The Gray images of linear codes over the ring $F_3+vF_3+v^2F_3$

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Abstract

In this work, we focus on the Gray images of the linear codes over the ring $R=F_3+vF_3+v^2F_3(v^3=1)$, which is a finite chain ring. Firstly, we give the generator matrix of the linear code and its dual code over the ring $F_3+uF_3+u^2F_3$. Secondly, we define an isomorphism from R to S and obtain the generator matrix of the linear code and its dual code over the ring R. Then, we define a Gray map ψ from R^n to F_3^{3n}, and obtain Gray image $\psi(C)$ from the generator matrix of the linear code C over the ring R. Finally, we prove that the Gray images $\psi(C)$ of cyclic codes C are quasi-cyclic codes over F_3.

Keywords: Linear codes, Generator matrix, Gray image, Dual code

1. Introduction

The study of linear codes and their Gray images over finite rings has obtained many useful results in coding theory[1-6]. The two main classes of rings that have been studied are Galois rings and rings of the $F_{2^m}+uF_{2^m}$ and some variations of these [1][2]. Codes over F_3+uF_3 were studied and improvements to the bounds on ternary linear codes [3]. In 2010, linear codes and cyclic codes over $F_2+uF_2+vF_2+uvF_2$ were studied by Bahattin.Yildiz and S.Karadeniz[7][8]. Linear codes and cyclic codes over the ring F_2+vF_2 were studied by Zhu Shixin, Wangyu and Shi Minjia[9][10] where the ring F_2+vF_2 is not a finite chain ring, In order to popularize the conclution of the coding theory over F_2+vF_2, we study the coding theory over the ring $F_3+vF_3+v^2F_3$ in this paper.

After presenting some notations and properties about linear codes , cyclic codes and quasi-cyclic codes over the finite chain ring $R=F_3+vF_3+v^2F_3$ in section 2. We study the structure of the linear code over the ring R and obtain the generator matrix of the linear code C and its dual code $C^{\perp}$ in section 3. In section 4, we study the gray image of the linear code and the cyclic code over the ring R.

2. Basic Concepts of the Codes over the Ring $F_3+vF_3+v^2F_3$

Let $R=\{a+bv+cv^2 \mid a,b,c\in F_3\}$, where $v^3=1$. Note that R is a finite chain ring with characteristic 3. The ideals can be listed as:

$$<0>\subseteq<(v+2)^2>\subseteq<v+2>\subseteq<1>=R,$$

Where

$$<(v+2)^2>=\{0,1+v+v^2,2+2v+2v^2\}$$

And

$$<v+2>=\{0,1+v+v^2,2+2v+2v^2,1+2v,1+2v^2,2+v,2+v^2,v+2v^2,2v+v^2\}.$$

$<2+v>$ is the uniquely maximal ideal of the ring R. The zero divisors in R are all in $<2+v>$. It is obvious that $2+v$ is a nilpotent of R with nilpotency 3. Let $R^*=R-<2+v>$, we can see that R^* consists of all units in R.

A linear code over the ring R of length n is an R-submodule of R^n. For any $x=(x_1,x_2,...,x_n)$,

$y=(y_1,y_2,...,y_n)\in R^n$, the inner product of x,y is defined as the following :

$$<x,y>=x_1y_1+x_2y_2+...+x_ny_n.$$

Let C be a linear code of length n over R, then we can prove that $C^\perp=\{x\,|<x,y>=0,\forall y\in C\}$ is also a linear code over R of length n. We call $C^\perp$ to be the dual code of C.

A cyclic code of length n over R is a linear code with the property that if $(c_0,c_1,\cdots,c_{n-1})\in C$ then

$$T(c_0,c_1,\cdots,c_{n-1})=(c_{n-1},c_0,\cdots,c_{n-2})\in C.$$

A k-quasi-cyclic code of length kn over R is a linear code with the property that if $(c_0,c_1,\cdots,c_{n-1})\in C$ then

$$T^k(c_{11},c_{12},\cdots,c_{1k},c_{21},c_{22},\cdots,c_{2k},\cdots,c_{n1},c_{n2},\cdots,c_{nk})$$
$$=(c_{1k},c_{11},\cdots,c_{1,k-1},c_{2k},c_{21},\cdots,c_{2,k-1},\cdots,c_{nk},c_{n1},\cdots,c_{n,k-1})\in C.$$

3. The structure of the linear code over the ring $F_3+vF_3+v^2F_3$

Let $\tilde{C}$ and C are all linear codes over the finite chain ring of length n. If the code C can be transformed to $\tilde{C}$ by the transformation of coordinates, we call C permutation-equivalent to $\tilde{C}$.

Lemma 1 Let

$$S=F_3+uF_3+u^2F_3=\{a+bu+cu^2\,|a,b,c\in F_3\},$$

Where $u^3=0$. Note that S is a finite chain ring with characteristic 3. Any linear code C of length n over the ring S is permutation-equivalent to a code with generator matrix of the form:

$$G=\begin{pmatrix} I_{k_1} & A_1 & A_2 & A_3 \\ 0 & uI_{k_2} & uA_{11} & uA_{12} \\ 0 & 0 & u^2I_{k_3} & u^2A_{22} \end{pmatrix}_{k\times n} \quad \cdots\cdots(1)$$

Where I_{k_1},I_{k_2},I_{k_3} are all unit matrixes with order k_1,k_2,k_3 respectively. Let $k=k_1+k_2+k_3$, where $A_i=B_{i1}+uB_{i2}+u^2B_{i3}(i=1,2,3)$, and A_{11},A_{12},A_{22}, $B_{i1},B_{i2},B_{i3}(i=1,2,3)$ are matrixes over the ring F_3. Then $|C|=3^{3k_1+2k_2+k_3}$.

Proof. Let $G_1=(g_{ij})_{k\times n}$ be the generator matrix of the linear code C over S.
If there exist invertible elements in G_1, by applying row transformation to G_1, we can transform the first column of the matrix G_1 to $(1,0,\cdots,0)^T$ and transform G_1 to G_2; Removing the first row and first column of G_2, if there also exist invertible elements in G_2, then, using the same method we can transform the second column of the matrix G_2 to $(0,1,\cdots,0)^T$ and also transform G_2 to G_3; After k_1 steps transformation, we can obtain the following matrix:

$$G_{k_1+1}=\begin{pmatrix} I_{k_1} & M_1 \\ 0 & M_2 \end{pmatrix},$$

Where I_{k_1} is a unit matrix with order k_1, M_1,M_2 are matrixes over the ring S, and there is not invertible elements in M_2;

Because there are not invertible elements in M_2, so M_2 is a matrix over uS. Then there exists a matrix $\tilde{M}_2$ over S such that $M_2=u\tilde{M}_2$. Using the similar method of (1), after applying k_2 steps row transformation to G_{k_1+1}, we can obtain the following matrix:

$$G_{k_1+k_2+1}=\begin{pmatrix} I_{k_1} & A_1 & M_3 \\ 0 & uI_{k_2} & M_4 \\ 0 & 0 & M_5 \end{pmatrix},$$

Where I_{k_2} is a unit matrix with order k_2, M_5 is a matrix over u^2S;

Applying k_3 steps row transformation to $G_{k_1+k_2+1}$, we can obtain the following matrix:

$$G=\begin{pmatrix} I_{k_1} & A_1 & A_2 & A_3 \\ 0 & uI_{k_2} & uA_{11} & uA_{12} \\ 0 & 0 & u^2I_{k_3} & u^2A_{22} \end{pmatrix}_{k\times n},$$

Where $I_{k_1}, I_{k_2}, I_{k_3}$ are all unit matrixes with order k_1, k_2, k_3 respectively. Let $k = k_1 + k_2 + k_3$, where $A_i = B_{i1} + uB_{i2} + u^2 B_{i3} (i = 1,2,3)$, and A_{11}, A_{12}, A_{22}, $B_{i1}, B_{i2}, B_{i3} (i = 1,2,3)$ are matrixes over the ring F_3.

From the above, we can prove the theorem.

Similar to the literature [6], the following lemma can be easily obtained.

Lemma 2 If C is an arbitrary linear code of S, then the generator matrix of the dual code $C^{\perp}$ is:

$$H = \begin{pmatrix} F & A_{12}^T + A_{22}^T A_{11}^T & A_{22}^T & I_{n-k} \\ u(A_2^T + A_{11}^T A_1^T) & uA_{11}^T & uI_{k_3} & 0 \\ u^2 A_1^T & u^2 I_{k_2} & 0 & 0 \end{pmatrix}_{(n-k_1)\times n} \cdots\cdots(2)$$

Where $F = A_{22}^T (A_2^T + A_{11}^T A_1^T) + A_{12}^T A_1^T + A_3^T$,

$A_i = B_{i1} + vB_{i2} + v^2 B_{i3} (i = 1,2,3)$, and A_{11}, A_{12}, A_{22},

$B_{i1}, B_{i2}, B_{i3} (i = 1,2,3)$ are matrixes over the ring F_3. Then $|C^{\perp}| = 3^{n-3k_1-2k_2-k_3}$.

Define the map ϕ from R to R by:

$$\phi(a + bv + cv^2) = (a+b+c) + (b+2c)(v+2) + c(v+2)^2.$$

It is obvious that ϕ is an automorphism map of the ring R.

Define the map φ from R to S by:

$$\varphi(a + bv + cv^2) = (a+b+c) + (b+2c)u + cu^2.$$

It is obvious that φ is a one to one map from R to S.

Theorem 3 The map φ is an isomorphism from R to S.

Proof. For any $\bar{x}, \bar{y} \in R$, where $\bar{x} = a_1 + b_1 v + c_1 v^2$, $\bar{y} = a_2 + b_2 v + c_2 v^2$. Then

$$\begin{aligned}
&\varphi(\bar{x}+\bar{y}) = \varphi((a_1+a_2)+(b_1+b_2)v+(c_1+c_2)v^2) \\
&= (a_1+a_2+b_1+b_2+c_1+c_2)+(b_1+b_2+2c_1+2c_2)u+(c_1+c_2)u^2 \\
&= (a_1+b_1+c_1)+(b_1+2c_1)u+c_1u^2 \\
&\quad +(a_2+b_2+c_2)+(b_2+2c_2)u+c_2u^2 \\
&= \varphi(\bar{x})+\varphi(\bar{y}),
\end{aligned}$$

$$\begin{aligned}
&\varphi(\bar{x}\cdot\bar{y}) \\
&= \varphi((a_1a_2+b_1c_2+c_1b_2)+(a_1b_2+b_1a_2+c_1c_2)v \\
&\quad +(a_1c_2+b_1b_2+c_1a_2)v^2) \\
&= a_1a_2+b_1c_2+c_1b_2+a_1b_2+b_1a_2+c_1c_2+a_1c_2+b_1b_2+c_1a_2 \\
&\quad +(a_1b_2+b_1a_2+c_1c_2+2a_1c_2+2b_1b_2+2c_1a_2)u \\
&\quad +(a_1c_2+b_1b_2+c_1a_2)u^2 \\
&= [(a_1+b_1+c_1)+(b_1+2c_1)u+c_1u^2] \\
&\quad \cdot[(a_2+b_2+c_2)+(b_2+2c_2)u+c_2u^2] \\
&= \varphi(\bar{x})\cdot\varphi(\bar{y}).
\end{aligned}$$

So

$$\varphi(\bar{x}+\bar{y}) = \varphi(\bar{x})+\varphi(\bar{y}) \cdots\cdots(3)$$

And

$$\varphi(\bar{x}\cdot\bar{y}) = \varphi(\bar{x})\cdot\varphi(\bar{y}) \cdots\cdots(4)$$

Thus, we have proved the theorem.

By the Lemma 1, Lemma 2 and the theorem 3, the following two theorems can be easily obtained.

Theorem 4 Any linear code C over R of length n is permutation-equivalent to a code with generator matrix of the form:

$$G = \begin{pmatrix} I_{k_1} & A_1 & A_2 & A_3 \\ 0 & (v+2)I_{k_2} & (v+2)A_{11} & (v+2)A_{12} \\ 0 & 0 & (v+2)^2 I_{k_3} & (v+2)^2 A_{22} \end{pmatrix}_{k\times n} \cdots\cdots(5)$$

Where $I_{k_1}, I_{k_2}, I_{k_3}$ are all unit matrixes with order k_1, k_2, k_3 respectively. Let $k = k_1 + k_2 + k_3$,

where $A_i = B_{i1} + vB_{i2} + v^2 B_{i3} (i = 1,2,3)$, and A_{11}, A_{12}, A_{22}, $B_{i1}, B_{i2}, B_{i3} (i = 1,2,3)$ are matrixes over the ring F_3. Then $|C| = 3^{3k_1+2k_2+k_3}$.

Theorem 5 If C is an arbitrary linear code of $F_3+vF_3+v^2F_3$, then the generator matrix of the dual code $C^{\perp}$ is:

$$H=\begin{pmatrix} F & A_{12}^T+A_{22}^TA_{11}^T & A_{22}^T & I_{n-k} \\ (v+2)(A_2^T+A_{11}^TA_1^T) & (v+2)A_{11}^T & (v+2)I_{k_3} & 0 \\ (v+2)^2A_1^T & (v+2)^2I_{k_2} & 0 & 0 \end{pmatrix}_{(n-k_1)\times n}, \qquad \cdots\cdots(6)$$

Where $F=A_{22}^T(A_2^T+A_{11}^TA_1^T)+A_{12}^TA_1^T+A_3^T$,

$A_i=B_{i1}+vB_{i2}+v^2B_{i3}(i=1,2,3)$, and A_{11},A_{12},A_{22}, $B_{i1},B_{i2},B_{i3}(i=1,2,3)$ are matrixes over the ring F_3.

4. The gray image of the linear codes over the ring $F_3+vF_3+v^2F_3$

For any $\overline{x}\in R$, then $\overline{x}=a+vb+v^2c(a,b,c\in F_3)$.
Define $\psi : R\to F_3^3$ by: $\psi(\overline{x})=(a+b+c,b+2c,c,)$. Then ψ is a ring homomorphism. The Lee weight of $\overline{x}$ are defined by $W_L(\overline{x})=W(\psi(\overline{x}))$. For any $\overline{x},\overline{y}\in F_3+vF_3+v^2F_3$, we have

$$\begin{aligned} W_L(\overline{x}-\overline{y}) &= d_L(\overline{x},\overline{y})=d(\psi(\overline{x}),\psi(\overline{y})) \\ &= W(\psi(\overline{x})-\psi(\overline{y})). \end{aligned}$$

The Gray map ψ can be extended to R^n. For any $x=(x_1,x_2,\cdots,x_n)\in R^n$, let $x_i=a_i+vb_i+v^2c_i\in R$, then, for any x, we have

$$\begin{aligned} \psi(x)=(&a_1+b_1+c_1,\cdots,a_n+b_n+c_n, \\ &b_1+2c_1,\cdots,b_n+2c_n,c_1,c_2,\cdots,c_n). \end{aligned}$$

It is obvious that ψ is a bijective from R^n to F_3^{3n}.

By the definition of the Gray map ψ, we can obtain the following lemma easily.

Lemma 6 The Gray map ψ is a distance preserving map from R^n to F_3^{3n}.

Theorem 7 Let C be a linear code of length n over the ring R with generator matrix of the form (5), $\psi(C)$ is the Gray image of C. Then, $\psi(C)$ is permutation-equivalent to a linear code of length $3n$ over F_3 with generator matrix of the form:

$$M=\begin{pmatrix} I_{k_1} & \tilde{B}_1 & \tilde{B}_2 & \tilde{B}_3 & 0 & B_{12}+2B_{13} & B_{22}+2B_{23} & B_{32}+2B_{33} & 0 & B_{13} & B_{23} & B_{33} \\ 0 & 0 & 0 & 0 & I_{k_1} & \tilde{B}_1 & \tilde{B}_2 & \tilde{B}_3 & 0 & B_{12}+2B_{13} & B_{22}+2B_{23} & B_{32}+2B_{33} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_{k_1} & \tilde{B}_1 & \tilde{B}_2 & \tilde{B}_3 \\ 0 & 0 & 0 & 0 & 0 & I_{k_2} & A_{11} & A_{12}^{'} & 0 & 0 & 0 & A_{12}^{''} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_{k_2} & A_{11} & A_{12}^{'} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_{k_3} & A_{22} \end{pmatrix},$$

Where $\tilde{B}_i=B_{i1}+B_{i2}+B_{i3}(i=1,2,3)$ and $A_{11},A_{12}^{'},A_{12}^{''},A_{22}$ $B_{i1},B_{i2},B_{i3}(i=1,2,3)$ are matrixes over the ring F_3.

Proof. By the theorem 4 and the definition of the Gray map ψ, $\psi(C)$ can be generated by linear combination of the Gray images of the row vector of the following matrix $\tilde{G}$.

$$\tilde{G}=\begin{pmatrix} I_{k_1} & B_{11}+vB_{12}+v^2B_{13} & B_{21}+vB_{22}+v^2B_{23} & B_{31}+vB_{32}+v^2B_{33} \\ (v+2)I_{k_1} & (v+2)(B_{11}+vB_{12}+v^2B_{13}) & (v+2)(B_{21}+vB_{22}+v^2B_{23}) & (v+2)(B_{31}+vB_{32}+v^2B_{33}) \\ (v+2)^2I_{k_1} & (v+2)^2(B_{11}+vB_{12}+v^2B_{13}) & (v+2)^2(B_{21}+vB_{22}+v^2B_{23}) & (v+2)^2(B_{31}+vB_{32}+v^2B_{33}) \\ 0 & (v+2)I_{k_2} & (v+2)A_{11} & (v+2)[A_{12}^{'}+(v+2)A_{12}^{''}] \\ 0 & (v+2)^2I_{k_2} & (v+2)^2A_{11} & (v+2)^2[A_{12}^{'}+(v+2)A_{12}^{''}] \\ 0 & 0 & (v+2)^2I_{k_3} & (v+2)^2A_{22} \end{pmatrix},$$

Because

$$\psi(I_{k_1}, B_{11}+vB_{12}+v^2B_{13}, B_{21}+vB_{22}+v^2B_{23}, B_{31}+vB_{32}+v^2B_{33})$$
$$=(I_{k_1}, B_{11}+B_{12}+B_{13}, B_{21}+B_{22}+B_{23}, B_{31}+B_{32}+B_{33}, 0,$$
$$B_{12}+2B_{13}, B_{22}+2B_{23}, B_{32}+3B_{33}, 0, B_{13}, B_{23}, B_{33});$$
$$\psi((v+2)I_{k_1}, (v+2)(B_{11}+vB_{12}+v^2B_{13}), (v+2)(B_{21}+vB_{22}$$
$$+v^2B_{23}), (v+2)(B_{31}+vB_{32}+v^2B_{33})) = (0,0,0,0, I_{k_1}, B_{11}$$
$$+B_{12}+B_{13}, B_{21}+B_{22}+B_{23}, B_{31}+B_{32}+B_{33}, 0, B_{12}+2B_{13},$$
$$B_{22}+2B_{23}, B_{32}+3B_{33});$$
$$\psi((v+2)^2 I_{k_1}, (v+2)^2(B_{11}+vB_{12}+v^2B_{13}), (v+2)^2(B_{21}+vB_{22}$$
$$+v^2B_{23}), (v+2)^2(B_{31}+vB_{32}+v^2B_{33})) = (0,0,0,0,0,0,0,0,$$
$$I_{k_1}, B_{11}+B_{12}+B_{13}, B_{21}+B_{22}+B_{23}, B_{31}+B_{32}+B_{33});$$
$$\psi(0, (v+2)I_{k_2}, (v+2)A_{11}, (v+2)[A_{12}^{'}+(v+2)A_{12}^{''}])$$
$$=(0,0,0,0,0, I_{k_2}, A_{11}, A_{12}^{'}, 0,0,0, A_{12}^{''});$$
$$\psi(0, (v+2)^2 I_{k_2}, (v+2)^2 A_{11}, (v+2)^2[A_{12}^{'}+(v+2)A_{12}^{''}])$$
$$=(0,0,0,0,0,0,0,0,0, I_{k_2}, A_{11}, A_{12}^{'});$$
$$\psi(0,0,(v+2)^2 I_{k_3}, (v+2)^2 A_{22}) = (0,0,0,0,0,0,0,0,0,0, I_{k_3}, A_{22});$$

Theorem 8 Let C be a cyclic code of length n over the ring R, $\psi(C)$ is a $3-$quasi-cyclic linear code of length $3n$ over F_3.

Proof. For any $x=(x_1, x_2, \cdots, x_n) \in C$, where

$$x_i = x_{i1} + x_{i2}v + x_{i3}v^2 (i=1,2,\cdots,n).$$

Then

$$\psi(x) = (x_{11}+x_{12}+x_{13}, \cdots, x_{n1}+x_{n2}+x_{n3},$$
$$x_{12}+2x_{13}, \cdots, x_{n2}+2x_{n3}, x_{13}, x_{23}, \cdots, x_{n3}).$$

Because C is a cyclic code of length n over the ring R, then

$$T(x) = (x_{n1}+x_{n2}v+x_{n3}v^2, x_{11}+x_{12}v+x_{13}v^2, \cdots,$$
$$x_{n-1,1}+x_{n-1,2}v+x_{n-1,3}v^2) \in C.$$

So,

$$\psi(T(x))$$
$$=(x_{n1}+x_{n2}+x_{n3}, x_{11}+x_{12}+x_{13}, \cdots, x_{n-1,1}+x_{n-1,2}+x_{n-1,3},$$
$$x_{n2}+2x_{n3}, x_{12}+2x_{13}, \cdots, x_{n-1,2}+2x_{n-1,3}, x_{n3}, x_{13}, \cdots, x_{n-1,3}).$$

Then,

$$\psi(T(x)) = T^3(\psi(x)).$$

Thus we have proved the theorem.

Conclusion

In this paper, we studied linear codes over the ring R. Another direction for research in this topic is of course the cyclic and constacyclic codes over the ring R.

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