Study on Bifurcation and Chaotic Motion of a Strongly Nonlinear Torsional Vibration System under Combination Harmonic Excitations

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Abstract

By using dissipative system Lagrange equation, the strongly nonlinear dynamic equation of torsional vibration system is deduced, which contains a class of square and cube nonlinear rigidity and combination harmonic excitations. Bifurcation characteristics of the strongly nonlinear system are analyzed in the autonomous and non-autonomous situations by means of singular point stability theory and singularity theory, respectively. The bifurcation diagram of system response corresponding to the change of torsional rigidity is derived by using numerical simulations, and evolution process of period, period doubling and chaotic motions is studied. Finally, chaotic motion is further verified by the maximum Lyapunov exponent, phase trajectory and Poincare map.

Keywords: *Strongly Nonlinear, Torsional Vibration, Bifurcations, Chaos*

1. Introduction

Torsional vibration system exists widely in rotating machinery equipment such as turbine generator, rolling mill and steam turbine. Torsional vibration may be due to torque fluctuations or due to unbalanced rotating parts or other mechanical reasons. Such vibrations, if not controlled may cause damage or destruction to the rotating shafts or their accessories. Torsional vibration has great influence on performance and the reliability of mechanical drive system. Therefore, torsional vibration instability mechanism and dynamics behaviors are the key issues to optimal design and vibration monitoring of system.

A lot of research on nonlinear torsional vibration system has been done in recent years[1-3]. The equilibrium stability, bifurcation and chaotic characteristics of several typical torsional vibration system were studied in[4-6]. Zhou[7] analysed the nonlinear gear meshing based on dynamics of gear system and the Hertz elastic theory, and the torsional vibration of the transmission system under speeding-up condition and comparisons with a real vehicle results were studied. M.S.tehrani et al[8]established the measurement model of cold tandem mill coupled torsional vibration system, and researched the influence of tension and rolling speed fluctuation of strip between frame on rolling mill drive system. Östman et al[9]studied the active torsional vibration control of reciprocating engines, and balanced the cylinder-wise torque contributions by utilizing the measured angular speeds of the crankshaft system. Jiang[10]developed a linear mathematical model of coupled drive system with multi-rotor and analyzed the vibration characteristics of multi-stage centrifugal pump. In [11], the authors studied the local dynamics near the Hopf bifurcation points with a direct linear time-delayed velocity feedback and the stability of trivial equilibrium is examined with the change of counting multiplicity of eigenvalue with positive real part. With precise symbolic computation and a completely mathematical analysis, Zhang[12] applied the normal form theory to investigate the Hopf bifurcation of the four dimensional autonomous hyperchaos and chaos system with whole parameter space completely.

Above papers better explained the vibration mechanism and dynamic characteristics of nonlinear system under the condition of weak nonlinear. However, the strongly nonlinear torsional vibration system is widespread in engineering, and its dynamic characteristics including bifurcation and chaos have received less attention. In this paper, the dynamics equation of strongly nonlinear torsional vibration system with a class of quadratic and cubic nonlinear rigidity and external excitation is established according to dissipative Lagrange equation. The bifurcation structures and chaotic behaviors of strongly nonlinear torsional vibration system are studied by theoretical analysis and numerical simulation. Some dynamical behaviors including period-m orbits, period-doubling and chaos are exhibited by bifurcation

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diagram, maximum Lyapunov exponent, phase trajectory and Poincare map. The paper provides a theoretical basis for further study of complex nonlinear dynamics behaviors and improving dynamic nature of mechanical drive systems.

2.Nonlinear Dynamic Equation of Torsional Vibration System

Torsional vibration system is widespread in engineering drive system. Considering a class of quadratic and cubic nonlinear rigidity, the kinetic and potential energy of two-mass system can be expressed as

$$E=\frac{1}{2}J_1\dot{\theta}_1^2+\frac{1}{2}J_2\dot{\theta}_2^2 \tag{1}$$

$$U=\frac{1}{2}a_1(\theta_1-\theta_2)^2+\frac{1}{3}a_2(\theta_1-\theta_2)^3+\frac{1}{4}a_3(\theta_1-\theta_2)^4 \tag{2}$$

Generalized damping force is

$$F_1^c=-c(\dot{\theta}_1-\dot{\theta}_2) \tag{3}$$

$$F_2^c=-c(\dot{\theta}_2-\dot{\theta}_1) \tag{4}$$

Generalized moment is

$$Q_j=\sum_{i=1}^{2}F_i^i\frac{\partial\theta_i}{\partial q_j}\quad(j=1,2) \tag{5}$$

Where J_i is inertia moment of concentrated mass , θ_i , $\dot{\theta}_i$ are rotation angle and angular velocity of concentrated mass, a_1 is linear torsional rigidity, a_2 , a_3 are nonlinear torsional rigidity, c is linear damping coefficient. $F_i^i=F_i+F_i^c$,where F_i is generalized external force, F_i^c is generalized damping force, q is generalized coordinate.

Substituting Eq. (3)and Eq. (4) into Eq. (5),yields generalized moment

$$Q_1=(F_1+F_1^c)\frac{\partial\theta_1}{\partial\theta_1}+(F_2+F_2^c)\frac{\partial\theta_2}{\partial\theta_1}=F_1-c(\dot{\theta}_1-\dot{\theta}_2) \tag{6}$$

$$Q_2=(F_1+F_1^c)\frac{\partial\theta_1}{\partial\theta_2}+(F_2+F_2^c)\frac{\partial\theta_2}{\partial\theta_2}=F_2-c(\dot{\theta}_2-\dot{\theta}_1) \tag{7}$$

Then substituting Eq. (6) and Eq. (7) into Lagrange equation

$$\frac{d}{dt}\frac{\partial E}{\partial\dot{q}_j}-\frac{\partial E}{\partial q_j}+\frac{\partial U}{\partial q_j}=Q_j\quad(j=1,2) \tag{8}$$

yields

$$J_1\ddot{\theta}_1+a_1(\theta_1-\theta_2)+a_2(\theta_1-\theta_2)^2+a_3(\theta_1-\theta_2)^3+c(\dot{\theta}_1-\dot{\theta}_2)=F_1 \tag{9}$$

$$J_2\ddot{\theta}_2+a_1(\theta_2-\theta_1)+a_2(\theta_2-\theta_1)^2+a_3(\theta_2-\theta_1)^3+c(\dot{\theta}_2-\dot{\theta}_1)=F_2 \tag{10}$$

Considering the variation of relative rotation angle in practical engineering, Eq.(9) minus Eq. (10),
yields

$$\ddot{\theta}_1-\ddot{\theta}_2+\frac{J_1+J_2}{J_1J_2}a_1(\theta_1-\theta_2)+\frac{J_2-J_1}{J_1J_2}a_2(\theta_1-\theta_2)^2+\frac{J_1+J_2}{J_1J_2}a_3(\theta_1-\theta_2)^3+\frac{J_1+J_2}{J_1J_2}c(\dot{\theta}_1-\dot{\theta}_2)=\frac{1}{J_1J_2}(J_2F_1-J_1F_2) \tag{11}$$

Suppose $x=\theta_1-\theta_2$, $\frac{J_1+J_2}{J_1J_2}a_1=\omega_0^2$, $\frac{J_2-J_1}{J_1J_2}a_2=k_1$, $\frac{J_1+J_2}{J_1J_2}a_3=k_2$, $\frac{J_1+J_2}{J_1J_2}c=\mu$, $\frac{1}{J_1J_2}(J_2F_1-J_1F_2)=F(t)$,
Eq. (11) can be simplified as

$$\ddot{x}+\omega_0^2x+k_1x^2+k_2x^3+\mu\dot{x}=F(t) \tag{12}$$

Eq. (12) is nonlinear dynamics equation of torsional vibration system, which is the basis for further study of dynamic behavior of torsional vibration system.

3.Bifurcation Characteristics of Strongly Nonlinear Torsional Vibration System

For the study of bifurcation characteristics of strongly nonlinear torsional vibration system, parameter ε is introduced, and ε is not be limited to a small parameter, then Eq. (12) can be written as

$$\ddot{x}+\omega_0^2x+\varepsilon k_1x^2+\varepsilon k_2x^3+\varepsilon\mu\dot{x}=\varepsilon F(t) \tag{13}$$

Eq. (13) is a strongly nonlinear dynamics equation of torsional vibration system, for ε is not a small parameter. Below, bifurcation analysis is carried out of autonomous system and nonautonomous system respectively.

3.1 Bifurcation Characteristics of Autonomous System

According to Eq.(13), when $F(t)=0$, autonomous equation of torsional vibration system is

$$\ddot{x}+\omega_0^2 x+\varepsilon k_1 x^2+\varepsilon k_2 x^3+\varepsilon\mu\dot{x}=0 \tag{14}$$

Eq. (14) can be reduced order for first-order equation

$$\begin{cases}\dot{x}=y\\ \dot{y}=-\omega_0^2 x-\varepsilon k_1 x^2-\varepsilon k_2 x^3-\varepsilon\mu y\end{cases} \tag{15}$$

Then Eq. (15) can be linearized as

$$\lambda^2+\varepsilon\mu\lambda+\omega_0^2=0 \tag{16}$$

at this point, singular point of system is $x=y=0$.The derivative operator of singular point is expressed as

$$\begin{vmatrix}0 & 1\\ -\omega_0^2 & -\varepsilon\mu\end{vmatrix}=0 \tag{17}$$

then characteristic equation is

$$\lambda^2+\varepsilon\mu\lambda+\omega_0^2=0 \tag{18}$$

and characteristic values are

$$\lambda_{1,2}=\frac{-\varepsilon\mu\pm\sqrt{\varepsilon^2\mu^2-4\omega_0^2}}{2} \tag{19}$$

According to singularity stability theory, Eq.(15) exists the following structures:

(1) When $\varepsilon^2\mu^2>4\omega_0^2$, $\omega_0^2<0$, characteristic values are two real roots of opposite sign, and singular point of system is saddle point.

(2) When $\varepsilon^2\mu^2>4\omega_0^2$, $\omega_0^2>0$, characteristic values are two real roots of the same sign. If $\varepsilon\mu<0$, characteristic values are two positive real roots, and singular point of system is unstable node; If $\varepsilon\mu>0$, characteristic values are two negative real roots, and singular point of system is stable node.

(3) When $\varepsilon^2\mu^2<4\omega_0^2$, characteristic values are two complex roots. If $\varepsilon\mu<0$, real part of characteristic value is positive, and singular point of system is unstable focus; when $\varepsilon\mu>0$, real part of characteristic value is negative, singular point of system is stable focus.

(4) When $\varepsilon^2\mu^2<4\omega_0^2$, $\varepsilon\mu=0$, characteristic values are two pure imaginary roots, and singular point of system is origin. at this time, oscillation curve is appeared, and Hopf bifurcation is occurred.

From the above stability analysis of singular points, when $\varepsilon\mu<0$, system is unstable; when $\varepsilon\mu>0$, system is stable; when $\varepsilon\mu=0$, system stability changes from unstable to stable.

3.2 Bifurcation Characteristics of Nonautonomous System

Suppose external disturbance excitation is a class of combination harmonic $F(t)=f_1\cos(\Omega t)+f_2\cos(2\Omega t)$, then nonautonomous equation of torsional vibration system can be written as

$$\ddot{x}+\omega_0^2 x+\varepsilon k_1 x^2+\varepsilon k_2 x^3+\varepsilon\mu\dot{x}=\varepsilon f_1\cos(\Omega t)+\varepsilon f_2\cos(2\Omega t) \tag{20}$$

Below, MLP method is employed for bifurcation analysis of nonautonomous system.

Introducing a new variable

$$\tau=\Omega t \tag{21}$$

substituting Eq.(21) into Eq.(20) yields

$$\Omega^2 x''+\omega_0^2 x+\varepsilon k_1 x^2+\varepsilon k_2 x^3+\varepsilon\Omega\mu x'=\varepsilon f_1\cos(\tau)+\varepsilon f_2\cos(2\tau) \tag{22}$$

where $x'=dx/d\tau$, $x''=d^2x/d\tau^2$,Ω^2 can be expand as power series of ε

$$\Omega^2=\omega_0^2+\varepsilon\omega_1+\varepsilon^2\omega_2+\cdots \tag{23}$$

a new parameter is defined

$$\sigma=\frac{\varepsilon\omega_1}{\omega_0^2+\varepsilon\omega_1} \tag{24}$$

such that

$$\varepsilon=\frac{\omega_0^2\sigma}{\omega_1(1-\sigma)} \tag{25}$$

and

$$\Omega^2=\frac{\omega_0^2}{1-\sigma}\left(1+\delta_2\sigma^2+\delta_3\sigma^3+\cdots\right) \tag{26}$$

$$\Omega=\omega_0\left[1+\frac{1}{2}\sigma+\left(\frac{3}{8}+\frac{\delta_2}{2}\right)\sigma^2+\cdots\right] \tag{27}$$

Expanding x into power series of σ, then substituting x into Eq. (22), comparing the coefficience of σ, and eliminating the secular term, one can obtain

$$\omega_1=\frac{3}{4}k_2\left(a_0^2+b_0^2\right)+\frac{2\mu\omega_0}{a_0^2-b_0^2}a_0 b_0-\frac{a_0 f_1}{a_0^2-b_0^2} \tag{28}$$

where a_0 is the initial condition of Eq.(22), $x_0(0)=a_0$,and b_0 is decided by the following equation

$$\mu\omega_0 b_0^2-f_1 b_0+\mu\omega_0 a_0^2=0 \tag{29}$$

Therefore, the new parameter σ will enable a strongly nonlinear system corresponding to ε be transformed into a small parameter system with respect to σ. Substituting Eq. (25)-(27) into Eq. (22), one can yield

$$\frac{\omega_0^2}{1-\sigma}\left(1+\delta_2\sigma^2+\delta_3\sigma^3+\cdots\right)x''+\omega_0^2x+\frac{\omega_0^2\sigma k_1}{\omega_1(1-\sigma)}x^2+\frac{\omega_0^2\sigma k_2}{\omega_1(1-\sigma)}x^3+$$
$$\frac{\omega_0^2\sigma}{\omega_1(1-\sigma)}\omega_0\mu\left[1+\frac{1}{2}\sigma+\left(\frac{3}{8}+\frac{\delta_2}{2}\right)\sigma^2+\cdots\right]x'=\frac{\omega_0^2\sigma}{\omega_1(1-\sigma)}f_1\cos\tau+\frac{\omega_0^2\sigma}{\omega_1(1-\sigma)}f_2\cos 2\tau \tag{30}$$

To study bifurcation characteristics of Eq.(30) , we use multiple scales method. Let x be expanded into power series of small parameter σ, namely

$$x=x_0\left(T_0,T_1\right)+\sigma x_1\left(T_0,T_1\right) \tag{31}$$

where $T_0=\tau$, $T_1=\sigma\tau$.

Suppose $\Omega=\omega_0$, substituting Eq.(31) into Eq.(30), then perturbation equations in this case are

$$D_0^2x_0+x_0=0 \tag{32}$$

$$D_0^2x_1+x_1=-2D_0D_1x_0+x_0-\frac{k_1}{\omega_1}x_0^2-\frac{k_2}{\omega_1}x_0^3-\frac{\mu\omega_0}{\omega_1}D_0x_0+\frac{f_1}{\omega_1}\cos T_0+\frac{f_2}{\omega_1}\cos 2T_0 \tag{33}$$

The solution of Eq. (32) is

$$x_0=A\left(T_1\right)e^{iT_0}+\bar{A}\left(T_1\right)e^{-iT_0} \tag{34}$$

where $\bar{A}\left(T_1\right)$ is the complex conjugate of $A\left(T_1\right)$.

Substituting equation (34) into (33), one can yield

$$D_0^2x_1+x_1=\left(-2iD_1A+A-\frac{3k_2}{\omega_1}A^2\bar{A}-i\frac{\mu\omega_0}{\omega_1}A+\frac{f_1}{2\omega_1}\right)e^{iT_0}+\mathrm{NST} \tag{35}$$

where NST indicates the other items which do not produce secular term.

Eliminating the secular term, one can obtain

$$-2iD_1A+A-\frac{3k_2}{\omega_1}A^2\bar{A}-i\frac{\mu\omega_0}{\omega_1}A+\frac{f_1}{2\omega_1}=0 \tag{36}$$

Setting $A=\frac{1}{2}r(T_1)e^{i\phi(T_1)}$, and substituting it into Eq.(36), and then separating real part and imaginary part, one can get average equations under polar coordinate

$$\frac{dr}{dT_1}=-\frac{\mu\omega_0}{2\omega_1}r-\frac{f_1}{2\omega_1}\sin\phi \tag{37}$$

$$r\frac{d\phi}{dT_1}=-\frac{r}{2}+\frac{3k_2}{8\omega_1}r^3-\frac{f_1}{2\omega_1}\cos\phi \tag{38}$$

Under stable condition, we set $\frac{dr}{dT_1}=\frac{d\phi}{dT_1}=0$, namely

$$\begin{cases}\frac{f_1}{2\omega_1}\sin\phi=-\frac{\mu\omega_0}{2\omega_1}r\\ \frac{f_1}{2\omega_1}\cos\phi=-\frac{1}{2}r+\frac{3k_2}{8\omega_1}r^3\end{cases} \tag{39}$$

eliminating $\sin\phi$ and $\cos\phi$, one can yield

$$\left(\frac{\mu\omega_0}{2\omega_1}r\right)^2+\left(\frac{3k_2}{8\omega_1}r^3-\frac{1}{2}r\right)^2=\left(\frac{f_1}{2\omega_1}\right)^2 \tag{40}$$

Eq.(40) is the bifurcation response equation of torsional vibration system under nonautonomous condition.

Setting $p=-\frac{8\omega_1}{3k_2}$, $q=\frac{16\left(\mu^2\omega_0^2+\omega_1^2\right)}{9k_2^2}$, $s=\frac{16f_1^2}{9\omega_1^2}$, Eq. (40) can be simplified to

$$G(r,s,p,q)=r^7+pr^5+qr^3-sr=0 \tag{41}$$

According to singularity theory, taking germ $g_0(r,s)=r^7-sr$, one can prove $G(r,s,p,q)$ is a universal unfolding of germ $g_0(r,s)=r^7-sr$ with unfolding parameters p 、q, and codimension is 2. To study the bifurcation topological structure of Eq. (41), and discuss the effect of unfolding parameters p, q on bifurcation diagram, we use transition set to decide qualitative behavior of bifurcation diagram when $G(r,s,p,q)$ is under small perturbation.

According to the definition of transition set, one can obtain $G_r=7r^6+5pr^4+3qr^2-s$, $G_s=-r$, $G_{rr}=42r^5+20pr^3+6qr$ .when $G=G_r=G_s=0$, system has a bifurcation point set $B_0\left(Z_2\right)=\varphi$ (empty set), $B_1\left(Z_2\right)=\varphi$ (empty set); when $G=G_r=G_{rr}=0$,system has a lag point set $H_0\left(Z_2\right)=\{q=0\}$, $H_1\left(Z_2\right)=\left\{q=p^2/3,p\le 0\right\}$; at the same time system has a double limit point set $D(Z_2)=\left\{q=p^2/4,p\le 0\right\}$ and transition set $\Sigma=B_0\cup B_1\cup H_0\cup H_1\cup D$.

4. Numerical study of chaotic motion

In order to study the chaotic motion evolution process of strongly nonlinear torsional vibration system, different kinds of numerical methods are applied such as bifurcation diagram, maximum Lyapunov exponent, phase trajectory and Poincare map. These methods are all very useful tools

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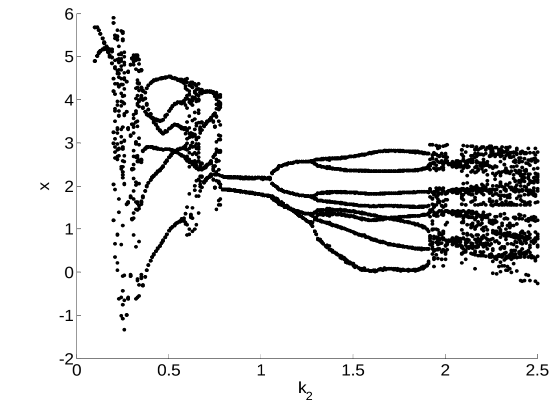
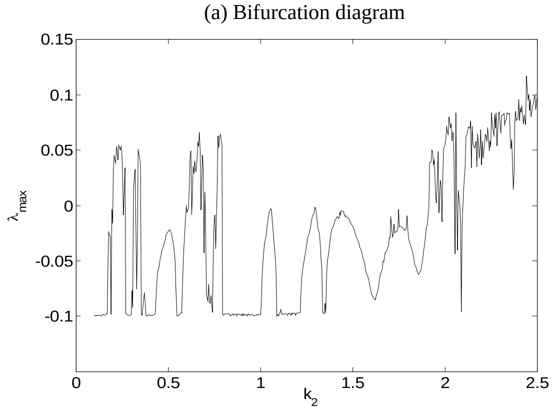
for examing chaotic properties and exploring chaotic attractors.

Fourth-order Runge-Kutta method is employed to numerical study of torsional vibration system. We fix $\omega_0 = 1$, $\Omega = 1$, $\mu = 0.1$, $\varepsilon = 2$, $k_1 = 0.1$, $f_1 = 5$, $f_2 = 10$, and let k_2 change in a wide range. The bifurcation diagram of Eq.(20) in (x, k_2) plane is shown in Fig.1(a) and the maximum Lyapunov exponent corresponding to Fig.1(a) is shown in Fig.1(b). From Fig.1(a), we can see that strongly nonlinear torsional vibration system exhibits periodic and chaotic behaviors when k_2 changes. The maximum Lyapunov exponent given by Fig.1(b) can be convince of occurrence of chaotic motion.



(a) Bifurcation diagram

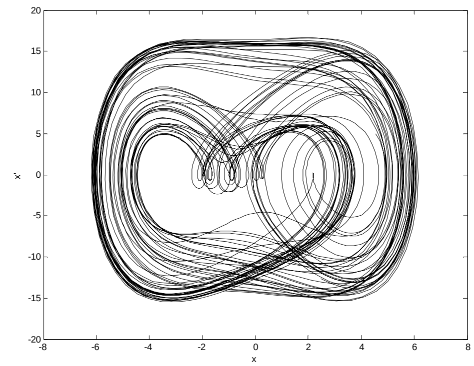


(b) Maximum Lyapunov exponent

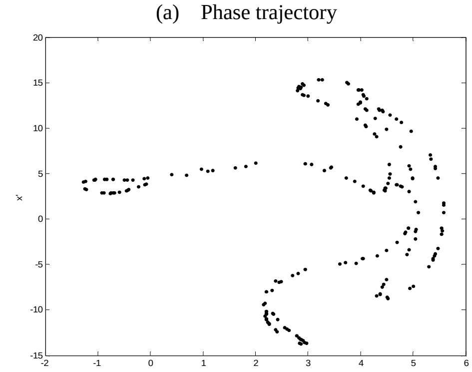
Fig.1 Bifurcation diagram and Maximum Lyapunov exponent

In Fig.1, periodic and chaotic motion are clearly visible. When torsional rigidity k_2 is small, system response is period-2 motion. With the increase of torsional rigidity, system jumps into chaotic motion. When $k_2 = 0.4$, system response is period-6 motion and then system jumps into chaotic motion. With further increase of torsional rigidity, system finally enters chaotic state after period-doubling bifurcation. From Fig.1, we can see that periodic and chaotic motion interval occur with the increase of torsional rigidity.

In order to further describe chaotic characteristics of torsional vibration system, phase trajectory and chaotic attractors are shown in Fig.3, Fig.4 and Fig.5 under $k_2 = 0.25, 0.65, 2.35$, respectively. We can see that Phase trajectory repeatedly winding in enclosed area but not closed, and Poincare section has the obvious fractal structure.

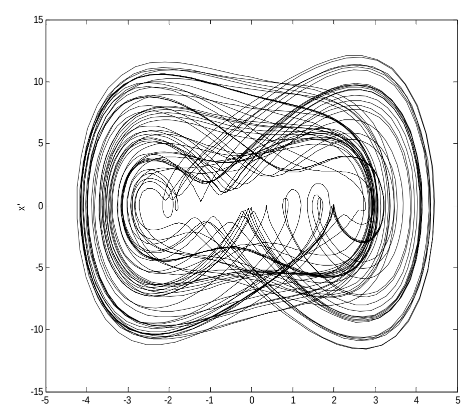


(a) Phase trajectory

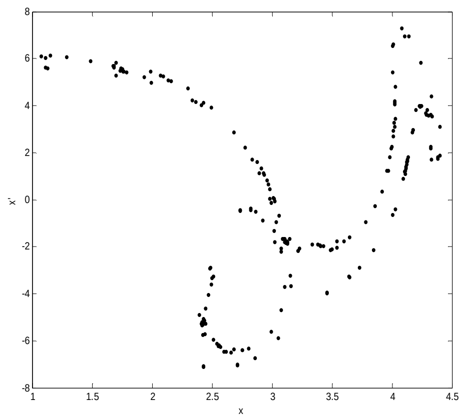


(b) Poincare map

Fig.2 Phase trajectory and Poincare map when k_2=0.25

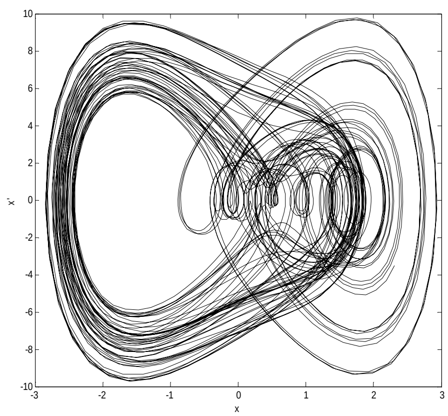


(a) Phase trajectory

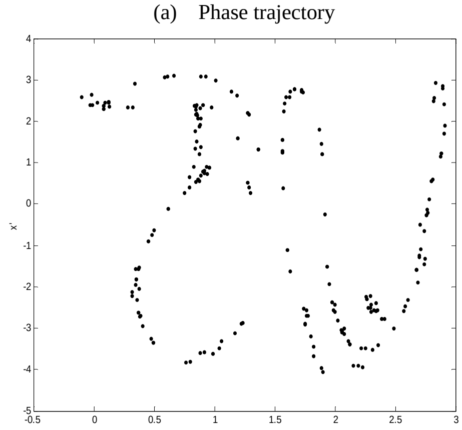


(b) Poincare map

Fig.3 Phase trajectory and Poincare map when k_2=0.65



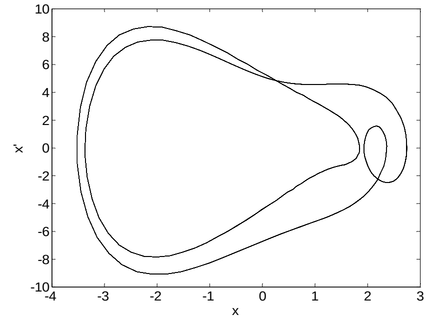
(a) Phase trajectory



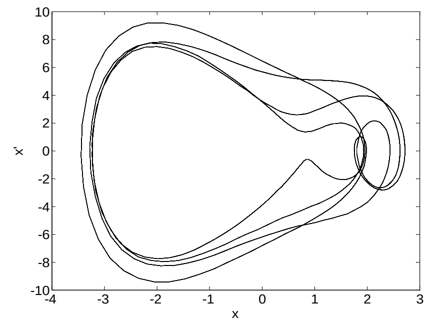
(b) Poincare map

Fig.4 Phase trajectory and Poincare map when k_2=2.35

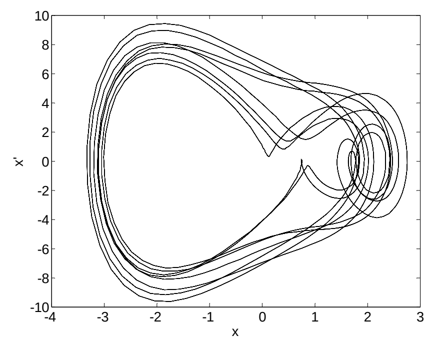
It can be seen from Fig.1 that system finally enters chaotic motion usually through period doubling, while period doubling is the most commonly known route to chaos at present. Phase trajectory and Poincare map are applied to depict the period doubling bifurcation motion in Fig.5 and Fig.6 respectively. When response is period-m motion, phase trajectory for m closed curves, and poincare map for m fixed points.



(a) Period-2 motion

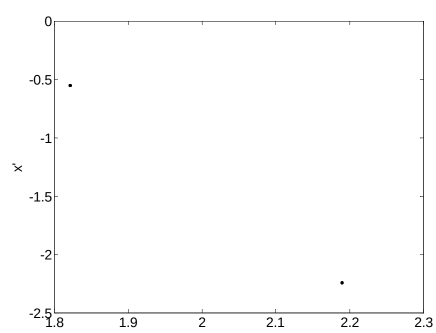


(b) Period-4 motion

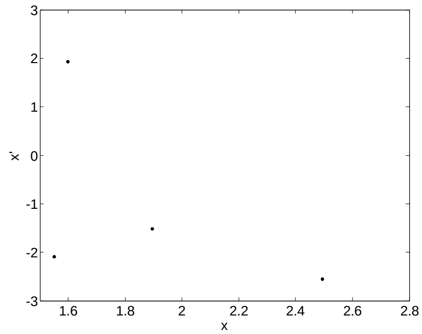


(c) Period-8 motion

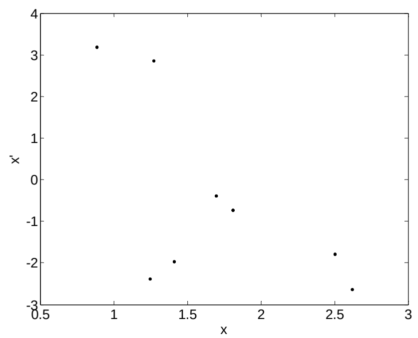
Fig.5 Phase trajectory of period doubling



(a) Period-2 motion



(b) Period-4 motion



(c) Period-8 motion

Fig.6 Poincare map of period doubling

5. Conclusion

Torsional vibration characteristics are important information for rotating machinery design and control. In this paper, the dynamic performance of nonlinear torsional vibration system has been studied by theoretical analysis and numerical simulation. The results are as follows:

(1) The strongly nonlinear dynamic equation of torsional vibration system is deduced by using dissipative system Lagrange equation, which contains a class of square and cube nonlinear rigidity and combination harmonic excitations.

(2) Bifurcation characteristics of the strongly nonlinear torsional vibration system are analyzed in the autonomous and nonautonomous situations, and bifurcation conditions of torsional vibration system are given.

(3) When system parameters and initial conditions are appropriately chosen, system bifurcation diagram is made by fourth-order Runge-Kutta method. It is found that with the increase of torsional rigidity, periodic motion and chaotic motion intervals occurs in torsional vibration system, and ultimately system enters into chaos after period-doubling bifurcation. Different shapes of chaotic attractors and period-doubling bifurcation motions are obtained by using phase trajectory and Poincare map. These results provide a reference for further studying complex nonlinear dynamics behaviors and improving dynamic nature of mechanical drive systems.

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