

Frequency-Domain Modeling and Simulation of Coupled Lossy Multiconductor Transmission Lines

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Abstract

This paper presents a study of modeling and simulation of the lossless and lossy multiconductor transmission lines in frequency domain. This study is based on a characteristics method, which permits modeling the line as a quadripole whose advantage is not to presuppose applied charge conditions in its extreme. This permits it to be introduced easily in the circuit simulators such as Spice, Esacap and Saber. The results presented here cover two types of multiconductor transmission lines, the "Ribbon Cable" and "2Wire Xtalck.

Keywords: *Characteristics method, Frequency domain, Multiconductor transmission line, Losses.*

1. Introduction

The problems related to the frequency mounting and the effects of interconnections in equipments and industrial applications are various (distortion, attenuation, crosstalk, etc...). Moreover the losses can play a very important role in the degradation and attenuation of signals moving through the line. Considering the framework of the inter-equipment electromagnetic compatibility (EMC), skin and proximity effects become more complicated in high frequency, which necessitates, therefore, a physical model adapted to the transmission lines.

The cascade model [1][2] allows modeling the transmission line in the form of a RLCG circuit. It can also be applied in case of MTL line, which becomes quite complicated when using more than two conductors. And it results not only in oscillations in time domain (phenomenon of Gibbs), but it needs an important calculating time, which makes it inefficient.

The finite differences method in time domain (FDTD) [2][3][4] is an analytic method. It consists of dividing the time and the space where the solution is searched for in a network of point spaced regularly to constitute a mesh.

The major difficulty then, is to have a distributed MTL line model valid in both time and frequency domains, with and without losses. The model presented in this paper for modeling and simulation of the lossless and lossy multiconductor transmission lines in frequency domain, it's valid also in the time domain [5].

This paper will expose a behavior study of the lossless and the lossy multiconductor transmission line –MTL- in frequency domain, based on the modelling of the MTL lines numerical method with the help of Branin model, that can analyze the MTL line by representing it in the form of a quadripole, this is the characteristics method [2][6][7][8], This method would permit the analysis of an MTL line without and with losses and present the advantage to avoid presupposing conditions of applied charges to its extremes. This permits it to be introduced easily in circuit simulators as Spice, Esacap and Saber. Diverse examples of applications are presented to validate this model and to show their interests and to understand the behavior of a line MTL in the frequency domain.

2. MTL Lines Modeling

2.1 Lossless Lines

Branin [6] was the first to propose a numerical model of a transmission line which permitted giving a simple schema equivalent to the ideal line. This schema includes two dipoles. In the input dipole ($z=0,t$) the tension is determined from the reflected tension in extremes of the line ($z=1$) at the precedent time $t-Tr$. The same interpretation is applied to the output dipole (Tr is the delay of the line).

Following [2][6], the equivalent schema of the lossless line ($R=G=0$) is depicted in the fig.1 :

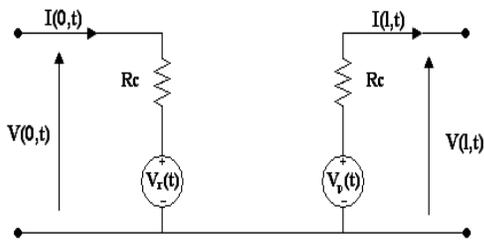


Fig.1 : Quadripole representation of the ideal line.

Indeed, it is easy to show that electrical parameters in extremities are linked by means of the following relations:

$$\begin{aligned} V(0,t) &= R_c I(0,t) + V(l,t-T_r) + R_c I(l,t-T_r) \quad (a) \\ V(l,t) &= -R_c I(l,t) + V(0,t-T_r) + R_c I(0,t-T_r) \quad (b) \end{aligned} \quad (1)$$

The equations (1.a) and (1.b) do not depend only on secondary parameters (the characteristic impedance and the delay of the line). Hence, the equivalent schema of the figure 1 is established by putting:

$$\begin{aligned} V_r(l,t) &= V(l,t-T_r) + R_c I(l,t-T_r) \quad (a) \\ V_r(0,t) &= V(0,t-T_r) + R_c I(0,t-T_r) \quad (b) \end{aligned} \quad (2)$$

$V_r(t)$ and $V_i(t)$ are then calculated using parameters observable at a known time $t - T_r$.

The features method can perhaps be spread to the multiconductor lines case [2][7][9][10]. This requires, on the other hand, uncoupled propagation modes on each line. This separation of modes is implemented using the modal method.

Keeping in mind electromagnetic couplings between the lines, the impedances matrix are not diagonal. The interest of the modal method is then to uncouple the equations to be able to diagonalise the matrix.

To model the transmission lines, it is necessary to solve the telegraphists equations. The time domain equations representation is:

$$\begin{cases} \frac{\partial V(z,t)}{\partial z} + RI(z,t) + L \frac{\partial I(z,t)}{\partial t} = 0 \\ \frac{\partial I(z,t)}{\partial z} + GV(z,t) + C \frac{\partial V(z,t)}{\partial t} = 0 \end{cases} \quad (3)$$

[V] and [I] represent, respectively, the tensions and currents matrix.

[R], [L], [C] and [G] represent, respectively, the resistances, inductances, conductance's and capacities matrix. These include implicitly of all informations concerning the transverse section, which permits to characterize a muticonductor structure.

The coefficients of these different matrix are obtained either by engineering practical techniques [11], or by numerical methods [9][10][12]. The modelling of a lossless and lossy MTL lines in frequency domain by the

Branin method will be exposed respectively in paragraphs A and B.

As we deal with the lossless lines case ($G=R=0$), the MTL lines equations in the time domain (3) become:

$$\begin{cases} \frac{\delta^2 V(z,t)}{\delta z^2} - LC \frac{\delta^2 V(z,t)}{\delta t^2} = 0 \\ \frac{\delta^2 I(z,t)}{\delta z^2} - CL \frac{\delta^2 I(z,t)}{\delta t^2} = 0 \end{cases} \quad (4)$$

The modal method introduces fictitious parameters V_m and I_m , by means of the following linear transformations:

$$\begin{cases} V(z,t) = T_v \cdot V_m(z,t) \\ I(z,t) = T_i \cdot I_m(z,t) \end{cases} \quad (5)$$

The modal matrix T_v and T_i , are to be determined to ensure the uncoupling of propagation modes of the MTL line [2][7][9][10]. If we apply the transformation to the system of equations (4), we then obtain:

$$\begin{cases} \frac{\delta^2 V_m(z,t)}{\delta z^2} - T_v^{-1} L C T_v \frac{\delta^2 V_m(z,t)}{\delta t^2} = 0 \\ \frac{\delta^2 I_m(z,t)}{\delta z^2} - T_i^{-1} C L T_i \frac{\delta^2 I_m(z,t)}{\delta t^2} = 0 \end{cases} \quad (6)$$

We can choose both matrix T_v and T_i in such way that the system (6) could be uncoupled. In our case of a lossless line placed in a homogeneous and isotropic medium, we can always find the transformation matrix which diagonalise simultaneously the matrix L and C.

The two transformation matrix T_v and T_i are bound by the following relation [2]:

$$T_i^t = T_v^{-1} \quad (7)$$

The system of equation of a lossless line spells in the modal base :

$$\begin{cases} \frac{\delta V_m(z,t)}{\delta z} = -L_m \frac{\delta I_m(z,t)}{\delta t} \\ \frac{\delta I_m(z,t)}{\delta z} = -C_m \frac{\delta V_m(z,t)}{\delta t} \end{cases} \quad (8)$$

With L_m and C_m are diagonal matrix of dimension $N \times N$

$$\begin{cases} L_m = T_v^{-1} \cdot L \cdot T_i \\ C_m = T_i^{-1} \cdot C \cdot T_v \end{cases} \quad (9)$$

The system of equations (8) represents an uncoupled MTL line, which has a characteristic impedance R_{C_m} and a

delay T_{rm} :

$$R_{C_{m_i}} = \sqrt{\frac{L_{m_i}}{C_{m_i}}}, \quad T_{r_{m_i}} = \frac{l}{\sqrt{L_{m_i} C_{m_i}}} \quad (10)$$

2.2 Lossy Lines

The losses can play a very important role in the degradation and attenuation of signals transported by line. The losses are caused both by a non-null conductivity and of the loss of the polarization medium or imperfect conductors. The losses presented by imperfect conductors are habitually more significant than those owing to the transmission line. For this reason, we often suppose that the surrounding medium is lossless ($G = 0$) in the MTL line equations. The resistance due to imperfect conductors is represented in the resistance matrix by a length unity [R].

The system of equations (3) becomes:

$$\begin{cases} \frac{\partial V(z,t)}{\partial z} + R I(z,t) + L \frac{\partial I(z,t)}{\partial t} = 0 \\ \frac{\partial I(z,t)}{\partial z} + C \frac{\partial V(z,t)}{\partial t} = 0 \end{cases} \quad (11)$$

The equations (11) expressed in the frequency domain by Laplace operator p gives us:

$$\begin{cases} -\frac{\partial V}{\partial z} = [R][I] + p[L][I] = [Z][I] \\ -\frac{\partial I}{\partial z} = p[C][V] = [Y][V] \end{cases} \quad (12)$$

In general, a line can be modeled by its serial impedance $Z = R + j\omega L$, and parallel admittance $Y = j\omega C$, (The dielectrics losses are supposed to be negligible $G=0$).

For the working frequencies well superior to a characteristic frequency of the line equal $R/2\pi L$ (low losses hypothesis) [2][7], and using a first development order, we obtain:

$$Z_c = \sqrt{\frac{R + j\omega L}{j\omega C}} \approx R_c + \frac{RR_c}{2j\omega L} \quad \text{where } R_c = \sqrt{\frac{L}{C}} \quad (13)$$

The characteristic impedance in that case is equivalent to characteristic impedance RC mounted serially with a capacity $C_{pf} = \frac{2L}{R.R_c}$, when the frequency increases ZC becomes equal to RC.

With the same approximation, the constant of propagation becomes: $\gamma = \alpha + j\beta = \frac{R}{2R_c} + j\omega\sqrt{LC}$

To take into account the attenuation of the wave the term $e^{-\alpha l}$ is introduced, it is enough, then, to modify Branin's generators of the quadripole representation of the ideal line (cf Fig 2). They become [2][7] :

$$\begin{cases} V'_r(t) = V_r(t) \cdot e^{-\alpha l} \\ V'_i(t) = V_i(t) \cdot e^{-\alpha l} \end{cases} \quad (4)$$

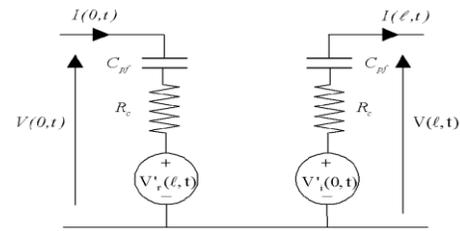


Fig.2 : Quadripole representation of the low loss line.

For a lossy multiconductor line, we do the same as in the case of a lossless MTL line. Therefore, in the case of coupled low loss lines, the characteristic impedance and the delay are given by:

$$Z_{C_m} \approx R_{C_m} + \frac{RR_{C_m}}{2j\omega L_m}, \quad R_{C_m} = \sqrt{\frac{L_m}{C_m}} \quad (15)$$

This supposes that the resistance's losses referred to the R matrix whose non-diagonal terms were null. Also, it is enough to bring the following modifications to the generators of the ideal lines, given by relations (2.a) (2.b), which become:

$$V_{m_i}(l,t) = T'_V \{ V_m(l,t - T_{m_i}) - Z_{C_m} I_m(l,t - T_r) \} \quad (a) \quad (16)$$

$$V_{m_i}(0,t) = T'_V \{ V_m(0,t - T_{m_i}) + Z_{C_m} I_m(0,t - T_r) \} \quad (b)$$

With $T'_V = T_V e^{-\alpha l}$ is the attenuated modal matrix.

The Electric parameters to the extremes of the MTL line are linked by the following relations:

$$V_m(l,t) = -Z_{C_m} I_m(l,t) + V'_{im}(0,t) \quad (a) \quad (17)$$

$$V_m(0,t) = Z_{C_m} I_m(0,t) + V'_{im}(l,t) \quad (b)$$

3. Simulation & Results

To verify the validity of this model in frequency domain, we simulate two types of multiconductor transmission lines, the "Ribbon Cable" and "2Wire Xtalck in ESACAP

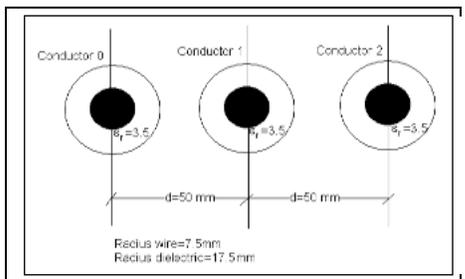
circuit simulator [13]. In this paragraph, we make a comparison between simulation results in the literature for example 1[2] and example 2 [10] and our simulation results using the ESACAP circuit simulator in frequency domain in the lossless and lossy. This model is valid also in the time domain [5].

3.1 Example 1: Ribbon Cable

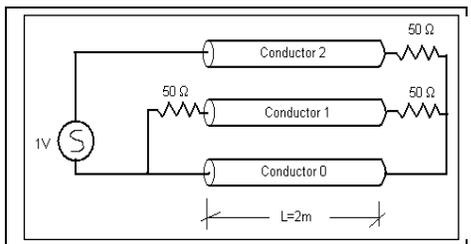
We consider a transmission line of three conductors with 2m of length and ligneous parameters R,L and C, fed by a tensional generator, the in and the out charges are equal to 50 Ohms, as indicated on the figure 3.

$$L = \begin{bmatrix} 750.37 & 508.69 \\ 508.69 & 750.37 \end{bmatrix} (nH/m) \quad R = \begin{bmatrix} 0.19 & 0 \\ 0 & 0.19 \end{bmatrix} (\Omega/m)$$

$$C = \begin{bmatrix} 37.21 & -18.60 \\ -18.60 & 37.21 \end{bmatrix} (pF/m)$$

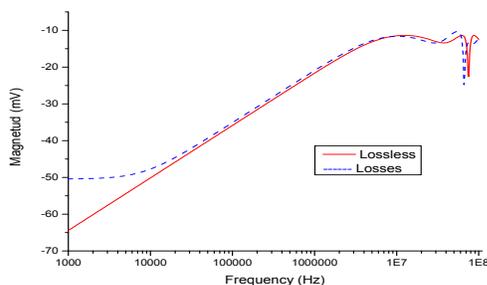


a) dimensions.

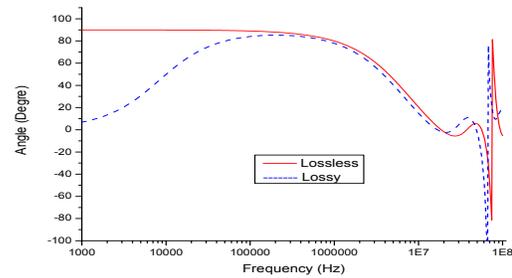


b) Electric representation.

Fig.3 : Ribbon Cables transmission line to 3 conductors.

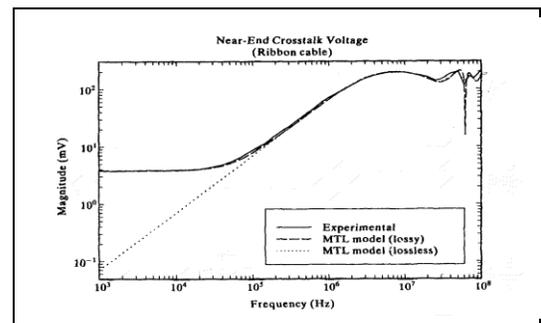


a) Magnitude.

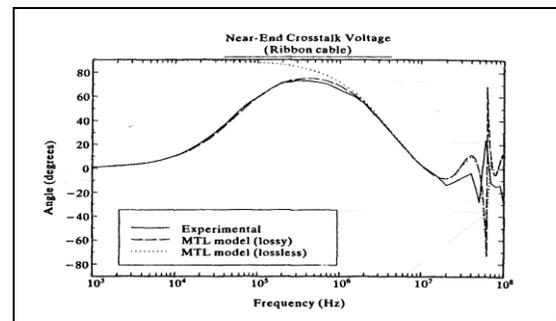


b) Phase.

Fig.4 : The near-End crosstalk voltage simulated with Esacap 2000.



a) Magnitude.



b) Phase.

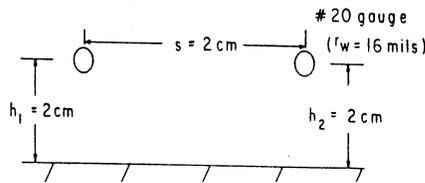
Fig.5 : Validation result of the near-End crosstalk voltage [2].

Simulation results: The experimental results from Ref [2] are compared to the predictions of our method, over frequency range of 1 kHz to 100 MHz. As from figure (4.a), for $F < 100$ kHz the losses in the line conductors are predominant. For $F > 100$ kHz, the inductive effect becomes predominant over resistive effect. In the Figure 4, the amplitude and the phase of near-end crosstalk, the predicted voltage induced at the source side of conductor n.1, are displayed. The corresponding values from Ref. [2] are displayed for comparison on figure 5. As example, the low-frequency value in Figure (4.a) is ~ -48 dBV, corresponding to ~ 4 mV in Figure (5.a).

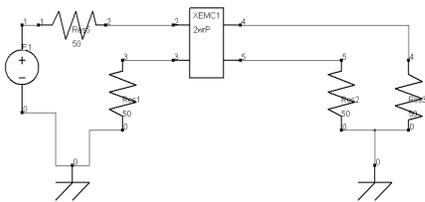
Physical interpretation: From (4.a) at low frequency ($R/DC > \omega L$), the resistive coupling is predominant, while at intermediate frequency, the classic inductive X-talk becomes predominant (slope +20dB/dec). At high frequency, the voltage is limited by the line's characteristic impedance. The sharp resonance at $F \sim 65\text{MHz}$ is the $\lambda/2$ resonance of 2m cable, shifted down by increased electrical length due to extra capacitance of dielectric gains. The $\lambda/2$ resonance is excited because of low R terminations, compared to common mode impedance $Z_{cm} \sim 130\Omega$.

3.2 Example 2: '2Wire Xtalk'

We consider a '2wire-xtalk' transmission line with length 4.674m and of dielectric constant $\epsilon = 1$, fed by a tensional generator, the in and the out charges are equal to 50 Ohms, as indicated on the figure 6.



a) Geometric configuration



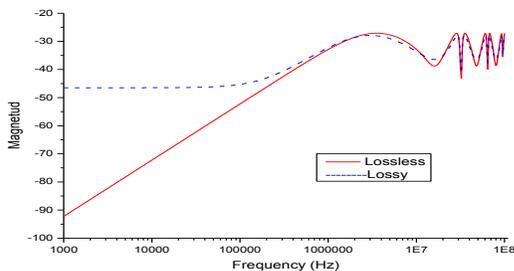
b) Electric model of the line

Fig.6 : configuration of the line '2wire-xtalk'.

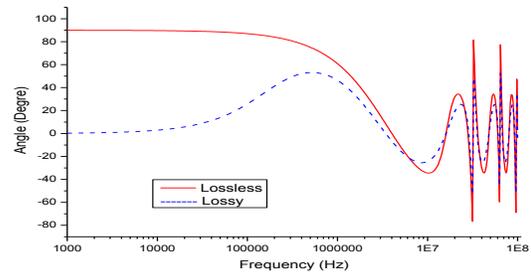
Figure 6 presents the geometric configuration of the line whose parameters are:

$$L = \begin{bmatrix} 918 & 161 \\ 161 & 918 \end{bmatrix} (nH/m) \quad R = \begin{bmatrix} 0.78 & 0 \\ 0 & 0.78 \end{bmatrix} (\Omega/m)$$

$$C = \begin{bmatrix} 12.50 & -2.19 \\ -2.19 & 12.50 \end{bmatrix} (pF/m)$$

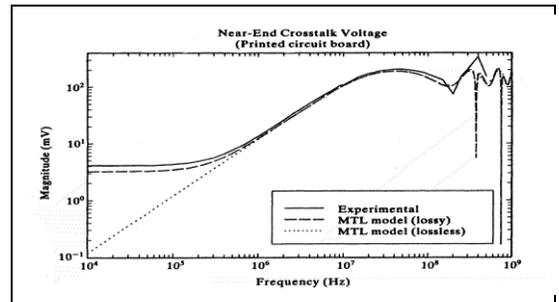


a) Magnitude

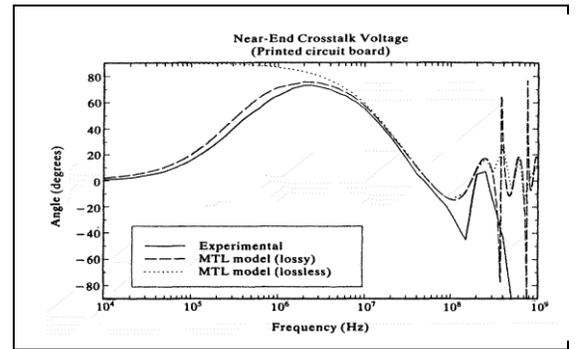


b) Phase

Fig.7 : The near-End crosstalk voltage simulated with Esacap 2000



a) Magnitude



b) Phase

Fig.8 : Validation result for the near-End crosstalk voltage [2].

Simulation results: The predictions of our method are compared to experimental results (Ref.[2]), for frequency range $F = 10 \text{ kHz} - 1\text{GHz}$. At $F < 200\text{KHz}$, the conductor losses become predominant.

In the following figures, the amplitude and phase of near-end crosstalk voltage on conductor n. 1 are shown, for comparison between EMCAP prediction results and reference canonical results (Ref. [2]).

For example, the low-frequency value in Fig.(7.a) is -49.7dBV, equivalent to 3.3 mV in the figure (8.a).

Physical interpretation: The same considerations of previous example in the Section 3.2 are applicable. The frequency of the sharp resonance is now shifted to $F \sim 400\text{MHz}$, compared to previous one $F \sim 65\text{MHz}$, with

scaling almost similar to length ratio (2m against 0.25 m) . The increased capacitance due to PCB substrate, compared to wire dielectric gains, explains the difference between the frequency and length ratios.

4. Conclusion

In this work, we show that the interest of the characteristics method modeling the MTL lines in frequency domain. This method can be introduced easily in circuit simulators as Spice, Esacap and Saber, but it is only valid in low loss MTL line in the time and the frequency domains. The future work will be devoted to the development model of both real losses using a model based on the Pade approximation, and the effect of disturbing EM wave disturbed on a MTL line.

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