Decision Making Processes Supported by a Set Theoretical Approach

Sylvia Encheva

Faculty of Technology, Business and Maritime Education, Stord/Haugesund University College Haugesund, 5528, Norway

Abstract

Current solutions for multi criteria decision making problems based on vague sets address static situations where a decision maker is presented with a number of alternatives and attributes. The attributes are placed in two disjoint sets related with 'OR' operation. The first set contains several attributes related with 'AND' operation while the second set contains a single attribute. We propose an extension of this approach that can handle situations where several decision makers are taking part in the decision making process and the second set of attributes consists of several elements.

Keywords: *Set theory, multi criteria, multy person decision making.*

1. Introduction

Multi-criteria fuzzy decision-making problems based on vague set theory have been originally addressed in [2]. The authors apply certain functions measuring the degree of suitability of each alternative. Since then a number of researchers have been working on these problems, see f. ex. [3], [6], [7].

Vague sets have been already exploitet for buiding models supporting multi criteria decision processes. These models are constructed to support the work of a single decision maker. This appears to be insufficent in complicated situations where expertiese from different domains is required and a number of financially involved parties have to agree on the final decision. We propose an extension of this approach that can handle situations where several decision makers are taking part in the decision making and the second set of attributes consists of several elements. Such a way to handle multicriteria decision problems is needed due to the constanly increasing amount of data to be considered as well as ever growing variates of technologies that have to be evaluated by new experts.

The rest of the paper is organised as follows. Section 2 contains definitions of terms used later on. Section 3 presents the main results of this article. Section 4 contains the conclusion of this work.

2. Background

Notations in this subsection are as in [6]. Let U be the universe of discourse, $U = \{u_1, u_2, ..., u_n\}$ with a generic element of U denoted by u_i. A vague set A in U is characterized by a truth-membership function t_A and a false-membership function f_A,

$$t_A : U \to [0,1], \qquad f_A : U \to [0,1],$$

where $t_A(u_i)$ is a lower bound on the grade of membership of u_i derived from the evidence for u_i, $f_A(u_i)$ is a lower bound on the negation of u_i derived from the evidence against u_i, and $t_A(u_i) + f_A(u_i) \le 1$. The grade of membership of u_i in the vague set A is bounded to a subinterval $[t_A(u_i), 1 - f_A(u_i)]$ of $[0,1]$. The vague value $[t_A(u_i), 1 - f_A(u_i)]$ indicates that the exact grade of membership $\mu(u_i)$ of u_i may be unknown. But it is bounded by $t_A(u_i) \le \mu(u_i) \le 1 - f_A(u_i)$, where $t_A(u_i) + f_A(u_i) \le 1$.

When the universe of discourse U is continuous, a vague set A can be written as

$$A = \int_U [t_A(u_i), 1 - f_A(u_i)]/u_i$$

When the universe of discourse U is discrete, a vague set can be written as

$$A = \sum_{i=1}^{n} [t_A(u_i), 1 - f_A(u_i)]/u_i$$

Definition 3 *Let x and y be two vague values, $x=[t_x,1-f_x]$ and $y=[t_y,1-f_y]$, where $t_x\in[0,1], t_y\in[0;1], f_x\in[0,1], f_y\in[0,1], t_x+f_x\le 1$ and $t_y+f_y\le 1$. The result of the maximum operation of the vague values x and y is a vague value c, written as $c=x\vee y=[t_c,1-f_c]$, where*

$$t_c = Max(t_x,t_y), \quad 1-f_c = Max(1-f_x,1-f_y)$$

Let A be a vague set of the universe of discourse U with truth-membership function and false-membership function t_A and f_A, respectively, and let B be a vague set of U with truth-membership function and false-membership function t_B and f_B, respectively. The notions of complement, union, and intersection of vague sets are defined as follows.

Definition 6 *The union of the vague sets A and B is a vague set C, written as $C=A\vee B$, whose truth- membership function and false-membership function are t_C and f_C, respectively, where $\forall u_i\in U$,*

$$t_C(u_i)=Max(t_A(u_i),t_B(u_i)),$$

$$1-f_C(u_i)=Max(1-f_A(u_i),1-f_B(u_i))$$

Definition 7 *The intersection of the vague sets A and B is a vague set C, written as $C=A\wedge B$, whose truth- membership function and false-membership function are t_C and f_C, respectively, where $\forall u_i\in U$,*

$$t_C(u_i)=Min(t_A(u_i),t_B(u_i)),$$

$$1-f_C(u_i)=Min(1-f_A(u_i),1-f_B(u_i))$$

3. Several Decision Makers

Decision making processes including several alternatives (objects) and a number of criteria (attributes) need to be considered also with respect to the number of decision makers being involved.

Suppose there are two disjoint sets of attributes $A=\{A_i, i=1,\ldots,n\}$ and $B=\{B_i, i=1,\ldots,m\}$, see Fig. 1.

Figure 1: Two disjoint sets of attributes containing more than two elements each

This case can be handled by first applying AND operation in both sets A and B resulting in A' and B' and then apply OR operation on A' and B'. Thus, if

(A_1 AND A_2 AND ... AND A_n) = A',

(B_1 AND B_2 AND ... AND B_m) = B'

then the outcome is (A' OR B'). A situation requires involvement of more than two disjoint sets of attributes will be hold in an analogous fashion, i.e.

(A' OR B' OR ... OR S').

Suppose there are two intersecting sets of attributes $A=\{A_1,\ldots,A_s,E_1,\ldots,E_k\}$ and $B=\{B_1,\ldots,B_t,E_1,\ldots,E_k\}$, see Fig. 2.

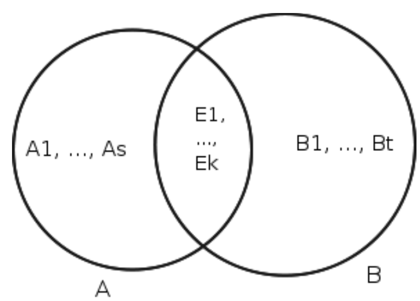

Figure 2: Two intersecting sets of attributes

In this case we propose to proceed in a way similar to the one described above. Thus we are looking at three disjoint sets,

(A_1 AND A_2 AND ... AND A_s) = A',

(B_1 AND B_2 AND ... AND B_t) = B',

and

(E_1 AND E_2 AND ... AND E_k) = E'.

The outcome is (A' OR B' OR E'). If a situation requires involvement of more than two intersecting sets of attributes will be hold in an analogous fashion.

In the case of $q > 1$ where the decision makers are with different degree of influence it is advisable that they work out a set of coefficients addressing these differences, i.e. $0 < w_i < 1, i = 1,...,q$, where

$$\sum_{i=1}^{q} w_i = 1.$$

The outcome is then ($A' \times w_1$ OR $B' \times w_2$) for two disjoint sets of attributes. The case with intersecting sets will be ($A' \times w_1$ OR $B' \times w_2$ OR E') since the attributes in E' are of interest to all decision makers.

4. Conclusions

Multi criteria decision making supported by vague sets has been presented. An approach applicable for involvement of several decision makers has been presented. The approach extends an existing one where a single decision maker is assumed.

References

[1] Calvo, T., Mesiar, R., Yager, R.: ’Quantitative weights and aggregation’, IEEE T. Fuzzy Systems, Vol. 12(1), (2004) pp. 62–69.
[2] Chen, S. M. and Tan, J. M.: Handling multicriteria fuzzy decision-making problems based on vague set theory, Fuzzy Sets and Systems, 67 (1994) pp. 163–172
[3] Chen, S. M.: Analyzing Fuzzy System Reliability Based on the Vague Set Theory, Computational Intelligence in Reliability Engineering (SCI) 40, (2007) pp. 347–362
[4] B. A. Davey, and H. A. Priestley, “Introduction to lattices and order”. Cambridge, Cambridge University Press, 2005.
[5] Dubois, D. and Prade, H.: A review of fuzzy sets aggregation connectives, Information Science, Vol. 36, (1985) pp. 85–121.
[6] Hong, D. H. , Chang-Hwan Choi, C-H.: Multicriteria fuzzy decision-making problems based on vague set theory, Fuzzy Sets and Systems 114, (2000) pp. 103–11
[7] Yager, R. and Rybalov, A.: ’Noncommutative self-identity aggregation’, Fuzzy Sets and Systems, Vol. 85, (1997) pp. 73–82.

Sylvia Encheva is professor in mathematics and informatics at Stord/Haugesund University College. Her research interests are within decision support systems, non-classical logics, and fuzzy systems.