Stochastic Model Predictive Control for optimization costs in multi-level supply chain

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Abstract

The Model Predictive Control is a methodology very used in systems with slow dynamics like chemical process plant and supply chain. The MPC is usually used in supply chain management, under constraints like buffer limits and shipping capacities limits, based on approximations which make the future values of disturbance predicted, thus no recourse is available in the future. However, most real life applications are not only subject to constraints but also involve stochastic uncertainty. In this paper we suggest to use a Stochastic Model Predictive Control to optimize the combined costs of storage and shipping in a multi-level supply chain, to take into account the stochastic demand.

Keywords: *Model Predictive Control, Stochastic demand, supply chain.*

1. Introduction

With the technology offered nowadays, and all the facilities to access to the information, we cannot deny the shift that have been appeared in the behavior of customers; they have become more discerning and demanding but less loyal than before. Under these circumstances managers should find the balance between minimizing their costs and fulfilling the customer satisfaction. That is why the companies are more interested than ever in hunting waste and optimizing their costs. The work of Perea-Lopez [7] showed that profit could increases of up to 15 per cent by proposing a model predictive control strategy to optimize supply chain..

In this paper we consider a multi-product and multi-stage supply chain, to optimize the combined cost of storage and shipping under a stochastic demand. In the first section we describe the problem to be optimized, then in the second part we present the Stochastic Model Predictive Control that will be used to model the problem. In the third part we show how the problem is solved by the dynamic programming. Finally we present in the last section the numerical results for the comparison between the Stochastic Model Predictive Control and some of the most used models in optimization.

2. Problem description

In general the structure of a supply chain consists on different levels of interveners as showed in Fig. 1. The first level is retailers, who take the customer demands and try to satisfy it according to their level stock, and make orders to the next level of the chain. The second level is the distribution centers (DC). The centers are the responsible for the distribution of the products to the retailers to fulfill their needs. They also make orders to the third level, which is the Plant Warehouses. The stock capacity of the warehouses is bigger than the distribution centers. Finally we have the plants as the fourth level. They procure the row materials from their suppliers.

We consider a multi-product supply chain, and we assume that the products are independents from each other. Each node in the supply chain has to manage its storage level and at the same time satisfy the demand of its corresponding nodes from the former level.

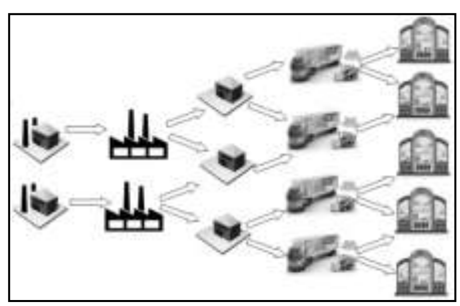

Figure 1: Structure of supply chain

The problem consists on optimizing the combined costs of the storage and the transportation of the products across all the nodes of the supply chain. In fact, to minimize the storage cost, managers tend to make more shifts which increases the cost of transportation. And in the other hand, to have less costs in shipping, they store big amounts of the products to apprehend the fluctuation of the demand. Then the problem could be stated as follows: minimizing the costs of the storage and the shipping in a multi-level and multi-product supply chain, under a stochastic demand over a finite time horizon.

3. Stochastic model predictive control

3.1. Description of the Model Predictive Control

Like many technical inventions, the idea of the Model Predictive Control (MPC) appears to have been proposed long before the model came to the forefront (Propoi, 1963; Rafal and Stevens, 1968; Nour-Eldin, 1971). The MPC was first implemented in industry under various names (dynamic matrix control, rolling horizon planning, and dynamic linear programming), long before a thorough understanding of its theoretical properties was available. Academic interest in the MPC started growing in the mid eighties of the twenty century, particularly after two workshops organized by Shell (Prett and Morari, 1987; Prett et al., 1990).

In the model proposed by Bellman (1975) to deal with multi-stage decision process, the future state of the system is totally defined by its history.

$$\boldsymbol{x_{t+1} = Ax_t + Bu_t} \quad (1)$$

The term x_t is the state of the system at time t, while u_t refers to the control action. In the mathematical theory, the objective is to minimize the cost function over the entire time horizon:

$$\min_{u_t} C_T(x_T) + \sum_{t=0}^{T-1} C_T(x_t\ , u_t\ ) \quad (2)$$

The algorithm of the Model Predictive Control is described below :

At each time τ:
1. Take the process measurements.
2. Solve the planning problem of Eq. (2) subject to the constraints, with the variables $x_{\tau+1}$, .. , x_T, $u_{\tau+1}$, .. , u_{T-1}
3. We interpret the solution as plan of action from τ+1 to T-1
4. Implement the control policy for the next step τ+1.

The Stochastic Model Predictive Control, is a closed-loop control, it takes into account the disturbance or the noise of the system in its expression of the state function. :

$$\boldsymbol{x_{t+1} = f_t(x_t, u_t, d_t)} \quad (3)$$

Then the objective function of the stochastic problem will be expressed like in the Eq. (4).

$$\boldsymbol{\min_{u_t} \mathbb{E}_d[C_T(x_T) + \sum_{t=0}^{T-1} C_T(x_t\ , u_t, d_t)]} \quad (4)$$

3.2. State function

In the case of the supply chain, the state function is expressed like below:

$$\boldsymbol{x_{t+1} = x_t + Bu_t + d_t \qquad t = 0, .., T-1} \quad (5)$$

The term $x_t \in \mathbb{R}_{N,P}$ is the state matrix of the system at the time t. It represents the amount of each one of the P products remaining in the stock of every node among the N nodes of the supply chain.

In the other hand, the term $u_t \in \mathbb{R}_{M,P}$ is the matrix of control at the time t, it represents the amount of the P products transported by the M links existing between the nodes of the supply chain.

The matrix $d_t \in \mathbb{R}_{N,P}$ expresses the noise applied to the system. In the case of the supply chain, this disturbance is caused by the stochastic demand for the P products in each one of the N nodes at the time t.

The final term to define in the state function is the matrix B. Note that this matrix is independent of time; it expresses the incoming and outgoing node incidence. The dimension of the matrix is $\mathbb{R}_{N,M}$; it is defined as showed in the equations Eq. (6) and Eq. (7).

$$B = B^{in} - B^{out} \quad ; t = 0, .., T-1 \quad (6)$$

$$B_{i,k}^{in(out)} = \begin{cases} 1 & , if\ the\ link\ S_k enters\ (exits)\ the\ node\ n_i \\ 0 & , otherwise \end{cases} \quad (7)$$

3.3. Cost functions

We consider the costs of warehousing and shipping as quadratic functions.

- At the each level L of the supply chain, the cost of warehousing is expressed like the equation Eq. (8). With the term $x_{t;L}$ is a bloc of the rows of the matrix x_t corresponding to the nodes of the level L of the supply chain.

$$W_l(x_{t;l}) = x_{t;l}^{\mathrm{T}} Q_{t;l}\, x_{t;l} \quad (8)$$

- At the each level L of the supply chain, the cost of shipping is expressed like the equation Eq. (9). With the term $u_{t;L}$ is a bloc of the rows of the matrix u_t corresponding to the nodes of the level L of the supply chain.

$$S_l(u_{t:l}) = u_{t:l}^{\mathrm{T}} R_{t:l}\, u_{t:l} \qquad (9)$$

- Then the storage cost function and the transport cost function will be the sum of the costs in each level of the supply chain. In general we express the costs as quadratic functions.

$$W(x_t) = x_t^{\mathrm{T}} Q_t\, x_t \qquad (10)$$

$$S(u_t) = u_t^{\mathrm{T}} R_t\, u_t \qquad (11)$$

- The cost function in a time t , for all the nodes in the multi-level supply chain is expressed in the Equation (10).

$$C_t(x_t, u_t) = W(x_t) + S(u_t); \quad t = 0,..,T-1 \quad (12)$$

- But at the end of the time horizon, we will not apply any control, so the cost function will be equal to the storage cost only.

$$C_T(x_T) = W(x_T) \qquad (13)$$

4. Solution

4.1. The objective function

The stochastic problem is to minimize the objective function as expressed in the Equation (14)

$$J = \mathbb{E}[C_T(x_T) + \sum_{t=0}^{T-1} C(x_t, u_t)] \qquad (14)$$

And the state function is expressed as follows:

$$x_{t+1} = x_t + B^{in} u_t - B^{out} u_t + d_t \;\; ; t = 0,..,T-1 \quad (15)$$

4.2. The control policies

Let be X_t the matrix of the matrixes, it is the representation of the history of the system until the time t.

$$X_t = (x_0, x_1, .., x_t) \qquad (16)$$

Then the control policy at time t, is the function which according to the history X_t it gives the control u_t to apply to minimize the objective function, under the constraints expressed in Equations (18), (19) and (20).

$$u_t = \Phi_t(X_t) = \boldsymbol{\psi_t(x_0, d_0, .., d_{t-1})} \qquad \boldsymbol{t = 0, .., T-1} \quad (17)$$

- Buffer limits: For each node n_i, we cannot store more than the maximum capacity from the product pj.

$$0 \le x_{i,j} \le x_{\max j} \qquad (18)$$

- Shipment capacities: For each link j, we cannot transport more than a maximum amount of the product j.

$$0 \le u_{k,j} \le u_{\max j} \qquad (19)$$

- And as we cannot transport more than we have in the stock, we have the constraint in the Equation (18)

$$B^{out}\, u_t \le x_t \qquad (20)$$

4.3. Dynamic programming

Using the dynamic programming we solve the problem of the stochastic model predictive control. We found using Bellman recursion that the control policy is a linear state feedback.

$$\Phi_t^*(x_t) = K_t x_t \qquad (21)$$

5. Numerical example

We consider a multi-level supply chain with N = 6 nodes and M = 5 links dealing with P = 3 products (p1, p2, p3).

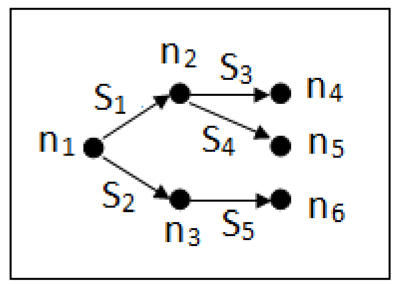

Figure 2: Example of a model of a supply chain

The links S_i, i=1,..,5 show the possibility and the direction of the products' shipping between the N nodes.

In that case the state matrix $x_t \in \mathbb{R}_{6,3}$ will be like showed in the equation Eq. (22). Then, the term $x_{i,j}$ is the amount of the product p_j stored at the node i at the instant t. Then, the column j describes the amount of the product p_j stocked in every node of the supply chain at time t. In the same way, we define a row i in the matrix x_t as the representation of the amount of all the products in the stock at the node i at the time t.

$$\boldsymbol{x_t} = \begin{pmatrix} \boldsymbol{x_{1,1}} & \boldsymbol{x_{1,2}} & \boldsymbol{x_{1,3}} \\ \boldsymbol{x_{2,1}} & \boldsymbol{x_{2,2}} & \boldsymbol{x_{2,3}} \\ \boldsymbol{x_{3,1}} & \boldsymbol{x_{3,2}} & \boldsymbol{x_{3,3}} \\ \boldsymbol{x_{4,1}} & \boldsymbol{x_{4,2}} & \boldsymbol{x_{4,3}} \\ \boldsymbol{x_{5,1}} & \boldsymbol{x_{5,2}} & \boldsymbol{x_{5,3}} \\ \boldsymbol{x_{6,1}} & \boldsymbol{x_{6,2}} & \boldsymbol{x_{6,3}} \end{pmatrix} \qquad (22)$$

The matrix of control $u_t \in \mathbb{R}_{5,3}$ is like showed in the equation Eq. (23). The term $u_{k,j}$ is the amount of the product p_j shipped by the link S_k at the instant t. Then, the column j describes the amount of the product p_j transported by every link in the supply chain at time *t*. Then a row *k* in the matrix u_t is the representation of the amount of all the products shipped by the link k at the time *t*.

$$\boldsymbol{u_t} = \begin{pmatrix} \boldsymbol{u_{1,1}} & \boldsymbol{u_{1,2}} & \boldsymbol{u_{1,3}} \\ \boldsymbol{u_{2,1}} & \boldsymbol{u_{2,2}} & \boldsymbol{u_{2,3}} \\ \boldsymbol{u_{3,1}} & \boldsymbol{u_{3,2}} & \boldsymbol{u_{3,3}} \\ \boldsymbol{u_{4,1}} & \boldsymbol{u_{4,2}} & \boldsymbol{u_{4,3}} \\ \boldsymbol{u_{5,1}} & \boldsymbol{u_{5,2}} & \boldsymbol{u_{5,3}} \end{pmatrix} \quad (23)$$

The matrix of the stochastic demand $d_t \in \mathbb{R}_{6,3}$ is like the state matrix x_t . The term $d_{i,j}$ is the amount of the product p_j demanded at the node *i* at the instant *t*. Then, the column *j* describes the amount of the product p_j demanded in every node of the supply chain at time *t*. In the same way, we define a row *i* in the matrix d_t as the representation of the amount of all the products demanded at the node *i* at the time *t*. Note that for the numerical simulation we suppose that the demand has a lognormal distribution, i.e., where $\log(d) \sim N(\mu,\sum)$.

Finally the dimension of the incidence matrix of the example is $\mathbb{R}_{6,5}$; like it is represented in the equation bellow:

$$B = \begin{pmatrix} -1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (24)$$

We will compare the stochastic model predictive control in the case of affine controller to other methods.

We first consider the naïve greedy controller where the demand at time t is supposed equal to its mean.

$$\bar{d}(t) = E(d(t)) \quad (25)$$

The objective is to choose the control *u(t)* to minimize the costs at time *t+1*.

The second method is the greedy controller. In this method we consider the demand is equal to its expected value.

$$\hat{d}(t) = E\big(d(t)\big|d(0),\dots,d(t-1)\big) \quad t = 1,\dots,T-1$$
$$\hat{d}(t) = E\big(d(0)\big) = \mu_0 \quad (26)$$

The result of the simulation shows that the Stochastic Model Predictive Control performs better than the naïve and the greedy control and the Model Predictive Control as showed in the table Table (1).

Table 1: Simulation Results

Control law	Cost mean	Standard deviation
Naïve greedy	2038.571	1507.3619
Greedy control	2091.5627	1503.1351
MPC	1440.532	885.5708
SMPC	1440.0691	885.6372

6. Conclusions

The model predictive control is widely used to optimize the costs in the supply chain. But taking into account the stochastic character of the demand, we presented in this paper the application of the Stochastic Model Predictive control to minimize the combined costs of shipping and storage multi-products in a multi-level supply chain. At the end of this paper, using Matlab, we compared the results of the SMPC with the naïve greedy control and the Greedy control, and finally the Model Predictive Control.

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