Path Tracking Control of Tracked Vehicle

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Abstract

In this paper, an improved time-optimal control algorithm is formulated to guarantee the tracked vehicle moves along the desired path. For tracked vehicles, the track is driven by the sprocket. During tracking, the velocity cannot change abruptly, or it will cause extra slippage of the track and mechanical damage of the whole system. Considering the dynamic constraints of the track, a cubic spline cure, which could meet the dynamic limits, is chosen as the tracking trajectory. Also the kinematic and positional error model of tracked vehicles is established. The control algorithm is implemented in MATLAB, and after a series of simulation, co-efficiency the curve is determined to better the validity and accuracy of the control system. Through comparison with PID control strategy in previous work, the superiority of this algorithm is confirmed.

Keywords: *Tracked vehicle, Path tracking, Algorithm*

1. Introduction

Tracked vehicles are broadly used in the area of unpredictable and terrain condition. For deep sea mining, tracked vehicles are superior to many other kinds of vehicles, for they have larger contact area of tracks and can provide better traction. During working, the deep sea tracked vehicle goes on the extremely cohesive soil, and because of the slippage from the track, for the track vehicle, it is difficult to move long the desired path. To avoid this phenomenon, path tracking control is of most significance.

As for the path tracking, a great number of studies have been carried out since Kanayama's pioneering work [1]. In his work, the kinematic model of mobile robots was established, and the proposed tracking method can be used for later studies. Since then, various different path planning methods have been addressed. The methods used can be sorted into two types, the first is Lyapunov function[2]-[4], and the second is backstepping method[5]-[6]. Also, many researchers paid much attention to the curvature continuity of the tracking path, and a lot of tracking curves are formulated such as clothoid pair curve[1], quadratic curve[7], polynomial curve[8], hyperbolic curve [3], and etc. At the same time, other scholars focused on the time-optimal trajectory planning. Yet, these studies concentrate on the mobile robots and the methods are too complicated.

For tracked vehicles, the key problem is the slippage of the track. Schiller and Ahmadi have done many studies on the analysis of the longitudinal slippage in the design of the path tracking control system[9]-[10]. The conclusion of their studies can be used in this paper. To better understand the interaction between the track and the soil, Hong has done studies on the soil mechanics, and so-called "line of sight" and vector pursuit method is proposed to control the vehicle moving along the desired path [11]-[13].

However, the tracked vehicle cannot change its velocity abruptly due to the dynamic constraints of the driving sprocket. Therefore, the acceleration limits of the track should be taken into consideration in path tracking design.

In this paper, a new path tracking algorithm is put forward considering the dynamic constraints of the tracked vehicle. To ensure time-optimality of tracking, the best control parameter is chosen. Also, via Lyapunov function, the stability of the control algorithm is proven. The quick and smooth tracking performance of the algorithm can be seen from the results of simulation, comparing with the PID control strategy.

2. Kinematics of the tracked vehicle

A tracked vehicle working on a horizontal plane is shown in Fig. 1. The vehicle's motion is described by kinematic equations written in the vehicle-fixed coordinate system. In Fig 1, the vehicle is turning to the right, (x_C, y_C) indicates the position of the vehicle with respect to the world coordinate system and the triplet (x_C, y_C, θ_C) defines the vehicle posture. The outside or left track is denoted by a subscript o, and i denotes the inside or right track.

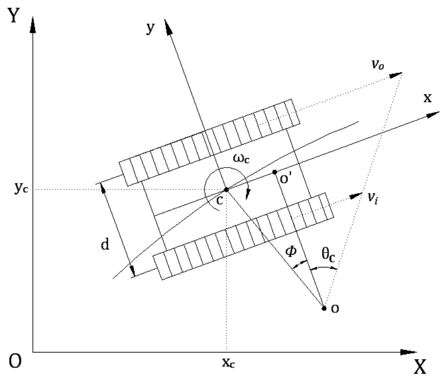

Fig. 1 Kinematic model of the track vehicle.

In the presence of the longitudinal slips i_o and i_i of the tracks, the speeds of the outer track v_o and inner track v_i would be

$$v_o = rw_o(1-i_o) \tag{1}$$

$$v_i = rw_i(1-i_i) \tag{2}$$

Where r is the track rolling radius, and w is the angular velocity of the track driving sprocket. Therefore, the linear and angular velocity of the vehicle is

$$v_C = (v_o + v_i)/2 \tag{3}$$

$$w_C = (v_o + v_i)/d \tag{4}$$

Where d is the tread of the vehicle. In order to better the traction force of the vehicle, the longitudinal slip i=10% is chosen according to the previous work [15].

Because of the difference between v_o and v_i, the angle θ_C is expressed in the form of an arctangent function

$$\theta_c = \arctan[(v_o - v_i)t/d] \tag{5}$$

In real working condition, the velocity of the tracked vehicle is very small (about 0.5 m/s), and the lateral friction is large enough. Then, we assume that the lateral slippage by centrifugal force is almost zero, that is, the slip angle Φ=0. So, the motion of the vehicle is thus described as follows:

$$\dot{x}_C = v_C \cos\theta_C \tag{6}$$

$$\dot{y}_C = v_C \sin\theta_C \tag{7}$$

$$\dot{\theta}_C = w_C \tag{8}$$

3. Path tracking system

3.1 Dynamic constraints of the vehicle

Since the vehicle is driven by the sprockets, any abrupt change in the vehicle motion may cause the slippage. Here we assume the angular acceleration of each sprocket wheel is limited by $\dot{\omega}_{\max}$. Therefore, the tangential acceleration a_C and angular acceleration α_C of the vehicle are limited by

$$|a_c| \le a_{\max} = r\dot{\omega}_{\max}/2 \tag{9}$$

$$|\alpha_c| \le \alpha_{\max} = r\dot{\omega}_{\max}/d \tag{10}$$

3.2 Error posture

Fig. 2 shows the coordinates system of tracked vehicle. Here, $P_d\ (x_d, y_d, \theta_d)$ is the desired posture and $P_C(x_c, y_c, \theta_c)$ is the current posture of the vehicle. $L(x)$ is the trajectory curve of the tracked vehicle. $P_t\,(x_t, y_t, \theta_t)$ is the posture on the trajectory curve.

We define an error posture $P_e\ (x_e, y_e, \theta_e)$ as this difference between P_d and P_C. Thus, P_e and $\dot{P}_e$ can be calculated by

$$P_e = \begin{pmatrix} x_e \\ y_e \\ \theta_e \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C & 0 \\ -\sin\theta_C & \cos\theta_C & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_d - x_c \\ y_d - y_c \\ \theta_d - \theta_c \end{pmatrix} \tag{11}$$

$$\dot{P}_e = \begin{pmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{pmatrix} = \begin{pmatrix} y_e\omega_c - \upsilon_c + \upsilon_d\cos\theta_e \\ \upsilon_d\sin\theta_e - x_e\omega_c \\ w_d - w_c \end{pmatrix} \tag{12}$$

The aim of path tracking control is the combination of a path and a profile of linear and angular velocities and accelerations with which this path has to be followed.

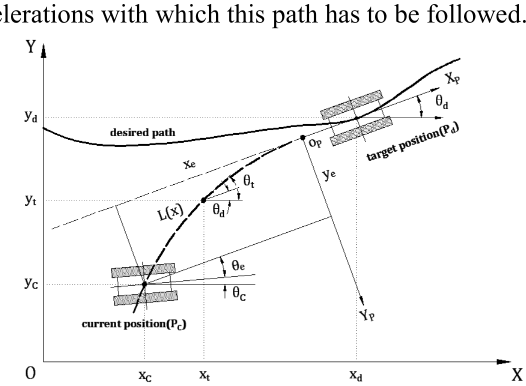

Fig. 2 Model of path error with temporary tracking path.

3.3 Path tracking algorithm

In order to make sure the tracked vehicle reaches the desired position smoothly, the trajectory to be chosen is of the most importance. Meanwhile, due to the dynamic constraints of the driving sprocket of the track, the trajectory should satisfy the dynamic condition to provide the continuities of the position, angle, and curvature:

$$L(x_L)\big|_{x_L=0} = 0 \tag{13}$$

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$$\dot{L}(x_L)\big|_{x_L=0}=0 \tag{14}$$

$$\ddot{L}(x_L)\big|_{x_L=0}=0 \tag{15}$$

It is evident that $L(x_L)=\lambda x^3$ is a cubic spline curve, and could satisfy the above limits. Here, λ is a positive constant denoting the curve co-efficiency. And the tangential angle and its differential of any point on the curve can be calculated as follows:

$$\begin{aligned}\theta_t &= \theta_d+\theta_e \\ &= \theta_d+\tan^{-1}[3\lambda(y_e/x_e)^{2/3}\operatorname{sgn}(y_e)]\end{aligned} \tag{16}$$

$$\dot{\theta}_t=\omega_t=\omega_d+\frac{2(y_e/\lambda)^{-1/3}}{1+\tan^2(\theta_t-\theta_d)}\dot{y}_e\operatorname{sgn}(y_e) \tag{17}$$

By using the point on this curve as a temporary target position to follow at the current posture, the vehicle could softly reach the target on the desired path. To guarantee the vehicle arrives at the target position in minimum time, the optimal-time control algorithm is then put forward:

$$\begin{Bmatrix}\upsilon_c(k+1)=\upsilon_c(k)+a_c\Delta t\\ \omega_c(k+1)=\omega_c(k)+\alpha_c\Delta t\end{Bmatrix} \tag{18}$$

$$\begin{cases}a_C=a_{\max}\operatorname{sgn}(v_j/\Delta t)\\ \alpha_C=\alpha_{\max}\operatorname{sgn}(w_j/\Delta t)\end{cases} \tag{19}$$

$$\begin{cases}v_j=\dot{x}_e+k_1(a_{\max}|x_e|)^{1/2}\\ w_j=\dot{\theta}_e+k_2(\alpha_{\max}|\theta_t-\theta_C|)^{1/2}\end{cases} \tag{20}$$

In this control system, the inputs are the linear and angular acceleration, and the output is the position of the vehicle. sgn(x) is the sign operator.

This control algorithm can be described as follows: if the vehicle lies behind the target, the positive maximal linear acceleration is desired, and if the vehicle is ahead of the target, the negative maximal acceleration is chosen, and so does the angular acceleration. However, any abrupt change in the robot motion may cause mechanical damage to the vehicle, thus v_j and w_j are introduced as the intermediate variable determining the acceleration inputs. We can see in equation (20) that the v_j is influenced by two factors, first is the rate in change of the tangential path error x_e, second is the the relative position between the current and target position considering the maximal linear acceleration. k_1 and k_2, which are positive constant, are introduced as the proportional co-efficiencies of these two factors respectively. It is the same as for the angular acceleration in equation (20). Due to the finite sampling time Δt, this control law can be digitally executed by using equation (18). Then the posture of the vehicle can be estimated from integration of Equation (6)-(8).

Using this control strategy, the time-optimal control goal can be guaranteed.

4. Effect of Control Parameters

Fig. 3 shows a series of curves with different values of λ. The co-efficiency λ has some effect on the tracking performance, for its value leads to the curvature of the smooth curve. It is apparent that the large value λ will improve the convergence rate of the tracking motion. For the path tracking control of the vehicle, we need a fast response; therefore, the optimal value of parameter λ should be determined.

As for the cubic spline curve $L(x)$, the curvature can be calculated as follows:

$$\mu(x)=\frac{6\lambda x}{(1+9\lambda^2x^4)^{3/2}} \tag{21}$$

Fig. 4 shows the curvature of the temporary path at different point. Apparently, the maximum value of the change rate of curvature occurs at the origin. Thus, the maximum value can be computed as

$$\dot{\mu}_{\max}=6\lambda\upsilon_t<\alpha_{\max}/\upsilon_t \tag{22}$$

$$\lambda<\alpha_{\max}/6\upsilon_t^2 \tag{23}$$

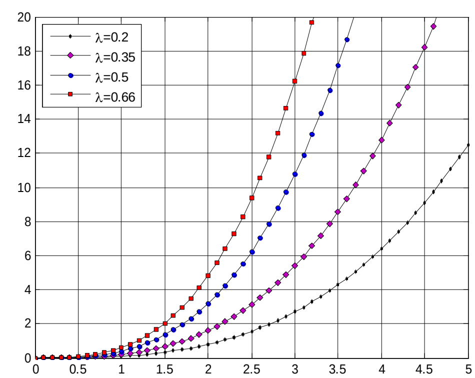

Fig. 3 Cubic spline curves with different co-efficiency.

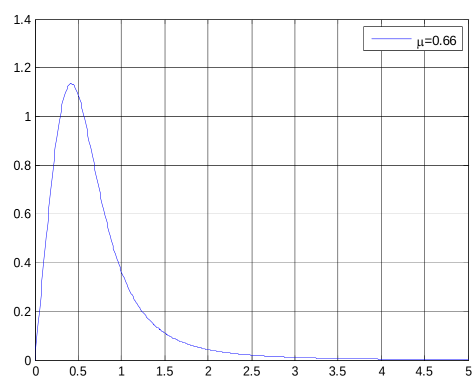

Fig. 4 Curvature of the temporary path (λ=0.66).

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5. Simulation

To investigate the tracking performance of the tracked vehicle, a series of the simulations were conducted with different tracking parameters. The control algorithm developed in section III.3 was implemented in MATLAB.

The simulations are mainly concerns two kinds of typical of path: first is straight path, second is a circular path. Fig. 5 shows the tracking paths obtained by performing the tracking motion for a straight path and Table 1 shows the simulation conditions. From equation (23) and the acceleration parameters in Table 1, λ should no more than 0.66.

Table 1 Simulation conditions for straight path

Parameters	NO.			
	1	2	3	4
λ	0.2	0.35	0.5	0.66
k_1	1.5			
k_2	0.8			
$a_{\max}$	0.25 m/s^2			
$\alpha_{\max}$	1 rad/s^2			
v_t	0.5 m/s			
w_t	0 rad/sec			
$(x_C(0), y_C(0), \theta_C(0))$	(0,2,0)			
$(x_t(0), y_t(0), \theta_t(0))$	(0,0,0)			

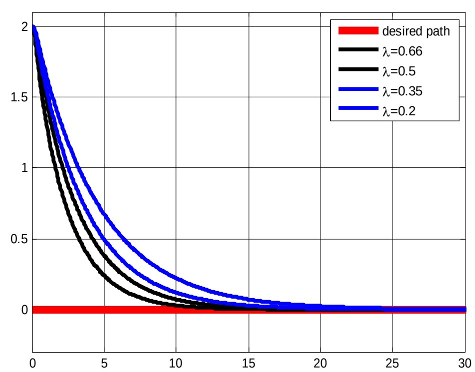

Fig. 5 Results of path tracking simulation with different λ (for a straight path).

In Fig. 5, although with different curvatures, all the tracking paths can smoothly tracking the desired path, thus, the validity of the control algorithm in III.3 is proved. However, with different curve co-efficiency λ, the actual tracking paths differ. It is obvious that as the value of λ decreases, the tracking time increases accordingly.

In order to evaluate the time-optimal performance with differentλ,a position error d is introduced, which shows the distance between the current position and the desired position, and d is defined as:

$$d = \sqrt{x_e^2 + y_e^2}$$

Fig. 6 shows the position errors with different λ when tracking a straight path.

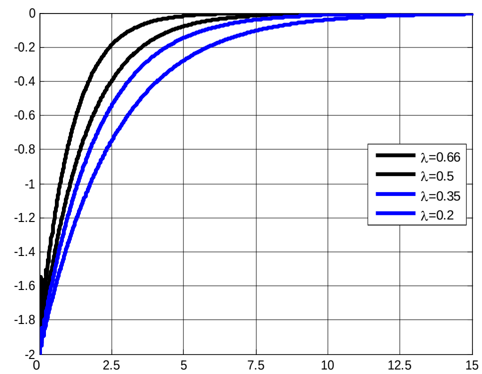

Fig. 6 Simulation results of the position error with different λ (for a straight path).

From Fig. 6 we can see that although the final value of position error can reach 0, which means the vehicle can track the desired path, it costs the least time when λ=0.66, comparing with the other tracking paths with different λ.

Therefore, for a straight path, the stability and validity of the proposed control strategy is confirmed, and λ=0.66 is chosen from the time-optimal respect.

In the real mining condition, it is a very complex trajectory that the tracked vehicle should moves along, therefore, a series of simulations with circular desired path was implemented. Fig. 7 shows the tracking paths obtained by performing the tracking motion for a circular path. The simulation conditions are shown in Table 2.

Table 2 Simulation conditions for circular path

Parameters	NO.			
	1	2	3	4
λ	0.2	0.35	0.5	0.66
k_1	1.5			
k_2	0.8			
$a_{\max}$	0.25 m/s^2			
$\alpha_{\max}$	1 rad/s^2			
v_t	0.5 m/s			
w_t	0.05 rad/sec			
$(x_C(0), y_C(0), \theta_C(0))$	(3,0,0)			
$(x_t(0), y_t(0), \theta_t(0))$	(0,0, π/4)			
R	10 m			

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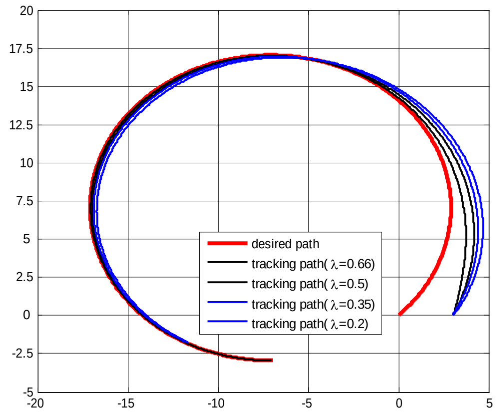

Fig. 7 Results of path tracking simulation (for a circular path).

From Fig. 7 we can see that with the proposed control algorithm, the vehicle can track the circular path well. Yet, with different value of λ, the tracking paths are different. Fig. 8 shows the simulation results of position error with different value of λ when tracking the circular path. The position error is 3 meters at the start, and decreases rapidly. With different λ, the slopes of the decrease differ. It is clear that when λ is 0.66 the rate of the decrease is the largest. Also, when λ=0.66, 0.5, or 0.35, the final position error is about 0.1m, while when λ=0.2, the final position error is about 0.2m. From the accuracy and efficiency perspective, λ=0.66 is suitable when tracking the circular path.

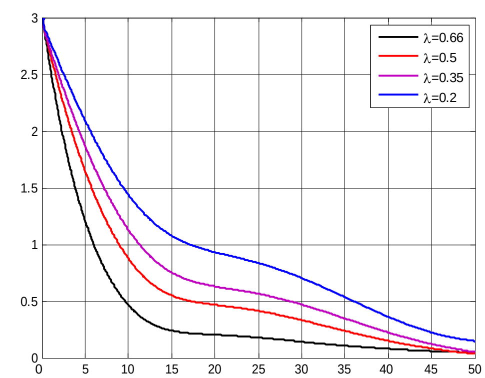

Fig. 8 Simulation results of the position error (for a circular path).

As for the simulation results discussed above, both for a straight path and circular path, the validity and rapidity of the control strategy is confirmed and the co-efficiency λ=0.66 is chosen.

6. Algorithm comparisons

To test the superiority of this proposed control algorithm, a comparison with the path tracking PID control strategy [14] has been done. For convenience, the proposed control algorithm in this paper is called S.

Fig. 9 and Fig. 11 show the simulation results of straight path tracking and circular path tracking under PID control strategy and S respectively. Fig. 10 and Fig. 12 show the correspondent position errors. The simulation conditions are the same in Table 1 and Table 2.

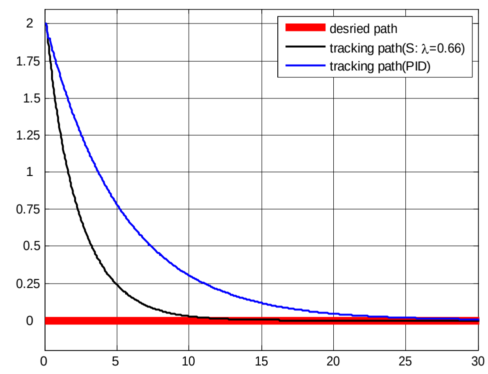

Fig. 9 Contrastive simulation results (for a straight path).

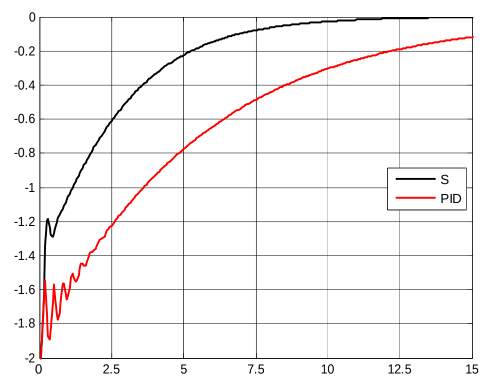

Fig. 10 Contrastive simulation results of the position error (for a straight path).

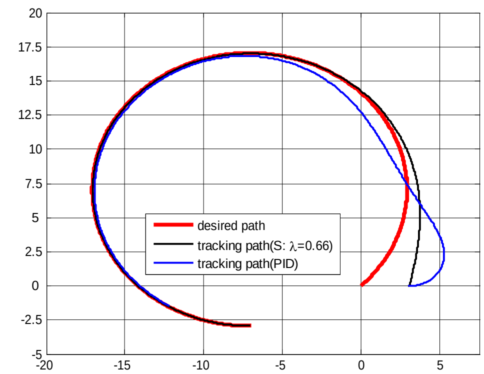

Fig. 11 Contrastive simulation results (for a circular path).

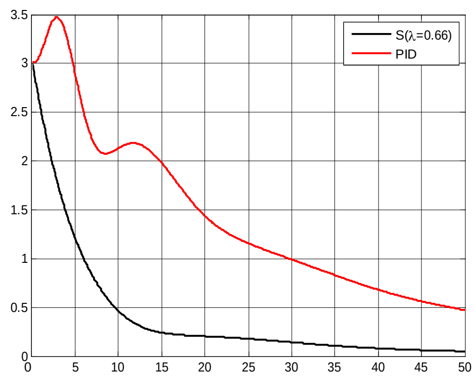

Fig. 12 Contrastive simulation results of the position error (for a circular path)

From Fig. 9 and Fig. 10, it is clear that although there is an initial positional deviation, both control strategies can ensure the vehicle go back to the desired path eventually. Thus, the validity of each control strategy can be confirmed. Yet, without taking dynamic constraints of the vehicle into account, an evident oscillation of position error occurs in the PID control strategy in Fig. 10, which is a bad impact on the smoothness of the tracking trajectory. This phenomenon can be also seen in Fig. 12.

Also, PID control strategy appears a poor rapidity than the S. For straight tracking path in Fig. 10, when the position error is -0.2 *m*, the time costs in these two control strategy are about 5s and 12.5s respectively. For circular tracking path in Fig. 12, when the position error is 0.5m, the time costs are 10s and 48s respectively.

In addition, S shows a better accuracy in the path tracking. By S, position error can be decreased to less than 0.1m both for straight and circular path at the end of simulation. By PID, position error is about 0.5*m* for the circular path, which is unacceptable.

Through comparison of S with the previous PID control strategy, the superiority of the proposed algorithm in the paper is confirmed.

7. Conclusions

1. According to the requirement of the dynamic constraints of the vehicle, a cubic spline curve is proposed as the temporary tracking path. By taking the smoothness and time-optimality of path tracking, an improved control algorithm is formulated.
2. After a series of simulations with different co-efficiency λ, it is clear that λ has an influential effect on the tracking path, and the method to determine the best value of λ is established. In this paper, λ=0.66 is chosen.
3. Through the comparison of the proposed algorithm and PID control strategy, a conclusion is gotten that the proposed algorithm is superior to PID control due to its rapidity and accuracy.
4. In the future, experiments of the path tracking needs to be carried out to testify this proposed algorithm.

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