,173278661	8,5788969	0
18	82,964671	119,38904	0	18,38904283	18,035329	0
19	93,73029	105,67632	0	4,676324941	7,2697095	0
20	91,528179	108,21883	0	7,218832009	9,471821	0
21	109,60614	90,369689	8,60613807	0	0	10,630311

Figure 4: The values of $S_i(T)$ et $V(T, S_i(T))$

	Call europ Oc(0)	Put europ Op(0)
	3,861009404	4,753944609
Oc(0) - Op(0)	-0,892935205	
Parité de call-put	-0,22608828	
= S(0) - Kexp(-r*T)		
erreur	0,666846925	

Figure 5: The values of Call et Put

Example 3 is applicable in the study of the evolution of variables GREEK of a European option. Indeed, suppose that the payoff $V(T, S(T))$ is that one of an option to evaluate. It is possible, once estimated its value $O_1(0)$, to calculate the Greek variables. To do this, a slightly different value is used for the support if we want to get the delta, or for a variable, such as volatility or interest rates, if we want to estimate a different sensitivity (such as Vega or rho). A second simulation is then performed with this new value, by maintaining the same number of periods N and of M trajectories and by using the same sequences U_i (and antitheticals $-U_i$) than the first simulation. Therefore, a second value $O_2(0)$ is obtained. Greek variable is calculated by the formula:

$$\frac{O_2(0) - O_1(0)}{\Delta S}$$

Where *Delta* S is the variation retained of the price of the underlying or of the concerned parameter (equal to 1 in our encrypted example).

Table (figure 6) below contains the evolution of GREEK variables of a European option.

	Call europ O1(0)	Put europ O1(0)
	3,861009404	4,753944609
	Call europ O2(0)	**Put europ O2(0)**
	5,061387695	5,89611705
Evolution des variable grecs d'une option europ		
	Call	**Put**
delta (ds)	1,200378291	1,142172441
(d_sigma)	120,0378291	114,2172441
(dr)	600,1891455	571,0862203
(dT)	12,00378291	11,42172441
(dK)	1,200378291	1,142172441

Figure 6: Evolution of GREEK variables of a European option

4 Conclusion

Our work aims to apply the Monte Carlo method in the field of finance. In fact, Monte Carlo simulations are often very greedy in calculation time. Indeed, in most applications, a compromise imposes itself between two antinomian objectives: precision and richness of empirical informations obtained (the maximum desired), which increases with the number of simulations operated, the computation time (the minimum desired). Effective procedure leads to a sufficient precision obtained at the cost of a limited computation time. Thus, we have developed so far Monte Carlo simulations in the presence of a single risk factor, a study we will strengthen by the development of Monte Carlo simulations in the presence of several risk factors.

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