New Algorithm For Edge Detection in Medical Images Based on Minimum Cross Entropy Thresholding

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Abstract

Edge detection in medical image is an important task for object recognition of the human organs, and it is an essential pre-processing step in medical image segmentation and 3D reconstruction. Successful results of image analysis extremely depend on edge detection. Up to now many edge detection methods have been developed. But, they are sensitive to noise. This paper proposes a new edge detection algorithm based minimum cross entropy thresholding for medical images corrupted with noise. The proposed method is tested under noisy conditions on several medical images and also compared with conventional edge detectors such as Sobel, LoG and Canny edge detectors. Experimental results reveal that the proposed algorithm exhibits better performance and may efficiently be used for the detection of edges in medical images corrupted by Salt-and-Pepper noise.

Keywords: Minimum Cross Entropy; Edge Detection; Threshold Value; Medical Images.

1. Introduction

The Edges in a digital image provide important information about the objects contained within the image since they constitute boundaries between objects in the image. Images are prone to artifacts and noise. Salt & pepper noise is a form of noise typically seen on images. It is typically manifested as randomly occurring white and black pixels. Salt & pepper noise creeps into images in situations where quick transients, such as faulty switching. Otherwise, White noise is additive in nature where the each pixel in the image is modified via the addition of a value drawn from a Gaussian distribution. To examine the generality of the results, the suggested edge detection algorithm was tested on medical images containing these type of noise. By medical image processing, researchers and clinicians can easily analyses the image data, thereby enhancing their ability to study, monitor, diagnose and treat medical disorders. Medical image processing is currently finding wide application in a large variety laboratory research and clinical practice. Biologists study cells and generate 3-D confocal microscopy data sets; virologists generate 3-D reconstructions of viruses from micrographs; radiologists identify and quantify tumors from MRI and CT scans; and neuroscientists detect regional metabolic brain activity from functional MRI scans and PET. Analysis of these diverse image types requires sophisticated computerized quantification and visualization tools.

Edge Detection techniques are classified as follows: the primary order by-product of selection in image process is that the gradient. The second order derivatives of selection in image process are typically computed exploitation Laplacian. For Sobel, a Prewitt & Roberts technique performs finding edges by thresholding the gradient for the LOG[1,2]. By default edge perform mechanically computes the edge to use. For Sobel & Prewitt strategies, we are able to opt to discover horizontal edges, vertical edges or each. Laplacian of a Gaussian (LOG) [3,4] finds edges by searching for zero crossing once filtering with a Gaussian filter. Zero crossing finds edges by searching for Zero crossing once filtering with a user-specified filter[4]. Clever finds by searching for native maxima of the gradient. The gradient is calculated exploitation the by-product of a Gaussian filter[5]. The strategy used 2 thresholds to discover sturdy & weak edges, and includes the weak edges within the output provided that they're connected to sturdy edges. Therefore; this technique is a lot of doubtless to discover true weak edges. Sobel edge detector technique is somewhat tough than Prewitt edge detector. Prewitt edge detector technique is slightly easier to implement computationally than the Sobel detector. However it tends to supply somewhat noisier results[6]. Parliamentarian edge detector is one amongst the oldest and simplest edge detectors in digital image process. It’s still used oftentimes in hardware implementations wherever simplicity and speed are dominant factors. This detector is employed significantly but the others. Attributable to partly to its restricted practicality. Log smooths the image (thus
where: 

\[ \text{cross entropy} = -\sum_{i=1}^{L} p_i \log q_i \]

The cross entropy between two distributions on the same set. The cross entropy between probability of each central pixel of image under the window is calculated as

\[ H(CP) = \sum_{i=1}^{p-1} h(i) \log(h(i)) - \sum_{i=1}^{p-1} i h(i) \log(\mu(1,i)) - \sum_{i=1}^{p-1} i h(i) \log(\mu(t,L+1)) \]

The Minimum Cross Entropy thresholding algorithm determines the optimal threshold by minimizing the cross entropy based on equation (4),

\[ t^* = \arg \min \{D(t)\} \]

3. Edge Detection Algorithm

A spatial filter mask may be defined as a matrix \( w \) of size \( m \times n \). So, we will use the usual masks for detecting the edges[1,12]. The process of spatial filtering consists simply of moving a filter mask \( w \) of order \( m \times n \) from point to point in an image. At each point \( (x,y) \), the response of the filter at that point is calculated a predefined relationship. Assume that \( m = 2a + 1 \) and \( n = 2b + 1 \), where \( a, b \) are nonnegative integers. For this purpose, smallest meaningful size of the mask is \( 3 \times 3 \), as shown in Fig. 1.

![Fig. 1: Mask coefficients showing coordinate arrangement](image)

![Fig. 2 Image region under the mask](image)

![Fig. 3 Window coefficients](image)

Move the window on the whole binary image and find the probability of each central pixel of image under the window. Then, the entropy [13-16] of each Central Pixel of image under the window is calculated as

\[ H(CP) = -p_c \ln(p_c) \]

Table 1 \( p \) and \( H(CP) \) of central pixel under window

<table>
<thead>
<tr>
<th>( P )</th>
<th>( 1/9 )</th>
<th>( 2/9 )</th>
<th>( 3/9 )</th>
<th>( 4/9 )</th>
<th>( 5/9 )</th>
<th>( 6/9 )</th>
<th>( 7/9 )</th>
<th>( 8/9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H )</td>
<td>0.244</td>
<td>0.334</td>
<td>0.366</td>
<td>0.360</td>
<td>0.326</td>
<td>0.270</td>
<td>0.195</td>
<td>0.104</td>
</tr>
</tbody>
</table>

where, \( p_c \) is the probability of Central Pixel CP of binary image under the window. When the probability of central pixel \( p_c = 1 \) then the entropy of this pixel is zero. Thus, if the gray level of all pixels under the window homogeneous, then \( p_c = 1 \) and \( H = 0 \). In this case, the central pixel is not an edge pixel. Other possibilities of entropy of central pixel under window are shown in Table 1.

The cross entropy thresholding is lost from gradient operators, where the image is convolved with the first-order derivatives of Gaussian filter for smoothing in the local gradient direction followed by edge detection by thresholding.

In this paper we proposed a new algorithm to detect edges of noisy medical images based on information theory, which is Minimum Cross Entropy based thresholding. The proposed approach is decrease the computation time as possible as can and the results were very good compared with the other methods. A brief introduction of minimum cross entropy thresholding is given in Section 2. Illustration of the proposed algorithm applied to gray scale images is presented in Section 3. In Section 4, some particular images will be analyzed using proposed method based algorithm and moreover, a comparison with some classical methods will be provided for these images. Conclusions is included in Section 5.

2. Cross Entropy Thresholding

The cross entropy was first developed for measuring information context [9-11]. Let \( F = \{ f_1, f_2, \ldots, f_N \} \) and \( G = \{ g_1, g_2, \ldots, g_N \} \) be two probability distributions on the same set. The cross entropy between \( F \) and \( G \) is defined by:

\[ D(F,G) = \sum_{i=1}^{N} f_i \log \frac{f_i}{g_i} \tag{1} \]

The Minimum Cross Entropy thresholding algorithm selects several thresholds by minimizing the cross entropy between the original image and the resulting image[9]. Let \( I \) be the original image and \( h(i), i=1,2,\ldots,L \) be the corresponding histogram with \( L \) being the number of gray levels. Then the resulting image, denoted by \( I_t \) using \( t \) as the threshold value is constructed by:

\[ I_t(x,y) = \begin{cases} \mu(1,t) & \text{if} \ (x,y) < t \\ \mu(t,L+1) & \text{if} \ (x,y) \geq t \end{cases} \tag{2} \]

where:

\[ \mu(a,b) = \frac{\sum_{i=a}^{b} h(i)}{\sum_{i=1}^{L} h(i)} \]

The cross entropy is then calculated by:

\[ D(t) = \sum_{i=1}^{p-1} i h(i) \log(i) - \sum_{i=1}^{p-1} h(i) \log(\mu(1,t)) - \sum_{i=1}^{p-1} i h(i) \log(\mu(t,L+1)) \tag{3} \]

The Minimum Cross Entropy thresholding algorithm determines the optimal threshold by minimizing the cross entropy based on equation (4),

\[ t^* = \arg \min \{D(t)\} \tag{4} \]
In cases $p_c = 8/9$, and $p_c = 7/9$, the diversity for gray level of pixels under the window is low. So, in these cases, central pixel is not an edge pixel. In remaining cases, $p_c \leq 6/9$, the diversity for gray level of pixels under the window is high. So, for these cases, central pixel is an edge pixel. Thus, the central pixel with entropy greater than and equal to 0.244 is an edge pixel, otherwise not.

The following Algorithm summarizes the proposed technique for calculating the optimal threshold values and the edge detector.

**Algorithm: Edge Detection**

1. **Input**: Image $I$ of size $M \times N$ and $t^\text{opt}$, that has been calculated by Minimum Cross Entropy.
2. Create a binary image; For all $x, y$,
   - if $f(x,y) \leq t^\text{opt}$ then $f(x,y) = 0$ else $f(x,y) = 1$.
3. Create a mask $w$ of order $m \times n$, in our case ($m = 3, n = 3$)
4. Create an $M \times N$ output image $g$: For all $x$ and $y$, Set $g(x,y) = f(x,y)$.
5. Checking for edge pixels:
   - Calculate: $a = (m-1)/2$ and $b = (n-1)/2$.
   - For all $y \in \{b+1, \ldots, N-b\}$, and $x \in \{a+1, \ldots, M-a\}$, $\sum = 0$;
   - For all $l \in \{-b, \ldots, b\}$, and $j \in \{-a, \ldots, a\}$,
     - if $(f(x,y) = f(x+j,y+l))$ then $\sum = \sum + 1$.
     - if $(\sum > 6)$ then $g(x,y) = 0$ else $g(x,y) = 1$.
6. **Output**: The edge detection image $g$ of $I$.

The stages of our proposed technique are as follows:

**Stage 1**: Find optimal threshold value ($t$) using Minimum Cross Entropy.

**Stage 2**: Applying Edge Detection Procedure with threshold value.

### 4. Experimental Results

In this section, we demonstrate the efficiency of the proposed scheme, the algorithm is tested over different noisy medical images and compared with traditional operators. The images detected by Canny, LOG, Sobel, and the proposed algorithm. The experiments results were implemented on Intel® Core™2 Duo 2.20GHz with 3 GB RAM using MATLAB R2012b without pre-processing. We run the Canny, LOG, Sobel and the proposed mixed algorithm 10 times for the test images and calculate the average of run time for the classical methods and proposed scheme Fig. 4 and 5. It has been observed that the proposed algorithm works effectively compare to the run time of other methods as shown in Fig. 4 and it produces good result compare with other algorithm and shows more resilience to noise as shown in Fig. 6.

### 5. Conclusion

An efficient algorithm based minimum cross entropy thresholding for detection of edges in noisy medical images is presented in this paper. The proposed method is compared with traditional edge detectors. On the basis of visual perception and edge counts of edge maps of various g images it is proved that our algorithm is able to detect highest edge pixels in images. The proposed method is decrease the computation time with generate high quality of edge detection. Also it gives smooth and thin edges without distorting the shape of images. Another benefit comes from easy implementation of this method.
Fig. 6: Performance of Proposed Edge Detector For Various Noisy Medical Images
References


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