Application of equivalent permeability in numerical simulation of horizontal wells in bottom water reservoirs

Li-feng Li[1], Bing-bing Wang[2], Xiang-an Yue[3]

[1] **CMOE Key Laboratory of Petroleum Engineering, China University of Petroleum(Beijing), Beijing, 102249, China**

[2] **Dongsheng Group Co, Ltd of Shengli Oilfield, Dongying 257000, China**

[3] **CMOE Key Laboratory of Petroleum Engineering, China University of Petroleum(Beijing), Beijing, 102249, China**

Abstract

Coupled flow model used in numerical simulation of horizontal well in bottom water reservoir is difficulty to resolve. In this paper, equivalent permeability was introduced to improve numerical simulation with considering pressure drop in horizontal well of bottom water reservoir. The pressure drop formula in horizontal well was derived according to material and momentum balance. Equivalent permeability of two phase flow in horizontal well was defined based on pressure drop formula. By use of equivalent permeability, the correlation between pressure drop and velocity was deformed to the same as Darcy's Law and horizontal well can be treated as the reservoir. Instead of coupled flow model, the same mathematics model of two phase flow in reservoir and horizontal well were established. In application, the reservoir and wellbore were given different data of porosity, permeability and saturation. In time region, the results got at the time step of *j* were transferred to time step of *j*+1, which was used to calculate equivalent permeability at time step of *i*+1. Case study indicated that numerical simulation based on equivalent permeability could be used in research of horizontal well.

Keywords: *Reservoir with bottom water, Horizontal well, Numerical simulation, Equivalent permeability, Pressure drop in horizontal well.*

1. Introduction

It is important to improve performance and ultimate recovery of every developing reservoir, for total petroleum resources are limited. Reservoirs with bottom water are widely distributed in the world, which are easily troubled by bottom water coning and breakthrough during development [1]. Horizontal wells are usually applied to suppress water cone in the reservoirs [2]. Compared to vertical wells, the drawdown pressure of horizontal wells are lower at the same production rate, which is disadvantage for water coning.

Initially, the horizontal well was simplified to be line source or infinite conductivity pipe without considering pressure drop in it by many researchers [3-7]. Based on the assumption, the productivity [5], breakthrough time [6] and performance [7] of horizontal wells were investigated. However, field reports indicated that the pressure drop had important impact for performance of horizontal well: the breakthrough of bottom water occurred near the heel end usually because of pressure drop along the well, which make the well section near toe end no contribution for production [8-9]. The coupled flow was applied to research influence of pressure drop by some researchers, in which pipe flow theory was used in horizontal well and seepage theory was used in reservoir. The influences of pressure drop for productivity were studied by Dikken [10] and Liu [11], and the researches revealed that productivity of horizontal well predicted by method ignoring pressure drop was higher than the practical result. The pressure distribution in horizontal well was investigated by Li [13]. Ozkan [14-15] researched the breakthrough time and presented build-up curve of cone at the condition of considering pressure drop. Some researchers applied the couple flow method to the numerical simulation. Huang [15] used the Navier-Stocks equation to stimulate the flow in the well, and numerical simulation method was established by coupling flow in reservoir. K-ε turbulence model was applied to flow along the well by Li [16]. Numerical simulation of two phase flow was developed by Cheng [17], based on four kinds of pressure losses in well and black oil simulator in reservoirs. It can be seen that pressure drop along the well should be incorporated in the numerical simulation for its important influence. However, the motion equation of well and reservoir are different in the couple flow model, which would increase the difficulty of resolving the model. The disadvantage may be more serious in numerical simulation of larger scale. Therefore, it is necessary to develop numerical simulation method with considering well drop in horizontal well, which is easier to resolve.

The equivalent hydraulic conductivity theory was presented by Chen [18-19] in the study of hydraulics, which was used to resolve the seepage-pipe coupling single phase flow. In this paper, equivalent permeability of two phase flow in horizontal well was investigated, and mathematics model of coupled flow problem was simplified by use of equivalent permeability.

2. Physical model and assumptions

Fig1 shows a horizontal well in the bottom water reservoir. The top boundary of the reservoir is impermeable and bottom boundary that is supported by bottom water has a constant pressure, the boundary condition of lateral edge is no-flow. The horizontal well is produced at constant production. The capillary force and gravity are considered with ignoring the compressibility of rock and fluid in the research. The variables of the problem are: oil column height h, m; permeability of reservoir K, m^2; porosity of reservoir Φ, dimensionless; length of horizontal well L, m; height between horizontal well and bottom water Z_w, m; oil density ρ_o, kg/m^3; water density ρ_w, kg/m^3; acceleration of gravity g, m/s^2; oil velocity v_o, m/s; water velocity v_w, m/s; fluid viscosity μ, Pa·s; oil saturation S_o, dimensionless; water saturation S_w, dimensionless; pressure of oil phase p_o, Pa; pressure of water phase p_w, Pa,; production of well Q, m^3/s; water cut f_w, dimensionless.

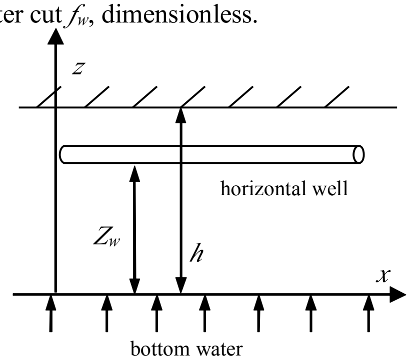


Fig 1: Schematic diagram of horizontal well in bottom water reservoir

3. Mathematics model

3.1 **Flow in horizontal well**

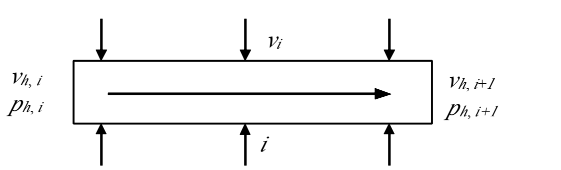


Fig 2: Schematic diagram of an infinitesimal section of horizontal well

Fluid of reservoir flows into the wellbore continually, which leads to variable mass flow in the horizontal well. An infinitesimal section i of horizontal well was taken as the object of study, shown in Fig2. The entry velocity of the section is $v_{h,i}$, and the pressure is $p_{h,i}$. Fluid of reservoir penetrates into the section with the velocity of v_i, which changes the velocity along the well and leads to pressure drop. So the exit velocity becomes $v_{h,i+1}$, and the pressure is $p_{h,i+1}$. On the basis of material balance, the relation $v_{h,i}$ of $v_{h,i+1}$ and is: $v_{h,i+1}=v_{h,i}+v_i\Delta x/r_w$.

According to momentum balance, the relation between pressure and velocity in horizontal well can be written as:

$$(p_{h,i}-p_{h,i+1})A-\tau(2\pi r_w)\Delta x=d(mv_{h,i}) \tag{1}$$

where A is sectional area of wellbore, m^2; r_w is radius of wellbore, m ; τ is frictional resistance, $\tau=f\rho v^2/8$, f is frictional factors. $d(mv)$ is difference of momentum in the element. The Eq.1 can be rewritten as:

$$(p_{h,i}-p_{h,i+1})A-\frac{f\rho v_{h,i}{}^2}{8}(2\pi r_w)\Delta x= \rho A(v_{h,i}{}^2-v_{h,i+1}{}^2) \tag{2}$$

replacing $(p_{h,i}-p_{h,i+1})$ of Eq.2 by $\Delta p_{h,i}$:

$$\Delta p_{h,i}=\frac{f\rho(\pi r_w)}{4A}v_{h,i}^2\Delta x+2\rho v_{h,i}\frac{v_i}{r_w}\Delta x+ \rho(\frac{v_i}{r_w})^2\Delta x^2 \tag{3}$$

The first part of right side in Eq.3 represents pressure drop by frictional resistance, the second and third part represent pressure drop by acceleration. It can be seen the third part is much smaller than the first two parts, so it can be ignored in application. Pressure drop from frictional resistance is about 80% of the total pressure drop approximately by literature[20]. Therefore, the relation between the total pressure drop along the horizontal well and the frictional pressure drop is:

$$0.8\Delta p_{h,i}=\frac{f\rho(\pi r_w)}{4A}v_{h,i}^2\Delta x \tag{4}$$

From Eq.4, the relationship between velocity and pressure gradient in the infinitesimal section can be got:

$$v_{h,i}^2=\frac{3.2r_w}{f}\frac{\Delta p_{h,i}}{\Delta x} \tag{5}$$

The equivalent permeability of horizontal well is defined as:

$$K_h=\frac{3.2\mu r_w}{fv_{h,i}} \tag{6}$$

For two phase flow, the parameter included in equivalent permeability is not constant for variation of water-oil ratio. The mix density is determined according to weighted average of volume:

$$\rho = S_w\rho_w + S_o\rho_o \tag{7}$$

According to literature[21], the mix viscosity can be written as:

$$\mu = \frac{n_w}{\mu_w} + \frac{n_o}{\mu_o} \tag{8}$$

where, $n_w = \frac{S_w\rho_w}{\rho}$, $n_o = \frac{S_o\rho_o}{\rho}$

The value of f is related to the Reynolds Number R_e, $R_e=\rho vd/\mu$. For laminar flow, $f=64/R_e$. There are four stages in turbulence flow, and the relation f and R_e differs in each stage. The relationship can be found in literature[22].

The mix velocity is:

$$v_h = v_{h,w} + v_{h,o} \tag{9}$$

The velocity v_h is not only the result to be resolved but also the parameter in equivalent permeability at the same time. So the special treatment for v_h is required in numerical simulation. The initial value of equivalent permeability is given at the assumption of laminar flow, where f has no relation with velocity. In time domain, the results at the time step of j is transferred to the time step of $j+1$, which is used to calculate the equivalent permeability at the time of $i+1$.

Taking equivalent permeability into Eq.5 and rewriting the formula in differential expression:

$$v_h = \frac{K_h}{\mu}\frac{dp_h}{dx} \tag{10}$$

Eq.10 is the correlation between velocity and pressure gradient along horizontal well, which is the same as the Darcy's Law in form.

3.2 **Governing equation of whole flow region**

By use of equivalent permeability, the motion equation in horizontal well obeys the Darcy's Law as well as the reservoir. So the coupled flow model is not needed yet for the problem and the same governing equation of whole flow region including horizontal well and reservoir can be established. The continuity equations for two-phase flow can be written as:

$$\nabla \cdot (\rho_o v_o) + \frac{\partial(\rho_o\phi S_o)}{\partial t} = 0 \tag{11}$$

$$\nabla \cdot (\rho_w v_w) + \frac{\partial(\rho_w\phi S_w)}{\partial t} = 0 \tag{12}$$

where ∇ is gradient operator.

According to Darcy's Law, the equations of motion are:

$$v_o = -K\frac{k_{ro}}{\mu_o}\nabla(p_o + \rho_o gD) \tag{13}$$

$$v_w = -K\frac{k_{rw}}{\mu_w}\nabla(p_w + \rho_w gD) \tag{14}$$

where k_r is relative permeability, D is elevation, m. Subsidiary equations are needed:

$$S_o + S_w = 1$$

$$p_c = p_o - p_w$$

The boundary conditions are:

$$v_o\Big|_{(z=h)} = 0,\ v_w\Big|_{(z=h)} = 0$$

$$v_o\Big|_{(x=\pm x_e)} = 0,\ v_w\Big|_{(x=\pm x_e)} = 0$$

$$p\Big|_{z=0} = p_e$$

$$v_o + v_w\Big|_{(x=L,z=z_w)} = \frac{Q}{\pi r_w^2}$$

replacing the velocity of continuity equations by motion equations:

$$\nabla \cdot (-K\frac{k_{ro}}{\mu_o}\nabla(p_o + \rho_o gD)) + \phi\frac{\partial(S_o)}{\partial t} = 0 \tag{15}$$

$$\nabla \cdot (-K\frac{k_{rw}}{\mu_w}\nabla(p_w + \rho_w gD)) + \phi\frac{\partial(S_w)}{\partial t} = 0 \tag{16}$$

According to subsidiary equations, Eq.16 can be deformed to:

$$\nabla \cdot (-K\frac{k_{ro}}{\mu_o}\nabla((p_w + p_c) + \rho_o gD)) - \phi\frac{\partial(S_w)}{\partial t} = 0$$

(17)

Eq.16 plus Eq.17:

$$\begin{aligned}&\nabla \cdot (-K(\frac{k_{rw}}{\mu_w} + \frac{k_{ro}}{\mu_o})\nabla p_w - K\frac{k_{ro}}{\mu_o}p_c) + \\ &\nabla \cdot (-K\frac{k_{rw}}{\mu_w}\rho_w gD - K\frac{k_{ro}}{\mu_o}\rho_o gD) = 0\end{aligned} \tag{18}$$

The capillary force is related to the water saturation, and the water saturation has correlation with distance x:

$$p_c = f(S_w)\ ,\ S_w = f(x) \tag{19}$$

So the Eq.18 becomes:

$$\begin{aligned}&\nabla \cdot (-K(\frac{k_{rw}}{\mu_w} + \frac{k_{ro}}{\mu_o})\nabla p_w - K\frac{k_{ro}}{\mu_o}\frac{dp_c}{dS_w}\nabla S_w) + \\ &\nabla \cdot (-K\frac{k_{rw}}{\mu_w}\rho_w gD - K\frac{k_{ro}}{\mu_o}\rho_o gD) = 0\end{aligned} \tag{20}$$

Eq.20 is the pressure differential equation of two phase flow in two flow regions. Eq.16 can be deformed to:

$$\phi\frac{\partial(S_w)}{\partial t} = -\nabla \cdot (-K\frac{k_{rw}}{\mu_w}\nabla(p_w + \rho_w gD)) \tag{21}$$

Eqs.20, 21 along with boundary conditions are the governing equations of two phase flow in the well and reservoir.
In above model, the horizontal well was treated as the reservoir, so the porosity, relative permeability in it is required in application. Porosity is set as 1 in horizontal well and relative permeability is set as follows:

$$k_{h,rw} = S_w, k_{h,ro} = S_o \quad (22)$$

In horizontal well, capillary force can be ignored and the pressure of oil phase equals to that of water phase.

3.3. Application method

The mathematics model presented above can be resolved by many commercial numerical simulators. The step of application is:
(1) Applying the mathematics model to both the reservoir and horizontal well. Setting the parameter of reservoir and horizontal model in simulator according to practical data. The initial equivalent permeability of horizontal well is determined at the basis of laminar flow.
(2) After meshing, resolving the model, calculating the pressure, saturation and velocity of the flow field. Transferring the results at the time step of j to the time of j+1, which is used to calculate the equivalent permeability at the time of j+1.
(3) Resolving until the required time.

4. Application and discussion

The numerical simulation method proposed in this paper was applied to a physical modeling of horizontal well in reservoir with bottom water. The physical modeling was carried out to research position of water breakthrough in horizontal well. The size of reservoir model was 0.8cm (length) ×0.1cm (height) ×0.05cm (width). The horizontal well was placed on the top of model, with length of 0.7m and diameter of 0.003m. Four pressure sensors were set along the wellbore to measure the pressure during the experiment. The basic data of physical model was shown in table 1.

Table 1 basic data of physical model

Variables	Value
Oil column height h, m	0.1
Reservoir permeability Ky, m2	1e-12
Porosityφ, dimensionless	0.25
Horizontal well length L, m	0.7
Wellbore radius rw, m	0.003
Oil density ρo, kg/m3	950
Water density ρw, kg/m3	1000
Oil viscosity μo，Pa·s	0.06
Water viscosity μw, Pa·s	0.001

The relative permeability relation of reservoir model was measured and shown in table 2.

Table 2 relative permeability relationship of reservoir model

S_w	k_{rw}	k_{ro}
0.5	0	0.8
0.55	0.02	0.7
0.6	0.05	0.6
0.65	0.08	0.5
0.7	0.11	0.4
0.75	0.15	0.25
0.8	0.2	0

Pressure along the horizontal well of simulation results and experimental results was shown in Fig 3. It can be seen that the simulation results matched the experiment well. From the toe end to the heel end, the pressure in well declined continually, and most pressure losses occurred at part near heel end. The results got from numerical simulation represented the feature of pressure distribution in horizontal well.

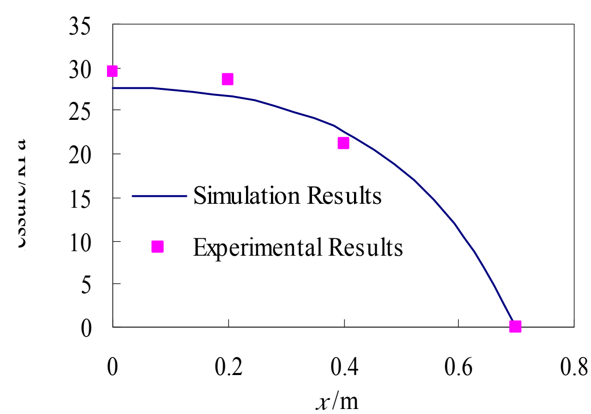
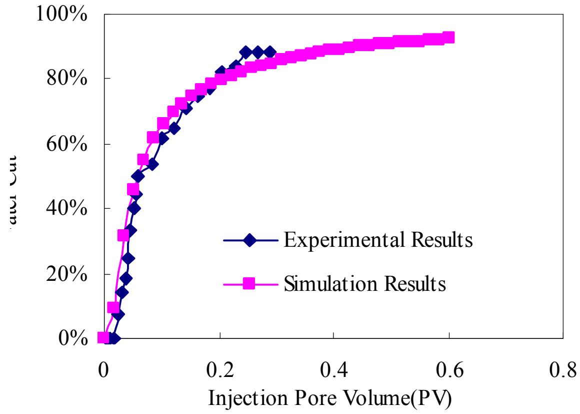
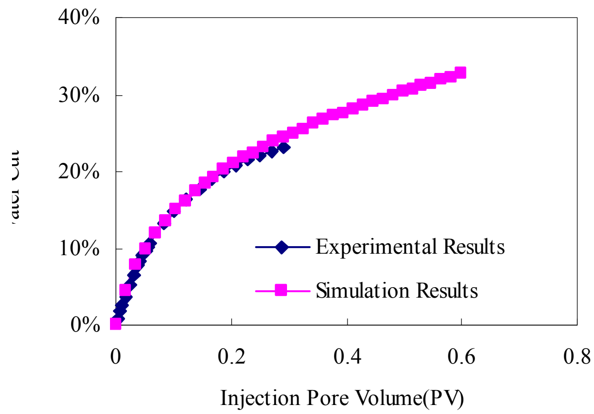


Fig.3. Pressure distribution along the horizontal well

Production performances of horizontal well were shown in Fig 4. The experimental results indicated that bottom water broke into the well early and the water cut increased rapidly, which made the recovery less than 25%. The simulation results agreed with results of experiment well. This meant the numerical simulation method in the paper was suitable for modeling development of horizontal well in reservoirs with bottom water. Meanwhile, the method did not increase resolving time a lot



(a) water cut



(b) recovery

Fig.4. Performance of horizontal well

Fig 5 showed the ratio between cumulative production and total production along the horizontal well from simulation. From toe end to heel end, cumulative production increased continually. And the cumulative production on half part of wellbore near toe end is less than 20% of total production, which revealed that the part of wellbore do little contribution to the production. The part of wellbore near heel end was the main producing section.

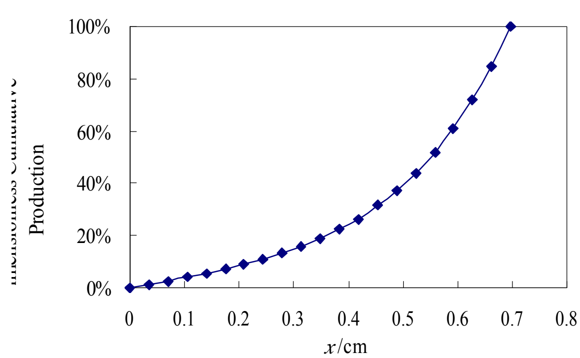


Fig .5. Ratio of cumulative production and total production along the horizontal well

Water saturation distribution of physical model when water cut reached 80% was shown in Fig6, which could represent movement of bottom water. It can be seen that the water near heel end in horizontal direction moved faster than others, which made the heel end to be water breakthrough position. The result was consistent with the experimental result of literature[8].

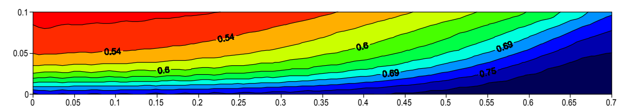


Fig. 6. Water saturation distribution of the model (fw=80%)

5.Conclusion

(1) By use of equivalent permeability, the coupled model was avoided and the mathematics model based on Darcy's Law could be applied to both reservoir and horizontal well.

(2) Numerical simulation based on model proposed by this paper matched the experimental results well without increasing resolving time and could be used in research of horizontal well.

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Li-feng Li was born in 1985. He is PhD candidate at China University of Petroleum (Beijing).His major is oil and gas development. His research interest is enhanced oil recovery and water breakthrough law of horizontal well in reservoir with bottom water.

Xiang-an Yue is a professor in China University of Petroleum (Beijing), His main research interest is Enhanced Oil Recovery in low permeability reservoir..