

# Image Restoration using Multilayer Neural Networks with Minimization of Total Variation Approach

Mohammed Debakla<sup>1</sup>, Khalifa Djemal<sup>2</sup> and Mohamed Benyettou<sup>1</sup>

<sup>1</sup>MOSIM Laboratory, University of USTO Oran Algeria

<sup>2</sup>IBISC Laboratory, University of Evry Val d'Essonne, France

## Abstract

Noise reduction is a very important task in image processing. In this aim, many approaches and methods have been developed and proposed in the literature. In this paper, we present a new restoration method for noisy images by minimizing the Total Variation (TV) under constraints using a multilayer neural network (MLP). Indeed, the obtained Euler-Lagrange functional is resolved by minimizing an error functional. The MLP parameters (weights) in this case are adjusted to minimize appropriate functional and provides optimal solution. The proposed method can restore degraded images and preserves the discontinuities. The effectiveness of our approach has been tested on synthetic and real images, and compared with known restoration methods

**Keywords:** Image restoration, Partial Differential Equations, Total variation, Multilayer neural network.

## 1. Introduction

Image restoration is an important operation in many processes of image analysis. Indeed, restored image can improve the accuracy of downstream processing operations such as segmentation. Degraded image restoration problems are largely treated in literature in many applications [1]-[5]. Image restoration was one of the first problems to attract attention. It seeks to correct the distortions that result in many degradations such reduction of contrast, blur and random or due to noise occurring during image formation.

Observed image modeling considering noise and blur is generally assumed to minimize established criteria in order to find the best original image. The main difficulty is that the restoration methods should preserve contours of objects and discontinuities contained in the degraded images. This needs the introduction of variational methods [6] or stochastic [2], which can smooth the homogeneous regions and preserve discontinuities of image. Methods based on partial differential equations (PDE) [8]-[10] and

anisotropic filtering techniques [11]-[13], well established, meet these requirements and have been particularly studied in recent years. Include also some nonlinear methods that compute a weighted average of intensity values in a local neighborhood [14]-[16], [9]

In recent years, a new emerging technique has grown considerably. It helps to process information more efficiently than conventional systems, to remove data previously unavailable to assist in decision making, it is the neural network. They have the specificity of learning by themselves to extract information hidden in a mass of data, and provide powerful models as well as to the knowledge of a given problem. In other words, they represent a class of powerful algorithms that are used for classification, prediction and aggregation of data. These methods have been successfully introduced in image processing and computer vision. Their applications are numerous, such as edge detection [17]-[18], [8], segmentation [18]-[19], stereovision [20], restoration [21]-[24].

In this aim, the function mapping to be minimized into energy of a given network is the commonly adopted strategy. Restoration of a high quality image from a degraded recording is a good application area of neural nets. Zhou et al. [24] are the first who proposed the use of the Hopfield neural network (HNN) in image restoration and showed the instability of the original (HNN) when applied to image restoration. They proposed an algorithm to ensure the stability of the HNN. They also proposed the use of simulated annealing algorithm that allows energy increase with a probability decreasing in time so as to converge to a better solution in stochastic sense. These two algorithms are time-consuming because energy change has to be checked step by step.

In [25], Paik et al. proposed a Modified Hopfield neural network (MHNN) model for solving the restoration problem which improves upon the algorithm proposed by Zhou et al. in [24]. The algorithms based on the MHNN ensure network stability without checking energy change step by step, and two new updating schemes (one sequential and the other parallel) are introduced. However,

the convergence proof for the parallel scheme is based on an almost never satisfied condition. Sun Yi, presents in [26], [22] a Generalized Updating Rule (GUR) of the MHNN for gray image recovery. The stability properties of the GUR are given. It is shown that the neural threshold set up in this GUR is necessary and sufficient for energy decrease with probability one at each update. In [23] the authors present two novel image restoration algorithms based on a modified Hopfield neural network and variational partial differential equations (PDE). The first algorithm is based on harmonic model and the other is based on total variation (TV) model [3]. Both algorithms can restore the degraded images and preserve the edges.

A novel neural network based multiscale restoration approach was proposed in [27] and improved in [28]. The method uses a Multilayer Perceptron (MLP) algorithm, trained with synthetic an 8-bit gray level image of artificially degraded co-centered circles, with 256 x 256 pixels. Recently in [29], authors provide an effective algorithm with Cellular Neural network and Contour matching that can be used to the inpainting digital images or video frames with very high noise ratio.

In this paper, we present a new image restoration method based on TV minimization proposed by Rudin et al. in [3]. In this aim, the obtained PDE of the TV model minimization under constraints is considered as an error function which is resolved by multilayer neural network. This approach rely on the function approximation capabilities of feedforward neural networks and results in the construction of a solution written in a differentiable, closed analytic form. In this approach, the multilayer neural network is considered as the basis of an approximation, whose parameters (weights) are adjusted to minimize an appropriate error function. To train the network we use optimization techniques, which require the calculation of the gradient of the error to set the network settings with respect to its parameters.

The paper is organized as follows: section 2 describes the proposed restoration method. In Section 3 we illustrate the method by presenting some examples of synthetic and real image restoration. The results are compared with know restoration methods such as Tichonov regularization [1], minimizing the total variation of ROF [3] methods and Multiscale Neural Network [28]. Finally, section 4 brings the conclusion for the work.

## 2. Proposed image restoration method

We present firstly in this section, the formulation of the image restoration problem by minimizing the TV model under constraints. The problem is characterized by a PDE functional, typically measuring some reconstruction error, and the solution is defined as the minimization of the

considered functional. Secondly, we propose to minimize this error function using MLP neural network approach and finally, a general algorithm is detailed.

### 2.1 Restoration Problem Statement

TV regularization has been extremely successful in a wide variety of restoration problems, and remains one of the most active areas of research in mathematical image processing and computer vision. By now, their scope encompasses not only the fundamental problem of image denoising, but also other restoration tasks such as deblurring, blind deconvolution, and inpainting [3], [30]-[33].

In all these approaches, a TV model is minimized in different ways. The typical problem in image restoration case were introduced by Rudin et al. in the pioneering work [3] on edge pre-serving image denoising with the minimization of the following functional :

$$F(u) = \int_{\Omega} |Du| + \lambda \|u_0 - u\|^2 dx dy \quad (1)$$

where  $\int_{\Omega} |Du|$  represents the TV model of the image  $u$ . If image  $u$  is regular, the equation (1) becomes only  $\int_{\Omega} |\nabla u| dx$ . In [3] the authors considered that the noise witch corrupted image is distinguished from noiseless one by the size of total variation, which is defined as  $\int_{\Omega} \sqrt{u_x^2 + u_y^2} dx dy$ , where denotes the image

domain  $u_x$  and  $u_y$  denote the corresponding partial differentiation. Consequently, they propose to restore a noisy and blurred image by minimizing total variation:

$$\min_u \int_{\Omega} \sqrt{u_x^2 + u_y^2} dx dy \quad (2)$$

Under constraints:

$$\begin{cases} \int_{\Omega} \frac{1}{2} (u(x, y) - u_0(x, y))^2 dx dy = \sigma^2 \\ \int_{\Omega} (u(x, y) - u_0(x, y)) dx dy = 0 \end{cases} \quad (3)$$

Where  $u_0(x, y)$  represents the given observed image, which is considered to be corrupted by a Gaussian noise of variance  $\sigma^2$  and  $u(x, y)$  denotes the desired clean image.

To minimize (2) Rudin et al. [3] have applied the Euler-Lagrange equation under the two constraints (3) they obtain the following equation:

$$\frac{\partial}{\partial x} \left( \frac{u_x}{\sqrt{u_x^2 + u_y^2}} \right) + \frac{\partial}{\partial y} \left( \frac{u_y}{\sqrt{u_x^2 + u_y^2}} \right) - \lambda (u - u_0) = 0 \quad (4)$$

Where the Lagrange multiplier is given by:

$$\lambda = \frac{1}{2\sigma^2} \int \left[ \sqrt{u_x^2 + u_y^2} - \left( \frac{(u_0)_x u_x}{\sqrt{u_x^2 + u_y^2}} \right) + \left( \frac{(u_0)_y u_y}{\sqrt{u_x^2 + u_y^2}} \right) \right] \quad (5)$$

Over the years, the TV model [3], has been extended to many other image restoration tasks, and has been modified in a variety of ways to improve its performance. The classical approach is then to use the associated Euler-Lagrange equation to compute the solution. Fixed step gradient descent [3], or later quasi-Newton methods [8], [34]-[36], have been proposed [37]-[39]. Iterative methods have proved successful [40]-[42]. Ideas from duality can be found in [43],[45]-[46]. In [45], [47], a graph cuts based algorithms could also be used. A combination of the primal and dual problems has been introduced in [48]. Recently, it has been shown that Neural Network for classification of noise followed by classification of filter is explored in [49]. Notice that all these works use an approximate numerical method for solving the associated PDE with the restoration problem.

An image restoration problem can be transformed to an optimization problem. In our assumption and from (4) we can formulate the image restoration problem as minimizing the following error function:

$$E(x, y) = \frac{\partial}{\partial x} \left( \frac{u_x}{\sqrt{u_x^2 + u_y^2}} \right) + \frac{\partial}{\partial y} \left( \frac{u_y}{\sqrt{u_x^2 + u_y^2}} \right) - \lambda(u - u_0) \quad (6)$$

where  $u_x$  and  $u_y$  are discretizations of the horizontal and vertical derivatives. A difficulty with TV is that it has a derivative singularity when  $u$  is locally constant. To avoid this, some algorithms regularize TV by introducing a small parameter  $\varepsilon > 0$  within the square root.  $\sqrt{u_x^2 + u_y^2 + \varepsilon}$

We propose to minimize the error function E in equation (6) using an MLP neural network approach. We present a generalization of the problem and then we introduce a weighting technique of weight for a term based on the data fidelity.

## 2.2 Error Function Minimization Based MLP Approach

The general idea of our method is to minimize an error function, taking in to account the TV approach as illustrated in equation (6). In this aim, we use MLP which shown by Lagaris et al. [50], where the authors were able to solve partial differential equations by neural networks. To minimize equation (6), a neural network (NN) with three layers is used, according to the MLP architecture

illustrated in Figure1. The input layer of the MLP consists of one neuron which is the pixel to be restored. The output layer contains one neuron correspond to the processed pixel. Next research has been done in evaluating the number of neurons in the hidden layer but no optimal number has been discovered. So the hidden layer consists of a varying number of neurons fixed by the user after several tests, in our experimentation case we have used ten neurons. The sigmoid function  $\delta$  is applied to each neuron in the hidden layer and output neuron.

In all previous works using neural networks in the field of restoration, the chosen network type is related to the assumptions given by the authors in the restoration process.

In our case we take the assumption that the output of the MLP is an image corresponding to the desired image  $u(x, y)$ . So we provided as inputs to the multilayer neural network the degraded image  $u_0(x, y)$  and at the output of the network, we have:

$$u(x, y) = N(u_0(x, y), w) \quad (7)$$

where  $u_0(x, y)$  is the noisy image represented by each intensity of pixel  $(x, y)$  and  $w$  is the weights vector of the MLP.

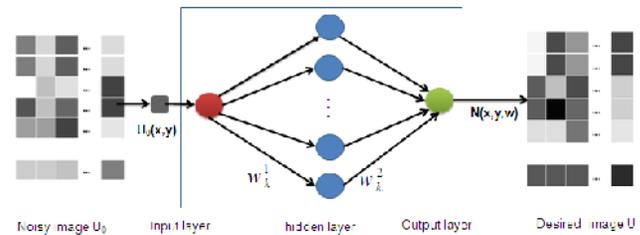


Figure 1. Multilayer neural network structure of our approach

We consider that all values of the sigmoid function are taken between 0 and 1; we must normalize the input and output values of the network. Indeed, each pixel of noisy image is coded by a one byte gray-level, the input values will be divided by 255, and the output results will be multiplied by 255.

The value of each neuron in the hidden layer is given by:

$$h_k = \delta(u_0(x, y).w_k^1) \quad (8)$$

where  $w_k^1$  are the weights from the input layer to the hidden layer.

The gray level of the output of each pixel is obtained by:

$$N = 255 \delta \left( \sum_{k=1}^n h_k w_k^2 \right) \quad (9)$$

where  $w_k^2$  are the weights from the hidden layer to the output layer, and  $n$  is the number of the neurons in the hidden layer.

$$N(x, y) = 255 \delta \left( \sum_{k=1}^n w_k^2 \delta(w_k^1 u_0(x, y)) \right) \quad (10)$$

Once all pixels of the noisy image are presented to the network, we get an image  $u$  aimed that satisfies the equation (4). This image is replaced in equation (5) to give  $\lambda$ , and in equation (6) for calculating the error function  $E$  provided by the network. The MLP is trained by the backpropagation algorithm [51] until convergence is obtained. Convergence is obtained when the fixed iterations number is exceed or the minimum value of error  $E$  is reached which corresponds to the convergence error  $E_c$ . To make changes the weights, we use the steepest gradient descent method. The adaptation of weights is done by the following equation:

$$w_k^i(t+1) = w_k^i(t) - \eta \Delta w_k^i(t) \quad (11)$$

Where  $w_k^i(t+1)$  and  $w_k^i(t)$  represent respectively the new and the last values of weights, with  $i=1$ (weights between input layer and hidden layer) or  $2$  (weights between hidden layer and output layer),  $k=1, \dots, n$  and  $\eta$  a learning positive constant.

The weight variation  $\Delta w_k^i$  is obtained by minimization of error  $E(x, y)$  presented in equation (6) using the following equation:

$$\Delta w_k^i = \sum_{x, y} \frac{\partial E(x, y)}{\partial w_k^i} \quad (12)$$

To compute  $\Delta w_k^i$ , and for the simplicity of the calculation, we denote the two terms of equation (6) as follow:

$$\frac{u_x}{\sqrt{u_x^2 + u_y^2}} = f(x, y) \quad \text{and} \quad \frac{u_y}{\sqrt{u_x^2 + u_y^2}} = g(x, y) \quad (13)$$

Then we have:

$$\frac{\partial}{\partial x} \left( \frac{u_x}{\sqrt{u_x^2 + u_y^2}} \right) = f_x, \quad \frac{\partial}{\partial y} \left( \frac{u_y}{\sqrt{u_x^2 + u_y^2}} \right) = g_y \quad (14)$$

Equation (6) now became

$$E(x, y) = f_x^2 + g_y^2 - \lambda(u - u_0) \quad (15)$$

From these considerations, the weight variation term  $\Delta w_k^i$ ,  $i=1, 2$  and  $k=1, \dots, n$  can be formulated as follow:

$$\frac{\partial E(x, y)}{\partial w_k^i} = \frac{\partial f_x}{\partial w_k^i} + \frac{\partial g_y}{\partial w_k^i} - \frac{\partial \lambda}{\partial w_k^i} (u - u_0) - \lambda \frac{\partial u}{\partial w_k^i} \quad (16)$$

All terms of the equation (16) are calculated from the value of  $\frac{\partial u}{\partial w_k^i}$ , so it is necessary to provide firstly these derivatives. Under our assumption and consideration in (7) and as  $N$  is considered in (10), in these conditions the derivatives can be given as follow:

$$\frac{\partial u}{\partial w_k^i} = \frac{\partial N}{\partial w_k^i} \quad (17)$$

So, the derivative of the desired image with respect to the weights from the input layer to the hidden layer is given by:

$$\frac{\partial u}{\partial w_k^1} = 255 \left[ u_0(x, y) w_k^2 \delta'(w_k^1 u_0(x, y)) \right] \quad (18)$$

$$\delta' \left[ \sum_{k=1}^n w_k^2 \delta(w_k^1 u_0(x, y)) \right]$$

In the same way, deriving with respect to the weights from the hidden to the output layer is given by:

$$\frac{\partial u}{\partial w_k^2} = 255 \delta(w_k^1 u_0(x, y)) \delta' \left[ \sum_{k=1}^n w_k^2 \delta(w_k^1 u_0(x, y)) \right] \quad (19)$$

From the obtained derivations  $\frac{\partial u}{\partial w_k^1}$  and  $\frac{\partial u}{\partial w_k^2}$  we can easily calculate  $\Delta w_k^1$  and  $\Delta w_k^2$

### 3. General algorithm of the method

**Input:** noisy image  $u_0$  of size  $(n,m)$ , MLP neural net (see sub-section 2.2)  
**Output:** desired clean image  $u$   
**Begin**  
 - Random initialization of network weights  $w_k^1$  and  $w_k^2$ .  
 - Normalization of the noisy image.  
 $Itt \leftarrow 1$  ;  
**Repeat**  
**For** each pixel  $u_0(x,y)$  of  $u_0$  **do**  
 - Activation of each neuron in the hidden layer:  
 $h_k \leftarrow \delta ( u_0 ( x , y ) w_k^1 )$   
 - Calculate the neuron value in the output layer which is the weighted sum of output neurons values  
 in the hidden layer:  $N ( x , y ) \leftarrow \delta ( \sum_{k=1}^n w_k^2 h_k )$   
**End For**  
 - calculate desired clean image  $u \leftarrow N$   
 - calculate the error function  $E$   
 - calculate  $\Delta w_k^i$  according to the delta rule:  
 $\Delta w_k^i \leftarrow -\eta \frac{\partial E}{\partial w_k^i}$   
 - Updating the weights  $w_k^i \leftarrow w_k^i - \eta \Delta w_k^i$   
 $Itt \leftarrow Itt + 1$   
**Until**  $(Itt > maxitt \text{ or } E \leq E_c)$  /\* Where  $E_c$  is closer to zero, maxitt is the maximum number of iteration \*/  
**End.**

Algorithm 1: General algorithm of the proposed image Restoration method.

### 4. Experimental results

In this section, we present some experimental results that evaluate the performance of our approach. We also chose to compare the denoising performance of our approach with other methods using their optimal parameters: Tichonov regularization [1], minimizing TV model of ROF [3] and Multiscale Neural Network (MNN) [28] for synthetic and real noisy images. For the purpose of objectively testing the performance of image restoration algorithm, the improvement ISNR is often used. It represents the amount of noise removed from the degraded image. If  $\square$ SNR increases, then the result of restoration is best. This metric, using the restored image, is given by:

$$ISNR(f,u)=10\log_0 \frac{\sum_{i,j} [f(i,j)-u_0(i,j)]^2}{\sum_{i,j} [f(i,j)-u(i,j)]^2} \quad (20)$$

Where  $f(i, j)$ ,  $u_0(i, j)$  and  $u(i, j)$  denote the original, degraded and restored images, respectively.

We also used the Normalized Mean Square Error (NMSE) as another measure of quality. If the value of NMSE decreases, the restoration is better. NMSE is given by:

$$NMSE(f,u)=\frac{\sum_{i,j} [f(i,j)-u(i,j)]^2}{\sum_{i,j} [f(i,j)]^2} \quad (21)$$

The first denoising experiment is shown in Figure 2. For this experiment, using a synthetic image, we added white Gaussian noise with three different standard deviations ( $\sigma = 15, 20, 25$ ). The noisy synthetic images are in the first column of Figure 2, and the denoised images by the Tichonov (with:  $n=100$ ,  $\epsilon=0.01$  and  $\alpha=5$ ), TV model (with:  $n=400$ ,  $dt=0.001$ ,  $\alpha=250$  and  $\epsilon=0.1$ ), multiscale neural network (MNN) methods and our approach (hidden layer with 10 neurons) are shown in the second, third, fourth and fifth columns, respectively.

The corresponding ISNR and NMSE values are shown in Table 1.

Figure 3 and 4, respectively, illustrates the evolution of the Lagrange multiplier  $\lambda$  and the error function  $E$  according to the iterations number. The considered noise standard deviation is  $\sigma = 25$ .

We can see clearly that the Tikhonov method tends to blur the image and can't preserve details (Figure 2 second column). In Figure 2 at the third column, the restored image obtained by the ROF method, suppress some details and blur is still apparent. For the MNN method we can observe an increase in brightness and contrast for the restored image (Figure 2 fourth column). The proposed method gives a good visual quality with strong noise suppression for all variations and also more details are preserved (Figure 2 fifth column).

In Table 1, values of ISNR and NMSE shows that our approach gives good performance compared to the other methods. Indeed, our approach with noise standard deviation  $\sigma = 25$ , obtains the best ISNR 13,81 in comparison with other methods.

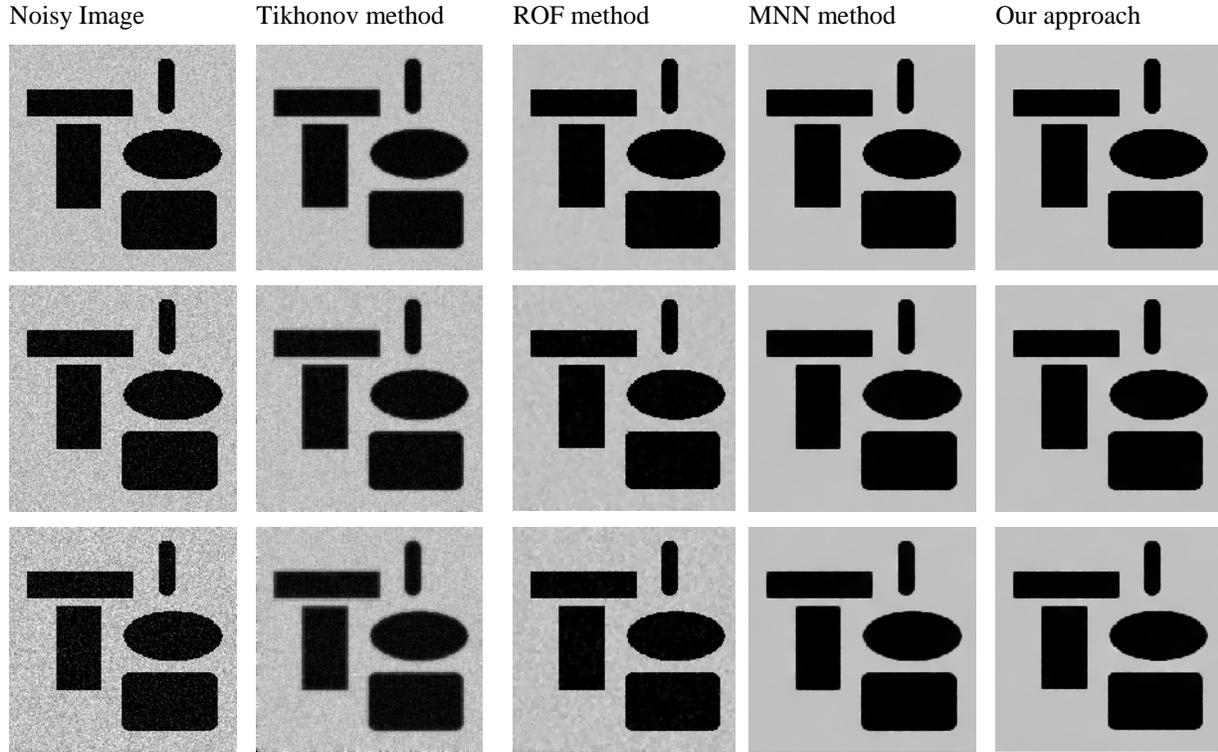


Figure 2 Examples of white Gaussian noise reduction: The columns from left to right show the noisy image and the restored images by Tichonov, the total variation, multiscale neural network and our approach. The rows from top to down are showing the experiments with different standard deviations ( $\sigma=15, 20, \text{ and } 25$ ).

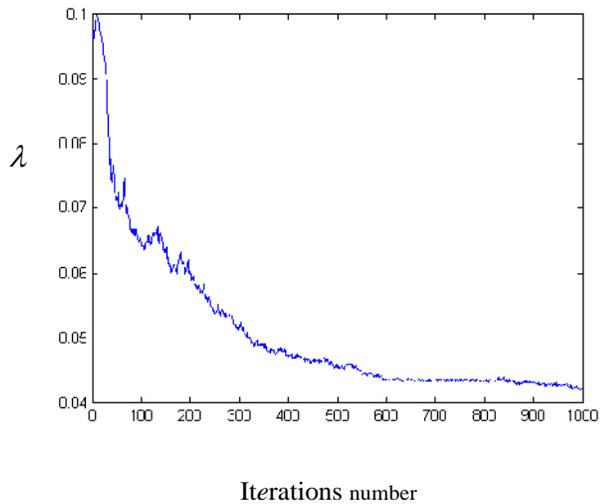


Figure 3 Evolution of  $\lambda$  according to the iterations number

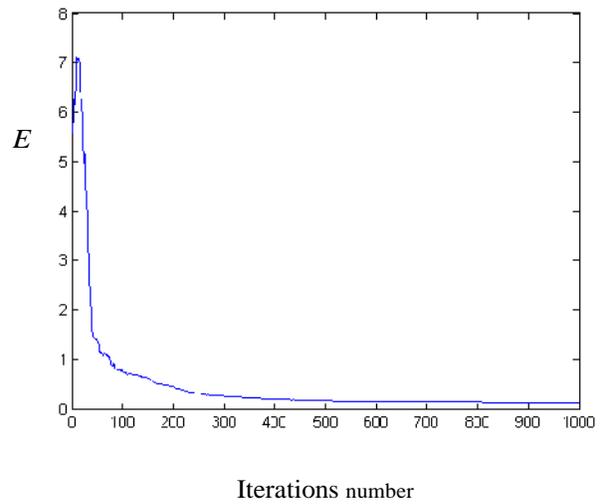


Figure 4 Evolution of  $E$  according to the iterations number

Table 1. The ISNR and NMSE values of white Gaussian noise reduction.

Image		$\sigma = 15$	$\sigma = 20$	$\sigma = 25$
Tikhonov algorithm	ISNR	0.86	2.03	2.21
	NMSE	0.0063	0.0085	0.0124
(ROF) approach	ISNR	12.33	10.91	9.38
	NMSE	0.004	0.0011	0.0024
Multiscale Neural Network	ISNR	11.85	12.25	12.57
	NMSE	0.0005	0.0008	0.0011
MLP with 400 iterations	ISNR	<b>14.06</b>	<b>13.37</b>	<b>13.81</b>
	NMSE	<b>0.0003</b>	<b>0.0008</b>	<b>0.0011</b>

We applied our approach on real known noisy images: House, Cameraman and Lena images. The comparisons with other methods are given in figure 5, 6 and 7 with the optimal parameters of each method.

The three images are disturbed by a Gaussian additive noise with zero mean and standard deviations  $\sigma = 15$ , restored images, obtained after 400 iterations of our approach are presented in Figure 5f, 6f and 7f.

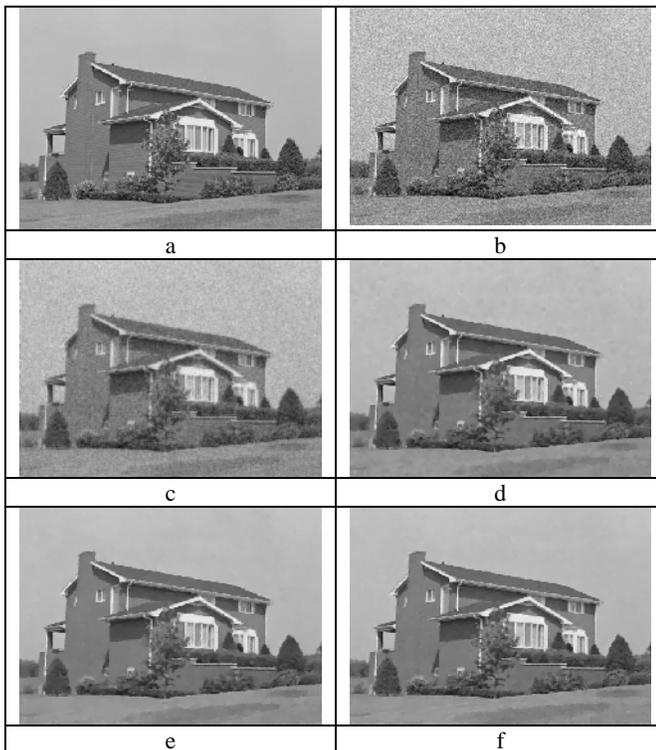


Figure 5 house garden image restoration : (a) original image, (b) Gaussian degraded image with  $\sigma = 15$ , restoration result using: (c) Tikhonov's algorithm, ISNR=2.52, (d) (ROF) Total Variation model, ISNR=5.35, (e) Multiscale neural network method, ISNR=5.79, (f) MLP approach with 400 iteration, ISNR=7.58

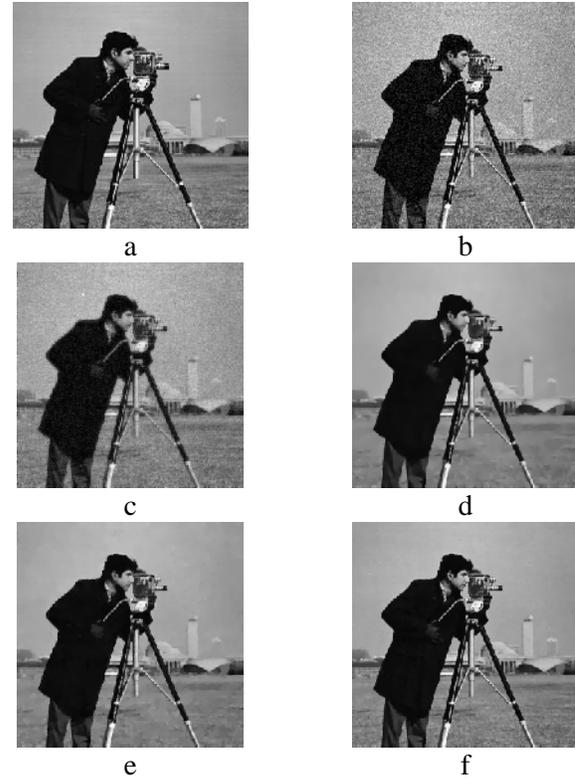


Figure 6 cameraman image restoration : (a) original image, (b) Gaussian degraded image with  $\sigma = 15$ , restoration result using: (c) Tikhonov's algorithm, ISNR=1.28, (d) (ROF) Total Variation model, ISNR=3.94, (e) Multiscale neural network method, ISNR= 4.59, (f) MLP approach with 400 iteration, ISNR=6.57

The results presented show the good performance of our algorithm, especially the preservation of discontinuities. Moreover the geometric characteristics such as corners and edges and originals contrast are well restored. For purposes of comparison, the results of restoration by the four methods chosen are summarized in Table2.

Table 2. The ISNR and the NMSE Values based on four methods

Image		House	Cameramen	Lena
Tikhonov algorithm	ISNR	2.52	1.28	1.26
	NMSE	0.0053	0.0088	0.0130
(ROF) approach	ISNR	5.35	3.94	5.12
	NMSE	0.0028	0.0047	0.0053
Multiscale Neural Network	ISNR	5.79	4.59	4.79
	NMSE	0.0025	0.0041	0.0058
MLP with 400 iterations	ISNR	<b>7.58</b>	<b>6.57</b>	<b>6.79</b>
	NMSE	<b>0.0016</b>	<b>0.0026</b>	<b>0.0036</b>



Figure 7 Lena image restoration : (a) original image, (b) Gaussian degraded image with  $\sigma = 15$ , restoration result using: (c) Tikhonov's algorithm, ISNR=1.26. (d) (ROF) Total Variation model, ISNR=5.12, (e) Multiscale neural network method, ISNR=4.79, (f) MLP approach with 400 iteration ISNR=6.79

## 5. Conclusions

We have presented in this paper a new image restoration method based on TV model minimization under constraints. Ended, we have shows that the obtained Euler-Lagrange function can be resolved by minimizing an error function using multi-layer neural network (MLP) approach. The developed algorithm improves noise reduction and preserves the original geometric characteristics and contrasts of the image well. The results comparison demonstrates the performance of our approach. In future work, we will investigate different neural network architectures such as recurrent or cellular neural networks, and consider different noise models.

## References

- [1] A.N Tikhonov, V.Y Arsenin, "Solutions of Ill-Posed Problems", Winston and Sons, Washington. 1977,
- [2] S. Geman, D. Geman, "Stochastic relaxation, gibbs distributions, and the bayesian restoration of images", IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 6, No. 6, 1984, pp. 721-741.
- [3] L. Rudin, S. Osher, E. Fatemi, "Nonlinear total variation based noise removal algorithms", Physica D 60, 1992, 259-268
- [4] M.R. Banham, A.K. Katsaggelos, "Digital image restoration", IEEE Signal Process. Mag. 14 (2),1997, pp 24-41.
- [5] A.K. Katsaggelos, J. Biemond, R.W. Schafer, R.M. Mersereau, "A regularized iterative image restoration algorithm", IEEE. Trans. Signal Processing, Vol. 39, 1999, p. 914-929.
- [6] D. Mumford, J. Shah, "Optimal approximations by piecewise smooth functions and variational problems", Communication on Pure and applied Mathematics, Vol. 42, No.5, 1989, pp. 577-685.
- [7] F. Catté, P.L.Lions, J.M. Morel, T. Coll, "Image selectivesmoothing and edge-detection by nonlinear diffusion", SIAM J. Numerical Analysis, Vol. 29, No.1, 1992, pp. 182-193.
- [8] P. Charbonnier, L. Blanc-Féraud, G. Aubert, M. Barlaud, "Deterministic edge-preserving regularization in computer imaging", IEEE Transactions on Image Processing, 6(2), 1997, pp 298-311.
- [9] T. Chan, S. Osher, J. Shen, "The digital TV filter and nonlinear denoising", IEEE Transactions on Image Processing, Vol. 10, No. 2, 2001, pp. 231-241.
- [10] F. Dibos, G. Koepfler, P. Monasse, "Total Variation Minimization for Scalar and Vector Image Regularization, Geometric Level Set Methods in Imaging", Vision and Graphics, Springer Verlag. 2002,
- [11] P. Perona, J. Malik, "Scale space and edge detection using anisotropic diffusion", IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 12, No. 7, 1990, pp. 629-239.
- [12] M.J. Black, G. Sapiro, D.Marimont, D. Heeger, "Robust anisotropic diffusion", IEEE Transactions on Image Processing, Vol. 7, No. 3, 1998, pp. 421-432.
- [13] M.J. Black, G. Sapiro, "Edges as outliers: Anisotropic smoothing using local image statistics In Scale-Space Theories" in Computer Vision, Kerkyra,Greece. 1999, pp. 259-270,
- [14] F. Godtliebsen, E. Spjotvoll, J.S. Marron, "A nonlinear gaussian filter applied to images with discontinuities", J. Nonparametric Statistics, Vol. 8, 1997, pp. 21-43.
- [15] C. Tomasi, R. Manduchi, "Bilateral filtering for gray and color images", In International Conference on Computer Vision, Bombay, India, 1998, pp. 839-846.
- [16] D. Barash, "A fundamental relationship between bilateral filtering, adaptive smoothing and the nonlinear diffusion equation", IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 24, No. 6, 2002, pp. 844- 847.
- [17] J.K Paik, "Image restoration and edge detection using neural networks", Ph.D.Dissertation, Dep.Elec.Eng., Computer Sci, North-Western Univ. 1990.
- [18] B Meftah. O.Lezoray A. Benyettou, "Segmentation and Edge Detection Based on Spiking Neuron Network", Neural Processing Letters, Vol. 32(2), 2010, pp 131-146.
- [19] B. Meftah, A. Benyettou, O. Lzoray, M. Debakla, " Image Segmentation with Spiking Neuron Network", Ist

- Mediterranean Conference on Intelligent Systems and Automation (CISA'08), Annaba, Algeria, AIP Conf. Proc. Volume 1019, 2008, pp. 15-19,
- [20] N. Nasrabadi, C. Choo, "Hopfield network for stereovision correspondence", *IEEE Trans. Neural Network*, Vol. 3, 1992, p. 5-13.
- [21] J. Paik, A. Kastagellos, "Edge detection using a neural network", *Proc of the Int.Conf.ASSP-ICASSP*, Albuquerque, NM, 1990, p. 2145-2148.
- [22] Y. Sun, "Hopfield neural network based algorithms for image restoration and reconstruction Part II: Algorithms and simulation", *IEEE Trans. Signal Processing*, vol. 48, 2000, pp. 2105–2118.
- [23] Y.D. Wu, Q.Z. Zhu, S.X. Sun, H.Y. Zhang, "Image restoration using variational PDE-based neural network", *Neurocomputing*, 69, No. 16-18, 2006, pp. 2364–2368.
- [24] Y.T. Zhou, A. Vaid, B.K. Jenkins, "Image restoration using a neural network", *IEEE Trans. Acoust. Speech Signal Processing*, vol.36, 1988, p. 1141-1151.
- [25] J K. Paik, A. Katsaggelos, "Image restoration using a modified Hopfield Network", *IEEE Transactions on image processing*, Vol 1, No.1, 1992, pp. 49-63.
- [26] Y. Sun, "A generalized updating rule for modified Hopfield neural network for quadratic optimization", *Neurocomput*, vol. 19, 1998, pp. 133–143.
- [27] A.P.A. Castro, J.D.S. Silva, "Neural Network-Based Multiscale Image Restoration Approach" In: *Proceeding on Electronic Imaging*, Vol. 6497, San Jose, 2007, pp. 3854-3859.
- [28] A.P.A. Castro, I.N. Drummond, J.D.S. Silva, "A Multiscale Neural Network Method for Image Restoration", *TEMA Tend. Mat. Apl. Comput*, 9, No. 1, 2008, pp. 41-50.
- [29] P. Elango, K. Murugesan, "Image Restoration Using Cellular Neural Network With Contour Tracking Ideas", *International Journal of Computer Theory and Engineering*, Vol. 2, No. 5. 2010.
- [30] S.Karthik et al, "Directional Total Variation Filtering Based Image Denoising Method", *IJCSI International Journal of Computer Science Issues*, Vol. 9, Issue 2, No 1, March 2012.
- [31] G. Aubert, P. Kornprobst, "Mathematical Problems in Image Processing", *Applied Mathematical Sciences*, vol. 147. Springer, New York, 2002.
- [32] T. Chan, J. Shen, "Image Processing and Analysis Variational, PDE", *Wavelet, and Stochastic Methods*, SIAM, Philadelphia. 2005,
- [33] K. Djemal, "Speckle Reduction in Ultrasound Images by Minimization of Total Variation", *IEEE International Conference on Image Processing*, Genova, Italya, Vol. 3, 2005, p.357-360.
- [34] A. Chambolle, P.L Lions, "Image recovery via total variation minimization and related problems". *Numer. Math.* 76(3), 1997, pp167–188.
- [35] M.K. Ng, L. Qi, Y.F. Yang, Y. Huang, "On semismooth Newton methods for total variation minimization", *J. Math. Imaging Vis.* 27, 2007, pp 265–276.
- [36] M. Nikolova, R. Chan, "The equivalence of half-quadratic minimization and the gradient linearization iteration", *IEEE Trans. Image Process*, 16(6), 2007, pp 1623–1627.
- [37] T.F. Chan, J. Shen, L.Vese, "Variational PDE models in image processing", *Notices Amer. Math. Soc.*, 50 1, 2003, pp 14-26.
- [38] T.F. Chan, S.Esedoglu, F.Park, A.Yip, "Total variation image restoration: overview and recent developments", *Handbook of mathematical models in computer vision*, 17-31, Springer, New York. 2006,
- [39] J. F. Aujol, "Some First-Order Algorithms for Total Variation Based Image Restoration", *Math Imaging Vis*, DOI 10.1007/s10851-009-0149-y, 2009.
- [40] I. Daubechies, M. Defrise, C. De Mol, "An iterative thresholding algorithm for linear inverse problems with a sparsity constraint", *Commun. Pure Appl. Math.* 57, 2004, 1413–1457.
- [41] J. Bect, L. Blanc-Fraud, G. Aubert, A. Chambolle, "A 11-unified variational framework for image restoration", In *ECCV 04. Lecture Notes in Computer Sciences*, vol. 3024, Springer, Berlin, 2004 pp. 1-13.
- [42] J. Bioucas-Dias, M.,Figueiredo, "Thresholding algorithms for image restoration", *IEEE Trans. Image Process*, 16(12), 2007, pp2980-2991.
- [43] T. Chan, G. Golub, P. Mulet, "A nonlinear primal-dual method for total variation-based image restoration", *SIAM J. Sci. Comput.* 20(6), 1999, pp 1964–1977,
- [44] A. Chambolle, "An algorithm for total variation minimization and applications", *J. Math. Imaging Vis.* 20, 2004, pp. 89–97.
- [45] A. Chambolle, "Total variation minimization and a class of binary MRF models", In: *EMMCVPR 05. Lecture Notes in Computer Sciences*, vol. 3757, Springer, Berlin, 2005, pp. 136–152.
- [46] P.L Combettes, V. Wajs, "Signal recovery by proximal forwardbackward splitting", *SIAM J. Multiscale Model. Simul.* 4(4), 2005, pp1168– 1200.
- [47] J. Darbon, M. Sigelle, "Image restoration with discrete constrained total variation part I: Fast and exact optimization". *J. Math. Imaging Vis.* 26(3), 2006, pp277–291.
- [48] M. Zhu, T.F. Chan, "An efficient primal-dual hybrid gradient algorithm for total variation image restoration", *UCLA CAM Report 08-34*. 2008,
- [49] T. Santhanam, S. Radhika, "Application of Neural Networks for Noise and Filter Classification to enhance the Image Quality", *IJCSI International Journal of Computer Science Issues*, Vol. 8, Issue 5, No 2, September 2011.
- [50] I Lagaris, L Aristidis, D. Fotiadis, "Artificial Neural Networks for Solving Ordinary and Partial Differential Equation", *IEEE Transaction on Neural Network*, Vol. 9, No. 5. 1998.
- [51] D. E. Rumelhart, G. E. Hinton and R. J. Williams "Learning internal representations by error propagation" *Parallel Distributed Processing* vol 1, Cambridge, MA: MIT Press, 1986, pp 318–362

**Debakla Mohammed** Is a PhD student in computer Science University of Science and Technology of Oran (USTO) - ALGERIA. He received the diploma of engineering in Computer Science from the University of Oran (USTO) - ALGERIA in 1996. He received the diploma of teaching in Computer Science from the University of Oran (USTO) - ALGERIA, in 2005. Is a teacher at the University of Mascara - Algeria, since 2005. His research interests are in the field of image processing and optimization methods.

**Khalifa Djemal** received his diploma degree in Optical, Image and Signal Processing in 1999 from the National School of Physics at the University of Marseille, France and his Ph.D. in Image and Signal Processing, 2002, from the University of Toulon, France.

Since 2003, he is an Associate Professor at the Electrical Engineering Department of the Institute of Technology at the University of Evry Val d'Essonne, France. He works now within the S.I.M.O.B team of the IBISC Laboratory. His current research interests are in the areas of image and data processing (Restoration, Segmentation, Clustering and CBIR). Dr. Djemal chaired the International Conference on Image Processing Theory, Tools and Applications IPTA, in 2008, 2010 and 2012, and also International Workshop on Medical Image Analysis and Description for Diagnosis Systems, MIAD, in 2009, 2010 and 2011. He was the chair of some special sessions in a number of conferences. He acted as technical chairman for a number of conferences. He is a reviewer for a number of international journals and conferences.

**Mohamed Benyettou** is a professor in department of computer science, University of Sciences and Technologies Mohamed Boudiaf USTO, ORAN-ALGERIA. He's also a director of laboratory of modeling and optimization LAMOSI, his main research interests include: artificial intelligence, image processing, optimization methods...