ANFIS Transient Control for Double Inverted Pendulum System

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Abstract

A double inverted pendulum system is a very complex and nonlinear system. In this paper, a new design method using adaptive networks based fuzzy inference system (ANFIS) is proposed to cope with this system. This is a single input multi outputs system which states are all nonlinear and the second state maybe need infinity control force in the steady state. So the transient states control is more important than steady state response. The first step is to derive every subsystem's state feedback gains, then by using ANFIS to calculate the optimal feedback gains by human knowledge. The results of simulations are revealed that the good performances are possessed.

Keywords: *ANFIS, Control, Double inverted pendulum.*

1. Introduction

The inverted pendulum is a nonlinear system and it is also a very interesting topic used to verify the control law [1-5]. Most articles are focus on single inverted pendulum for its simplicity. In [4, 5], the double inverted pendulum is discussed and by using sliding mode fuzzy neural networks technologies. This will yield the requirements of high speed and large memory processors in practical implementation. In this paper, the easy implementation and good performance are considered.

Fuzzy logic control (FLC) has been developed many years. It has advantages of easy implementation, model free, human knowledge based and robustness. In this area, the adaptive network based fuzzy inference system (ANFIS) [6] is more easily design and it has been embedded into the platform of MATLAB[TM] [7], meanwhile, many applications of ANFIS have been proposed [8-10]. The ANFIS use hybrid learning algorithm to construct a set of input-output pairs of fuzzy if-then rules with appropriate membership functions such as to generate the stipulated fuzzy associated memory (FAM) which can be look as a look-up table which is easily cooperated with other language of software such as C language in empirical implementations. The input-output pairs and fuzzy rules can be transferred into fuzzy inference system (FIS) matrices to be combined with MATLAB to do simulations. By using these FIS matrices, a reliable controller can be achieved by human knowledge. In this paper, the uncomfortable controller is designed first, then from these data, we can train ANFIS to be better; especially, for this time variant, nonlinear systems. By the simulation results, some suitable factors of state feedback gains are achieved.

The proposed ANFIS-based transient design method is applied for the double inverted pendulum system. This is a one input three outputs control system, we observe that the critical problem is the transient response control. If we miss the timing of this point, the system will become uncontrollable. Besides, the single input force is not reached at the same time for every states. Actually, they are time variant that is u(t) is forced in first state but there exist a very short time interval forced on second state and vice versa. From simulation results, we have coped with this problem and the good performances of control are also possessed.

2. Double Inverted Pendulum Model

A simplified double inverted pendulum model can be expressed as bellow.

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f} + \boldsymbol{g}u + \boldsymbol{d} \\ \boldsymbol{y} &= \boldsymbol{x} \end{aligned} \tag{1}$$

where $\boldsymbol{x}$ is the state vector; $\boldsymbol{f}$ and $\boldsymbol{g}$ are nonlinear functions; $u(t)$ is the single control input; and $\boldsymbol{d}$ is the external disturbances. The x_1 is the angle of the pole 1 with respect to the vertical axis, x_2 is the angular velocity of the pole 1 with respect to the vertical axis, x_3 is the angle of the pole 2 with respect to the vertical axis, x_4 is the angular velocity of the pole 2 with respect to the vertical axis, x_5 is the position of the cart, x_6 is the velocity of the cart, u is the applied force to move the cart and d is the disturbance. The detail parameters can be found in [4, 5].

This system can be treated as three subsystems with second order canonical form. The transient control tries to design a single input u to simultaneously control the states (x_1, x_2, x_3) to be stable and to achieve desired performance. But from (2), we see the b_2 includes the term $\sin(x_3 - x_1)$, this means at the moment of $x_3 = x_1$, the control input of subsystem will need infinity control force. This is an unreasonable result, so in this paper the control law is designed by traditional state feedback control. For the total

control force, the factors of these three subsystems are calculated by ANFIS.
The control law can be designed as

$$u = a_1 K_1 x_1 - a_2 K_2 x_3 + a_3 K_3 x_5 . \quad (2)$$

where the a_i $i = 1,2,3$ are tuned by ANFIS, the K_i $i = 1,2,3$ is state feedback gains of these three subsystems.

3. ANFIS Controller Design

The ANFIS uses a hybrid learning algorithm to identify the membership function parameters to generate Takagi-Sugeno type fuzzy inference systems (FIS). It uses the method of combination of least-squares and back-propagation gradient descent methods to train FIS membership function parameters to model a given set of input/output data. The principle of ANFIS is briefly described as follows [6].

$$R_i : If\ x\ is\ A_1\ \ldots and\ y\ is\ B_1$$
$$then\ h_1 = p_1 x + q_1 y + r_1 \quad (3)$$

where R_i denotes the *i*th fuzzy rules, *i*=1, 2,..,*r*; A_i is the fuzzy set in the antecedent associated with the *k*th input variable at the *i*th fuzzy rule, and $p_{i1},..., p_{in}$, r_i are the fuzzy consequent parameters.
Based on the *weighted averaged method* of defuzzification. The output u can be calculated as

$$h = \frac{w_1}{w_1 + w_2} h_1 + \frac{w_2}{w_1 + w_2} h_2$$
$$= \overline{w}_1 u_1 + .. + \overline{w}_2 u_n \quad (4)$$

where w_i is the *i*th node output firing strength of the *i*th rule, and $\overline{w}_1 = \frac{w_1}{w_1 + .. + w_n}$,.., $\overline{w}_n = \frac{w_n}{w_1 + .. + w_n}$.
Because the fuzzy inference system is a Takagi-Sugeno type, i.e., $h_i = p_i x + q_i y + r_i$, Eq. (4) can be rewritten as

$$h = \overline{w}_1 h_1 + \overline{w}_2 h_2$$
$$= (\overline{w}_1 x_1) p_{i1} + .. + (\overline{w}_1 x_n) p_{in} + (\overline{w}_1) r_1$$
$$+$$
$$\vdots$$
$$+ (\overline{w}_n x_1) p_{i1} + .. + (\overline{w}_n x_n) p_{in} + (\overline{w}_n) r_n . \quad (5)$$

The hybrid learning algorithm developed in [6] can be applied to (5) directly. A two inputs neural network structure of ANFIS is shown in Fig. 1. In the *forward pass* of the hybrid algorithm, functional signals go forward till layer 4 of Fig. 1 and the consequent parameters p_{i1}, p_{i2}, r_i are identified by the *least squares estimate* (LSE) approach. In the *backward pass*, the error rates propagate backward and the premise parameters x_1, x_2 are updated by the gradient descent approach. As the values of these parameters change, the membership functions vary accordingly; thus exhibits various forms of membership functions on linguistic labels A_{i1} and A_{i2} . In this paper, because we have three states should be controlled, and five membership functions are given for every state, total have 125 fuzzy rules should be designed. This is a heavy burden, for simplicity, we use three ANFISs for every states with ten rules, thus total just need 30 rules.

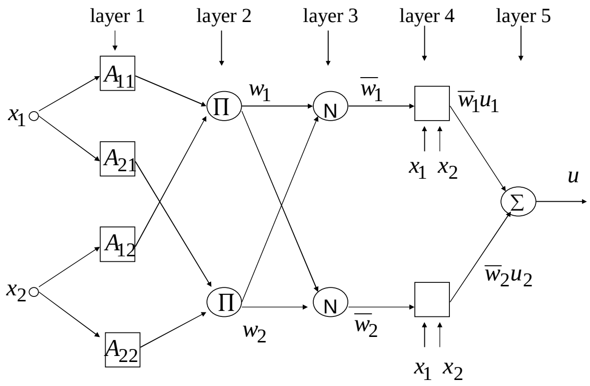

Fig. 1. A two-inputs one-output ANFIS architecture

4. Simulation Results

The double inverted pendulum system is simulated by using models of (1). The system parameters are given as $m_1 = m_2 = m_c$ =1Kg; $l_1 = l_2$ =1m; g =9.8 m/s^2 and $d = 0.05$ $\deg/s^2$. For the initial conditions, the values are set as $x_1 = 30^\circ$, $x_2 = 0^\circ / s$, $x_3 = 10^\circ$, $x_4 = 0^\circ / s$, $x_5 = 0$ m and $x_6 = 0$ m/s . For the factors of control law (2) are tuned by three ANFISs. In order to get trained data, the rough control simulations are proceed and shown in Fig. 3. From the results of Fig. 2, the performances are very bad. The transient control is very important such as the state of x_5 can be converged. The ANFIS is designed as Fig. 3. Finally, the control results are shown in Fig. 4. From the simulation results of Fig. 4, the first transient response is controlled in good conditions. Especially for x_3, it must be stable earlier than other states for the reason of it couple with x_1 and x_5. Just need the state of x_3 is controlled in stable at first, then the states of x_1 and x_5 will be converged rapidly.

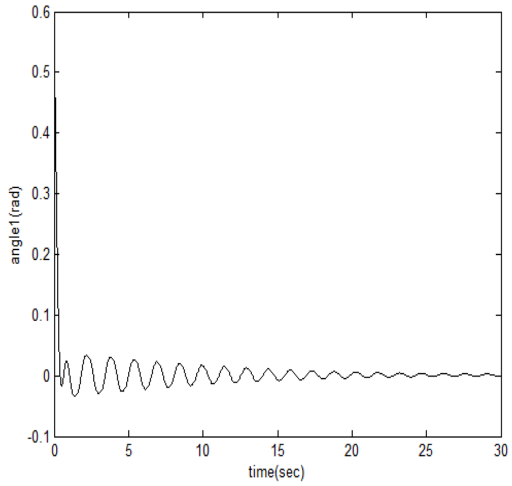

Fig. 2 (a) state trajectory of x_1

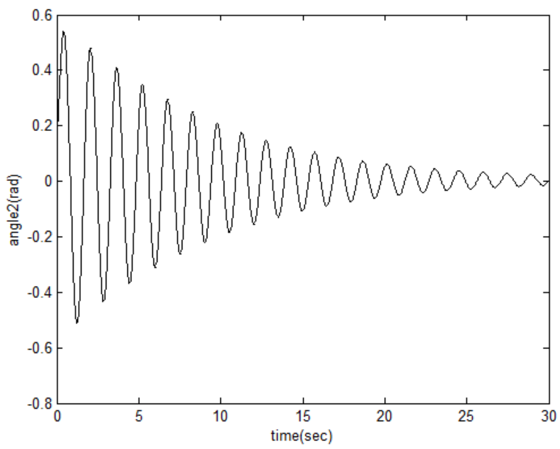

Fig. 2 (b) state trajectory of x_3

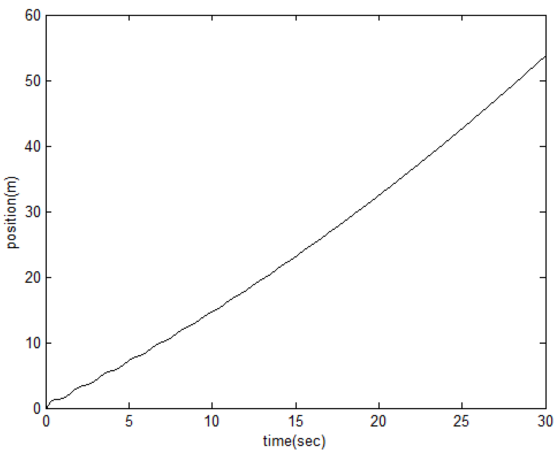

Fig. 2 (c) state trajectory of x_5

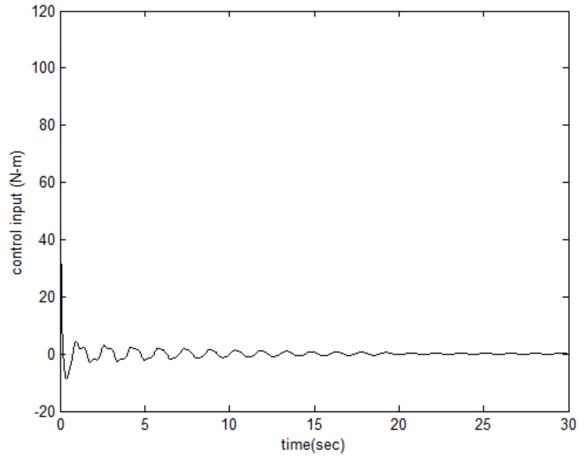

Fig. 2 (d) Control input

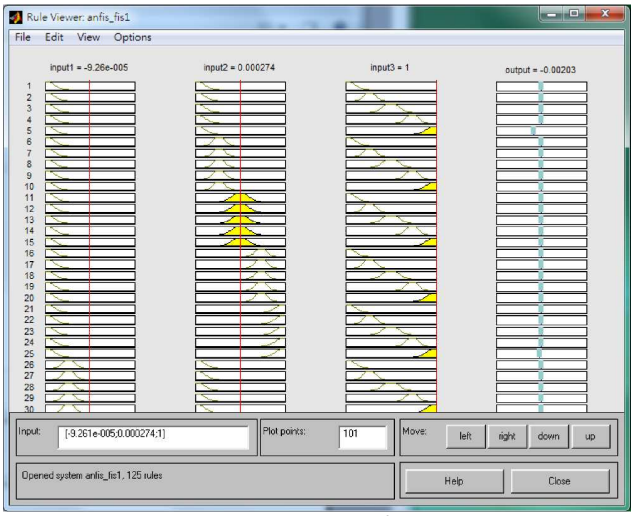

Fig. 3 The design result of ANFIS

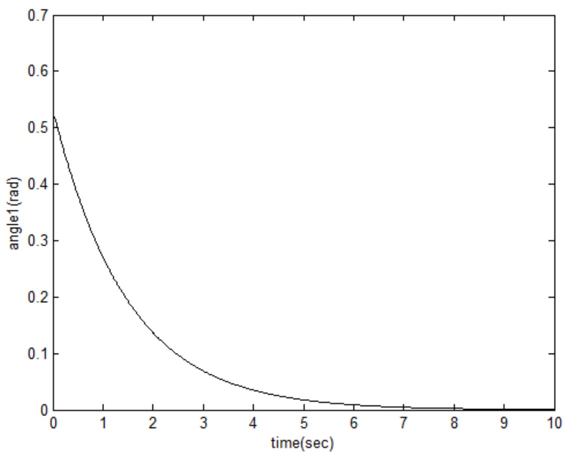

Fig. 4 (a) state trajectory of x_1 under ANFIS control

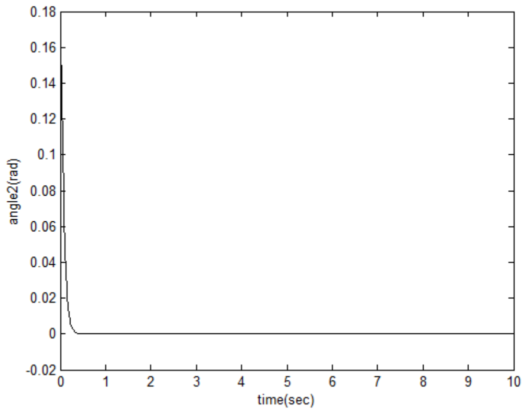

Fig. 4 (b) state trajectory of x_2 under ANFIS control

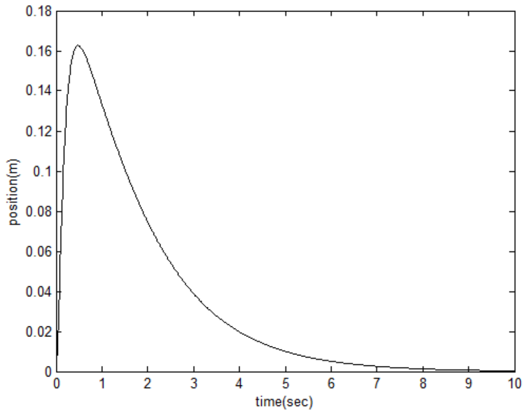

Fig. 4 (c) state trajectory of x_5 under ANFIS control

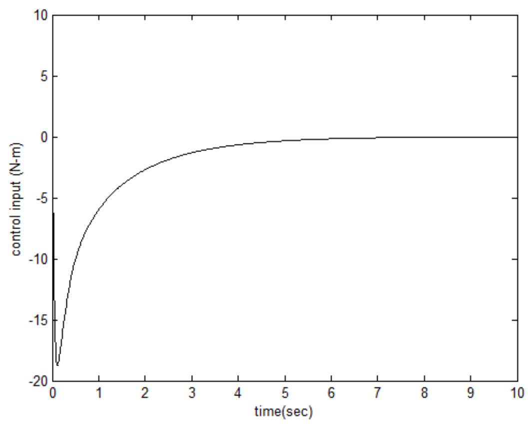

Fig. 4 (d) Control input of ANFIS control

5. Conclusions

The nonlinear coupled double inverted pendulum is discussed and controlled in this paper. The design method is by using adaptive networks based fuzzy inference system (ANFIS) to cope the transient response. The control factors of every subsystem are given by ANFIS, especially for coupled state x_3 need to be controlled stable at first. Then the overall system will be converged rapidly. From the simulation results, the good and stable performances are possessed.

Acknowledgments

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