Robust controller design of sliding mode control for a class of nonlinear system with parametric uncertainty via Takagi Sugeno Fuzzy Model

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Abstract

This paper presents the studied of robust sliding mode control for a class of nonlinear system with parametric uncertainty based on Takagi Sugeno (T-S) fuzzy model approach. The method utilized the concept of state feedback control and Lyapunov functional approach to determine the condition of asymptotically stability. The condition for asymptotically stability is presented in the forms of linear matrix inequalities (LMI), thus the global asymptotically stability of robust sliding mode control via Takagi Sugeno fuzzy model for a class of nonlinear system with parametric uncertainty is determined. An example is presented to shows the feasibility and the functionality of the proposed method.

Keywords: *nonlinear system, T-S Fuzzy Model, LMI, Sliding Mode Control, parametric uncertainty..*

1. Introduction

It's commonly known that a highly complex control system often consist uncertainty and nonlinearity characteristics which are not easy to presents in mathematical model. Furthermore the mathematical approach of nonlinear systems is less cater and understood by the used of classical linear systems approach. Ever since Takagi and Sugeno presented an ingenious method to represents a nonlinear systems as a group of linear time invariant (LTI) model mixed with nonlinear functions to form a fuzzy model representation [1]-[2], thus the used of Takagi-Sugeno (T-S) fuzzy model approach has been applied in numerous applications and attracted numerous studied [3]-[8]. This is due to its efficiency to control highly and complex nonlinearity in the system. Utilizing the state feedback approach, the same principles is applied to parallel distributed compensation (PDC) which is used to interpolated with the feedback gains in each of the determined Takagi-Sugeno (T-S) fuzzy rules [3]. Furthermore the global linearized fuzzy model is made up from set of local linearized models which are derived from set of membership functions.

Over the years there has been a lot of studied done to cater the parametric uncertainties of nonlinear system as well as its stability issues. There are studied that proposed an analysis method of linearization control approach which derived from fuzzy models. Nevertheless, each of this stability approaches still relying on determining a common positive definite matrix P in which often obtained from a derived condition of linear matrix inequalities (LMIs) terms. [9]-[15]. Furthermore the task of determined the common positive definite matrix is tedious and not easy if the control system consisting a great number of fuzzy rules.

Sliding mode control (SMC) systems theory have been widely been studied to cater the nonlinear dynamic control problems arise from uncertainty parameter, time varying delay and external disturbances [16]-[20]. The main concept of SMC is designing a control law which guides the system state to reach and remain on the switching surface.

In this paper, we utilized the concept of state feedback and Lyapunov functional approach to obtain a sufficient stability condition for designing a robust sliding mode plane. By our approach, it can be seen that the derivation of the controller is straight forward and the approach of finding the required parameters are reduced to solving linear matrix inequalities (LMIs), which are often solved by the MATLAB/Simulink LMI toolbox. We proposed yet another alternative by means of improving the T-S fuzzy model based control for a class of nonlinear system with parametric uncertainty. To be precise, we present a systematic design procedure of T-S fuzzy model based control with guaranteed stabilization for a class of nonlinear system with parametric uncertainty. A numerical example is simulated using the proposed algorithms to show the effectiveness and the feasibility of the controller.
This paper is presented and organized as follows. Section 2 presents the problem formulation of this intended paper. Section 3 presents the main results of the studied of robust

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sliding mode control for a class of nonlinear system with parametric uncertainty via Takagi Sugeno fuzzy model. Section 4 presents the numerical example of the control approach with the simulation results and analysis. Lastly, the conclusion is briefed in the section 5
.

2. Problem Formulation

Consider a class of nonlinear system with parametric uncertainty represented by Takagi and Sugeno fuzzy model:

Model Rule i:
IF $z_i(t)$ is M_i^1 and … and $z_p(t)$ is M_p^i
THEN

$$\dot{x}(t) = \left(A_i + \Delta A_i x(t)\right) + B_i u(t) + f_i(x,t) \qquad (1)$$

where $i = 1,2 \ldots, r$. r is the number of IF-THEN rules. $x(t) \in R^n$ is the state vector and $u(t) \in R^m$ is the input vector and $y(t) \in R^q$ is the output vector. and $B_i \in R^{n\times m}$ and $A_i \in R^{n\times n}$ are the system input matrices and the systems matrices and respectively. ΔA_i represents the parameters uncertainties and $f_i(x,t)$ is bounded external disturbance. $M_j^i, j = 1,2 \ldots, p$ is denoted as the j^{ith} fuzzy set for the i^{ith} rule and $z_1(t), \ldots, z_p(t)$ are the known variables functions of state variables. $w_j^i(z_j)$ is denoted as the membership function for the j^{ith} fuzzy set M_j^i in the i^{ith} and $w_i(z(t) = \prod_{j=1}^{p} M_j^i\,(z(t))$ $i = 1, \ldots, r$

Let's denotes the pair of $x(t), u(t)$, as the fuzzy systems output represented as [15]:

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} w_i\,(t)\left(\left(A_i + \Delta A_i x(t)\right) + B_i u(t) + f_i(x,t\right)}{\sum_{i=1}^{r} w_i\,(t)} \qquad (2)$$

$z(t)$ is the premise vector for $z(t) = z_1, z_2 \cdots z_p$

As the $z(t)$ is regards as the combination of linear and state vector, thus the weight function can be represented as:

$$h_i\big(z(t)\big) = \frac{w_i(z(t)}{\sum_{i=1}^{r} w_i\,(t)} \quad , i = 1, \cdots, r \qquad (3)$$

for all *t*.
$w_j^i\big(z_j\big)$ denoted as the membership grade for $z_j(t)$ in M_j^i from Eq.(2), noting

$$\begin{cases} \sum_{i=1}^{r} h_i\big(z(t)\big) = 1 \\ h_i\big(z(t)\big) \geq 0 \end{cases} \qquad i = 1, \cdots, r$$

Therefore the Takagi Sugeno fuzzy model can be represents as:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i\,(t)\,\left(\left(A_i + \Delta A_i\, x(t)\right) + B_i\, u(t) + f_i(x,t\right) \qquad (4)$$

and the output of Takagi Sugeno fuzzy model can be represents as:

$$y(t) = \sum_{i=1}^{r} h_i\,(t)\; C_i\, x(t) \qquad (5)$$

Before proceeding, the following assumptions are needed.
Assumption 1: There exists $K \in R^{m\times n}$ for the pair $(A_i\,, B)$ is stabiliseable such that $\overline{A_i} = A_i - BK$ is stable.
Assumption 2: The uncertain matrices $\Delta A_i\,(t)$ is denoted in the form of:

$$\Delta A_i \;= M_i F_i(t) E_i \qquad (6)$$

and are norm-bounded, where M_i and E_i are known constant matrices of appropriate dimension and $F_i(t)$ satisfying:

$$F_i^T(t) F_i(t) \leq I, \quad \forall t \qquad (7)$$

where $F_i(t)$ are Lebesgue measureable.
Assumption 3: $B_1 = B_2 = \cdots = B_n := B$ and B is a full column rank matrices.
Assumption 4: The external disturbances satisfying

$$f_i(x,t) = B\overline{f_i}(x,t), \qquad (8)$$

$$\left\|\overline{f_i}(x,t)\right\| \leq \delta(t), \qquad (9)$$

First step in designing the sliding mode control is to determine the sliding mode plane. Therefore we choose the sliding mode plane to be:

$$S = B^T P x(t) \qquad (10)$$

where $P \in R^{m\times n}$ is a determined positive definite matrices.
The main purpose of this section is to predetermine the stabilization for a class of nonlinear Takagi Sugeno fuzzy model with parametric uncertainty into two parts. The first part is to derive an appropriate sliding mode plane that guaranteed the system trajectories from any given initial states converges to sliding mode plane within certain time. Second part is to derive a sufficient stability conditions to guarantee an asymptotically stability of the control system. In order to obtain the main results, we recall this lemma.

Lemma 1: Given matrices M and E with appropriates dimensions and matrices $F(t)$ satisfying $F^T(t)F(t) \leq I$, then for any scalar $\varepsilon > 0$, then the following inequality holds:

$$MFE + (MFE)^T \leq \varepsilon HH^T + \varepsilon^{-1} E^T E \qquad (11)$$

3. Main Results

The researches of derived a sliding plane using Lyapunov functional approach for SMC theory has been studied in [16]. Therefore we applied this concept of approach for a class of nonlinear with parametric uncertainty via Takagi Sugeno fuzzy model.

Theorem 1: Noting on the assumptions 1-4, the trajectories for a class of nonlinear Takagi Sugeno fuzzy system with parametric uncertainty at any given initial states, are brought within the sliding plane in certain time with the given control as:

$$u(t) = u_{eq} + u_n \quad (12)$$

where u_{eq} is the equivalent control described as:

$$u_{eq} = -\sum_{i=1}^{n} h_i\left(z(t)\right)(B^T PB)^{-1}[B^T PA_i x(t)] \quad (13)$$

and u_n is the switching control described as:

$$u_n = -\sum_{i=1}^{n} h_i\left(z(t)\right)\{(B^T PB)^{-1}[\|B^T PH_i\| \cdot \|E_i x(t)\| + \|B^T PB\| \cdot \delta + \varepsilon_0] sgn(s)\} \quad (14)$$

where ε_0 is a small positive constant.

Proof: Consider the positive definite Lyapunov function for all $S \neq 0$.

$$V = 0.5 S^T S \quad (15)$$

Thus derivative of the Lyapunov functional Eq.(15) along the trajectory of the system Eq.(2) is

$$\begin{aligned}\dot{V} &= S^T \dot{S} = S^T B^T P \dot{x}(t) \\ &= \sum_{i=1}^{n} h_i(z(t))\, S^T B^T P[\left(A_i + \Delta A_i(t)\right)x(t) \\ &\quad + B_i u(t) + f_i(x,t)]\end{aligned}$$

Substituting Eq.(12) into the above equation, yields

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{n} h_i(z(t))\, S^T B^T P[\left(A_i + \Delta A_i(t)\right)x(t) \\ &\quad + B_i u(t) + f_i(x,t)] \\ &= \sum_{i=1}^{n} h_i(z(t))\, [S^T B^T P \Delta A_i x(t) + S^T B^T PB \bar{f}_i(x,t) \\ &\quad + S^T B^T PB u_n]\end{aligned}$$

Considering Eq.(6)-Eq.(9) and Eq.(3), $\dot{V}$ can be expressed as

$$\begin{aligned}\dot{V} &\leq \sum_{i=1}^{n} h_i(z(t))\, \|S^T\| [\|B^T PM_i\| \cdot \|E_i x(t)\| \\ &\quad + \|B^T PB_i\| \cdot \delta_f(t)] + S^T B^T B u_n \\ &= \sum_{i=1}^{n} h_i\left(z(t)\right) \varepsilon_0 S^T sgn(S) \\ &= \varepsilon_0 \|S\| \\ &\leq 0\end{aligned}$$

This proved that all the trajectories will arrive at within the sliding plane in a certain time. Therefore the proof is obtained.

The next part is to derive the switching control so that all the trajectories stays in the sliding plane once in reached and robust stable even with the appearance of disturbance. Thus yields the following.

Theorem 2: Based on assumptions 1-4, the Takagi Sugeno fuzzy sliding mode control system Eq.(1) is asymptotically stable in Eq.(10) with $P = Q^{-1}$ under the control Eq.(12) if there exist symmetric positive definite defined by matrices $Q > 0, J > 0$, general matrix L and a scalar $\varepsilon > 0$ that satisfied the following LMIs.

$$\begin{bmatrix} \Phi & 0 \\ * & -J \end{bmatrix} < 0, \quad i = 1,2 \cdots n \quad (16)$$

where $\Phi = A_i Q + Q A_i^T - B_i L_i - L_i^T B_i^T + J + \varepsilon M M_i^T + \varepsilon^{-1} Q E_i^T E_i Q$

Proof: Consider the Lyapunov function

$$V(x,t) = x^T P x + \int_0^t x^T(s) R x(s) ds \quad (17)$$

where P and Q are symmetric positive definite matrices.

Let us consider the controller given by Eq.(12) as

$$u(t) = -Kx + v(t) \quad (18)$$

where $v(t) = Kx + u_{eq} + u_n$. Substituting Eq.(18) into Eq.(1), the closed loop T-S fuzzy system can be described as:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{n} h_i\left(z(t)\right)[\left(\bar{A}_i + \Delta A_i(t)\right)x(t) \\ &\quad + B_i v(t) + f_i(x,t)] \end{aligned} \quad (19)$$

where

$$\bar{A}_i = A_i - BK \quad (20)$$

$$\begin{aligned}\dot{V}(x,t) &= 2x^T(t)\, P\dot{x}(t) + x^T(t) R x(t) \\ &= \sum_{i=1}^{n} \{2x^T P \left[\left(\bar{A}_i + \Delta A_i(t)\right)x(t)\right] \\ &\quad + 2x^T PB(v + \bar{f}_i(x,t) + x^T(t) R x(t)\end{aligned}$$

Refer to Eq.(10), $\dot{V}(x,t)$ is reduced to

$$\dot{V}(x,t) = \sum_{i=1}^{n} h_i\, [x(t)]^T M [x(t)] \quad (21)$$

once the trajectories of the system reach the sliding mode plane.

where

$$M = \begin{bmatrix} Y & 0 \\ * & -R \end{bmatrix} \quad (22)$$

with $Y = (A_i + \Delta A_i)^T P + P(A_i + \Delta A_i) + R$

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Solving Eq.(22) by multiplying with X^T where $X = diag\{P^{-1}, P^{-1}\}$ respectively and defining $P^{-1} = Q, J = QRQ$, yeilds:

$$X^T M X = \begin{bmatrix} Y & 0 \\ * & -J \end{bmatrix} = \begin{bmatrix} Y & 0 \\ * & -J \end{bmatrix} + T \qquad (23)$$

where

$$T = \begin{bmatrix} I \\ 0 \end{bmatrix} Q \Delta A_i^T [I \quad 0] + \begin{bmatrix} I \\ 0 \end{bmatrix} Q \Delta A_i [I \quad 0] \qquad (24)$$

By lemma 1 and Eq.(6), yields the following inequalities for scalar $\varepsilon > 0$.

$$T \leq \begin{bmatrix} \varepsilon M_i M_i^T + \varepsilon^{-1} Q E_i^T E_i Q & 0 \\ 0 & 0 \end{bmatrix} \qquad (25)$$

Substituting Eq.(25) into Eq.(23) and if $X^T M X < 0$, therefore $M < 0$ in addition $\dot{V}(x,t) < 0$.

$$X^T M X \leq \begin{bmatrix} Y & 0 \\ * & -J \end{bmatrix} \qquad (26)$$

Then substituting $L = KQ$ in Y, the conclusive LMI Eq.(16) is obtained. Thus the proof is completed.

4. Numerical Example

In this example, consider an uncertain nonlinear system describes by T-S fuzzy model Eq.(1) with membership function given as:

$$M_1(x_2(t)) = 1 - \frac{x_2^2(t)}{2.25} \quad ; \quad M_2(x_2(t)) = \frac{x_2^2(t)}{2.25}$$

and the fuzzy model rules as:

Model Rule 1: IF x_2 is M_1, THEN

$$\dot{x}(t) = (A_1 + \Delta A_1 x(t)) + B_1[u(t) + f_1(x,t)]$$

Model Rule 2: IF x_2 is M_2, THEN

$$\dot{x}(t) = (A_2 + \Delta A_2 x(t)) + B_2[u(t) + f_2(x,t)]$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ 17.2941 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ 12.6305 & 0 \end{bmatrix}$$

$$B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$\Delta A_1 = \Delta A_2 = MF(t)E,$$

$$M = [0\ 0.1125]^T, \quad E = [0\ 1]$$

$$F(t)^T F(t) \leq I, f_i(x,t) = 0.2 \sin(x_1 + x_2)$$

$$\text{for } i = 1, \cdots, r$$

By using LMI toolbox, solving the LMI Eq.(16) there exists a feasible solution with the symmetric positive definite P as:

$$P = \begin{bmatrix} 6.4996 & 2.9657 \\ 2.9657 & 1.9853 \end{bmatrix}$$

From Theorem 1 and Theorem 2, the given class for uncertain nonlinear system is stable and therefore asymptotically stability is guaranteed.
The simulation results with given initial $x(0) = [-1.2; 1.4]$ are shown in Figure 1-3.

Remarks 1: The chattering control effect can be alleviated by substituting $sgn(S)$ with $S/(\|S\| + 0.1)$ as method in [21].

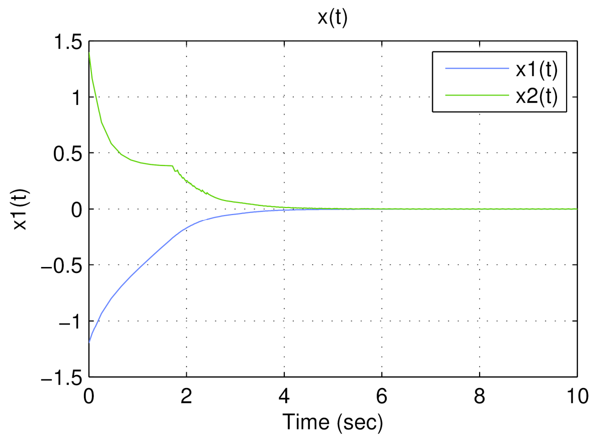

Fig. 1 System state under proposed SMC

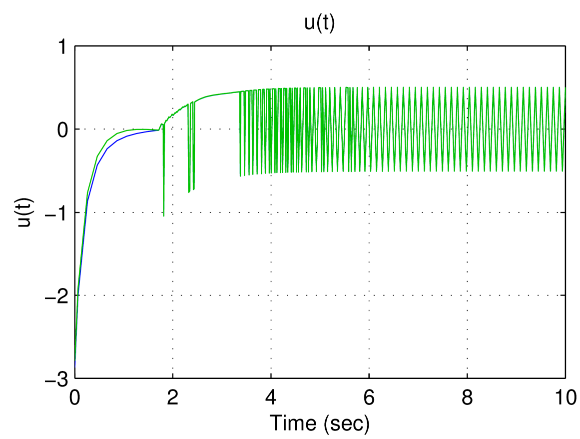

Fig.2 The proposed SMC

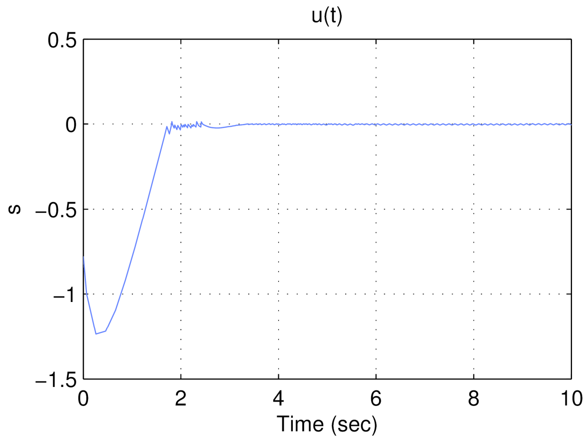

Fig. 3 The sliding response of the system.

5. Conclusions

In this paper we present the studied of sliding mode control for a class of nonlinear system with parametric uncertainty via Takagi Sugeno fuzzy model. The approach design is conceptually simple thus reduce the conservatism and computational efforts even to complex uncertain nonlinear system. Furthermore, the analysis for system stability and controller design approach is formulated into Linear Matrix Inequality (LMI) terms. Finally, an example used to show the feasibility and effectiveness of this control scheme.

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