Supply Chain Coordination Research under a Fuzzy Decision Environment

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Abstract

In a two-stage supply chain composed of a supplier and a retailer, the supply chain models under a fuzzy environment are researched. The parameters of market demand function, the costs of the supplier and retailer are treated as trapezoidal fuzzy numbers. The decision processes are analyzed under the decentralized and centralized decision, and the decision model with revenue sharing contract is also built by the method of fuzzy cut sets theory. It is shown that in fuzzy decision environment, the double marginalization effect is still existed in supply chain system; the profits of members can be coordinated by revenue sharing contract. Finally, a numerical is given to illustrate the models and the solution process.

Keywords: *Supply Chain, Revenue Sharing, Fuzzy Environment, Trapezoidal Fuzzy Number*

1. Introduction

Over the last decade or so, supply chain management has emerged as a key area of research among the practitioners of operations research. In recent years, more and more researchers have applied the fuzzy sets theory and technique to develop and solve the supply chain models problem.

Chang [1] presented a fuzzy EOQ model with imperfect quality item, where the defective rate and annual demand were considered as the triangular fuzzy numbers. Dutta et al. [2] presented a single-period inventory problem in an imprecise and uncertain mixed environment. Tan and Tang [3] studied the fuzzy safety stock model, where the market demand was considered as Gauss fuzzy variable. Mandal and Roy [4] developed a multi-item displayed inventory model under shelf-space constraint in fuzzy environment. Recently, Xu and Zhai [5, 6] assumed the demand to be a triangular fuzzy number and dealt with the newsboy problem in a two stage supply chain. Hu et al. [7] presented two EPQ models with fuzzy defective rate, where the defective rate was considered as the LR-fuzzy number. Zheng and Liu [8] constructed two models for newsboy problem with fuzzy random demand in both non-cooperation and cooperation situations. Wei and Zhao [9] considered a fuzzy closed-loop supply chain with retail competition. Ye and Li [10] developed a Stackelberg model with fuzzy demand. Zhao et al. [11, 12] analyzed the pricing problem of substitutable products in a fuzzy supply chain. Akinribido et al. [13] presented a fuzzy-ontology based information retrieval system for relevant feedback. The above models with fuzzy supply chain problem have focused on the cases that the actors do not make cooperation. In fact, in order to improve overall chain performance, the coordination and integration activities are critical factors for an effective supply chain.

Revenue sharing contract as an important kind of popular contract is an instrument for supply chain coordination. Giannoccaro and Pontrandolfo [14] showed that revenue sharing could coordinate members in the newsboy channel with three stages: supplier, manufacturer and retailer. Gupta and Weerawat [15] designed a revenue-sharing contract to maximize the centralized revenue by choosing an appropriate inventory level. Cachon and Lariviere [16] intensively discussed a revenue sharing contract between a single supplier and a single retailer in a single period newsboy problem. Yao et al. [17] investigated a revenue-sharing contract for coordinating a supply chain comprising one manufacturer and two competing retailers. The above literatures discuss the revenue sharing contact with probabilistic demand, in that probability distributions are estimated from historical date. However, in practice, especially for new products, the probabilities are not known due to lack of history data. Thus, the fuzzy sets theory, rather than the traditional probability theory is well suited to the revenue sharing contract problem.

In this paper, the demands are approximately estimated by the expert, and regarded as fuzzy variable. The decentralized decision-making model, the centralized decision-making model and the revenue sharing contract in fuzzy linear demand environment will be discussed and the impact of main parameter on the models will also be analyzed.

2. Preliminaries

2.1 Fuzzy set theory

Definition 1 The fuzzy set $\tilde{A}=(a_1,a_2,a_3,a_4)$, where $a_1<a_2<a_3<a_4$ and defined on R , is called the trapezoidal fuzzy number, if the membership function of $\tilde{A}$ is given by

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, & a_1<x\le a_2,\\ 1, & a_2<x\le a_3,\\ \dfrac{x-a_4}{a_3-a_4}, & a_3<x\le a_4,\\ 0, & \text{otherwise.}\end{cases} \tag{1}$$

where a_1 , a_2 , a_3 and a_4 are the lower limit, lower mode, upper mode and upper limit respectively of the fuzzy number $\tilde{A}$. The triangular fuzzy number $\tilde{A}$ is called the positive trapezoidal fuzzy number if $a_1>0$.

Definition 2 The set $\tilde{A}_\lambda=\{x\,|\,\mu_{\tilde{A}(x)}\ge\lambda\}$, where $\lambda\in[0,1]$ is called the λ-cut of $\tilde{A}$. $\tilde{A}_\lambda$ is a non-empty bounded closed interval contained in the set of real numbers, and it can be denoted by $\tilde{A}_\lambda=[\tilde{A}^L_\lambda,\tilde{A}^R_\lambda]$. Where, $\tilde{A}^L_\lambda$ and $\tilde{A}^R_\lambda$ are respectively the left and right boundary of $\tilde{A}$, with

$$\tilde{A}^L_\lambda=\inf\{x\in R:\mu_{\tilde{A}(x)}\ge\lambda\},$$
$$\tilde{A}^R_\lambda=\sup\{x\in R:\mu_{\tilde{A}(x)}\ge\lambda\}.$$

For $\forall\lambda\in[0,1]$, the λ-cut of a trapezoidal number $\tilde{A}=(a_1,a_2,a_3,a_4)$ is

$$\tilde{A}^L_\lambda=a_1+(a_2-a_1)\lambda,$$
$$\tilde{A}^R_\lambda=a_4-(a_4-a_3)\lambda\ . \tag{2}$$

Proposition 1 (Liu and Liu [18] and Zhao et al [19]) Let $\tilde{A}$ and $\tilde{B}$ be two positive independent trapezoidal fuzzy numbers, for $\forall\lambda\in[0,1]$, we have

$$(\tilde{A}+\tilde{B})^L_\lambda=\tilde{A}^L_\lambda+\tilde{B}^L_\lambda,\ (\tilde{A}+\tilde{B})^R_\lambda=\tilde{A}^R_\lambda+\tilde{B}^R_\lambda,$$
$$(\tilde{A}-\tilde{B})^L_\lambda=\tilde{A}^L_\lambda-\tilde{B}^R_\lambda,\ (\tilde{A}-\tilde{B})^R_\lambda=\tilde{A}^R_\lambda-\tilde{B}^L_\lambda,$$
$$(\tilde{A}\cdot\tilde{B})^L_\lambda=\tilde{A}^L_\lambda\tilde{B}^L_\lambda,\ (\tilde{A}\cdot\tilde{B})^R_\lambda=\tilde{A}^R_\lambda\tilde{B}^R_\lambda. \tag{3}$$

Proposition 2 Let k be a positive real number, then

$$(k\xi)^L_\lambda=k\xi^L_\lambda,\ (k\xi)^R_\lambda=k\xi^R_\lambda. \tag{4}$$

Proposition 3 (Liu and Liu [20]) Let $\tilde{A}$ be a positive trapezoidal number, the expected value of $\tilde{A}$ is

$$\mathrm{E}\left[\tilde{A}\right]=\tfrac{1}{2}\int_0^1(\tilde{A}^L_\lambda+\tilde{A}^R_\lambda)\,\mathrm{d}\lambda. \tag{5}$$

Proposition 4 (Liu and Liu [18]) Let $\tilde{A}$ and $\tilde{B}$ be two independent fuzzy variables with finite expected values. Then for any real numbers a and b, we have

$$E\left[a\tilde{A}+b\tilde{B}\right]=aE\left[\tilde{A}\right]+bE\left[\tilde{B}\right]. \tag{6}$$

2.2 Problem description

Consider a two-stage supply chain consisting of a supplier and a retailer. The retailer sells short life products, such as personal computers, consumer electronics or fashion items, with high uncertain demand. The products are sold only in one period. As the lead times of such goods are much longer than their selling season, the actors have no chance to place a second order.

In this paper, the linear demand function under a fuzzy decision environment can be written as

$$\tilde{Q}(p)=\tilde{\alpha}-\tilde{\beta}p\ . \tag{7}$$

where p is the unit price of the product, $\tilde{\alpha}$ represents the fuzzy size of the market, $\tilde{\beta}$ stands for the fuzzy sensitivity of demand to the price of the product offered to the market. $\tilde{\alpha}$ and $\tilde{\beta}$ are positive and independent trapezoidal fuzzy numbers.

The following notations are used for a product in the models:

$\tilde{c}_s$: the per unit product fuzzy cost incurred to the supplier,
$\tilde{c}_s$: the per unit fuzzy cost incurred to the retailer,
w: the wholesale price,
Φ : the fraction revenue of the retailer in revenue sharing contract and $\Phi\in(0,1)$,
$\tilde{\Pi}_S$: the fuzzy profit of the supplier,
$\tilde{\Pi}_R$: the fuzzy profit of the retailer,
$\tilde{\Pi}_{SC}$: the fuzzy profit of the supply chain.

In this paper the supplier and the retailer are assumed to be risk neutral and pursue fuzzy expected value of profit maximization.

We can express the fuzzy profit of the retailer, the supplier and the supply chain as follows

$$\tilde{\Pi}_R=\left(p-\tilde{c}_r-w\right)\left(\tilde{\alpha}-\tilde{\beta}p\right), \tag{8}$$

$$\tilde{\Pi}_S=\left(w-\tilde{c}_s\right)\left(\tilde{\alpha}-\tilde{\beta}p\right), \tag{9}$$

$$\tilde{\Pi}_{SC}=\left(p-\tilde{c}_r-\tilde{c}_s\right)\left(\tilde{\alpha}-\tilde{\beta}p\right). \tag{10}$$

3. Supply chain models with fuzzy demand

3.1 Fuzzy decentralized decision-making model

As usual, we firstly begin the analysis with the fuzzy decentralized supply chain. The behavior of the supplier and the retailer in fuzzy decentralized system can be characterized by the Stackelberg game. In the other words, at first the supplier sets the wholesale price w as the leader, and then the retailer set its price of the product p as the follower, which solves the following model

$$\begin{aligned}&\mathrm{Max}_w\, E\left[\tilde{\Pi}_S\right]=E\left[\left(w-\tilde{c}_s\right)\left(\tilde{\alpha}-\tilde{\beta}p\right)\right]\\ &\mathrm{s.t.Max}_p\, E\left[\tilde{\Pi}_R\right]=E\left[\left(p-\tilde{c}_r-w\right)\left(\tilde{\alpha}-\tilde{\beta}p\right)\right]\,. \end{aligned}\tag{11}$$

Theorem 1 Let $E\left[\tilde{\Pi}_R\right]$ and $E\left[\tilde{\Pi}_S\right]$ be the fuzzy expected value of the profit for retailer and supplier, then the optimal reaction function of the retailer is

$$p^*(w)=\frac{w}{2}+\frac{E[\tilde{\alpha}]+E\left[\tilde{c}_r\tilde{\beta}\right]}{2E\left[\tilde{\beta}\right]}$$

Proof: The fuzzy expected profit of the retailer is

$$\begin{aligned}E\left[\tilde{\Pi}_R\right]&=\tfrac{1}{2}\int_0^1\left[\left(\tilde{\Pi}_R\right)_\lambda^L+\left(\tilde{\Pi}_R\right)_\lambda^R\right]\mathrm{d}\lambda\\
&=\tfrac{1}{2}\int_0^1\left(\left((p-\tilde{c}_r-w)\left(\tilde{\alpha}-\tilde{\beta}p\right)\right)_\lambda^L+\left((p-\tilde{c}_r-w)\left(\tilde{\alpha}-\tilde{\beta}p\right)\right)_\lambda^R\right)\mathrm{d}\lambda\\
&=\tfrac{1}{2}\int_0^1\left((p-\tilde{c}_r-w)_\lambda^L\left(\tilde{\alpha}-\tilde{\beta}p\right)_\lambda^L+(p-\tilde{c}_r-w)_\lambda^R\left(\tilde{\alpha}-\tilde{\beta}p\right)_\lambda^R\right)\mathrm{d}\lambda\\
&=\tfrac{1}{2}\int_0^1\left(\left(p-(\tilde{c}_r)_\lambda^R-w\right)\left(\tilde{\alpha}_\lambda^L-\tilde{\beta}_\lambda^R p\right)+\left(p-(\tilde{c}_r)_\lambda^L-w\right)\left(\tilde{\alpha}_\lambda^R-\tilde{\beta}_\lambda^L p\right)\right)\mathrm{d}\lambda\\
&=-\tfrac{1}{2}p^2\int_0^1\left(\tilde{\beta}_\lambda^L+\tilde{\beta}_\lambda^R\right)\mathrm{d}\lambda+\tfrac{1}{2}p\int_0^1\left(\tilde{\alpha}_\lambda^L+\tilde{\alpha}_\lambda^R\right)\mathrm{d}\lambda+\tfrac{1}{2}p\int_0^1\left((\tilde{c}_r)_\lambda^L\tilde{\beta}_\lambda^L+(\tilde{c}_r)_\lambda^R\tilde{\beta}_\lambda^R\right)\mathrm{d}\lambda\\
&\quad+\tfrac{1}{2}pw\int_0^1\left(\tilde{\beta}_\lambda^L+\tilde{\beta}_\lambda^R\right)\mathrm{d}\lambda-\tfrac{1}{2}w\int_0^1\left(\tilde{\alpha}_\lambda^L+\tilde{\alpha}_\lambda^R\right)\mathrm{d}\lambda-\tfrac{1}{2}\int_0^1\left((\tilde{c}_r)_\lambda^L\tilde{\alpha}_\lambda^R+(\tilde{c}_r)_\lambda^R\tilde{\alpha}_\lambda^L\right)\mathrm{d}\lambda\\
&=-E\left[\tilde{\beta}\right]p^2+\left(E[\tilde{\alpha}]+E\left[\tilde{c}_r\tilde{\beta}\right]+E\left[\tilde{\beta}\right]w\right)p-E[\tilde{\alpha}]w\\
&\quad-\tfrac{1}{2}\int_0^1\left((\tilde{c}_r)_\lambda^L\tilde{\alpha}_\lambda^R+(\tilde{c}_r)_\lambda^R\tilde{\alpha}_\lambda^L\right)\mathrm{d}\lambda\,.\end{aligned}\tag{12}$$

Notice that the second-order derivative $\frac{\mathrm{d}^2E\left[\tilde{\Pi}_R\right]}{\mathrm{d}p^2}=-2E\left[\tilde{\beta}\right]<0$ since $\tilde{\beta}$ is a positive fuzzy variable.

Consequently, $E\left[\tilde{\Pi}_R\right]$ is a concave function of p. Hence, for any given w, the optimal reaction function of the retailer can be obtained by solving $\frac{\mathrm{d}E\left[\tilde{\Pi}_R\right]}{\mathrm{d}p}=0$, which gives

$$p^*(w)=\frac{w}{2}+\frac{E[\tilde{\alpha}]+E\left[\tilde{c}_r\tilde{\beta}\right]}{2E\left[\tilde{\beta}\right]}\,.$$

Theorem 2 The optimal strategy of the supplier in Fuzzy decentralized decision-making model is

$$w^*=\frac{E[\tilde{\alpha}]+E\left[\tilde{c}_s\tilde{\beta}\right]-E\left[\tilde{c}_r\tilde{\beta}\right]}{2E\left[\tilde{\beta}\right]}\,.$$

Proof: The fuzzy expected profit of the supplier is

$$\begin{aligned}E\left[\tilde{\Pi}_S\right]&=\tfrac{1}{2}\int_0^1\left(\left((w-\tilde{c}_s)\left(\tilde{\alpha}-\tilde{\beta}p\right)\right)_\lambda^L+\left((w-\tilde{c}_s)\left(\tilde{\alpha}-\tilde{\beta}p\right)\right)_\lambda^R\right)\mathrm{d}\lambda\\
&=E[\tilde{\alpha}]w-E\left[\tilde{\beta}\right]wp+E\left[\tilde{c}_s\tilde{\beta}\right]p-\tfrac{1}{2}\int_0^1\left((\tilde{c}_s)_\lambda^L\tilde{\alpha}_\lambda^R+(\tilde{c}_s)_\lambda^R\tilde{\alpha}_\lambda^L\right)\mathrm{d}\lambda\,.\end{aligned}\tag{13}$$

Substituting $p^*(w)$ into (13), we can get the first-order and second-order derivatives of $E\left[\tilde{\Pi}_S\right]$ as

$$\frac{\mathrm{d}E\left[\tilde{\Pi}_S\right]}{\mathrm{d}w}=-E\left[\tilde{\beta}\right]w+\tfrac{1}{2}\left(E[\tilde{\alpha}]+E\left[\tilde{c}_s\tilde{\beta}\right]-E\left[\tilde{c}_r\tilde{\beta}\right]\right)w\,,$$

$$\frac{\mathrm{d}^2E\left[\tilde{\Pi}_S\right]}{\mathrm{d}w^2}=-E\left[\tilde{\beta}\right].$$

Notice that the second-order derivative $\frac{\mathrm{d}^2E\left[\tilde{\Pi}_S\right]}{\mathrm{d}w^2}=-E\left[\tilde{\beta}\right]<0$ since $\tilde{\beta}$ is a positive fuzzy variable.

Consequently, $E\left[\tilde{\Pi}_S\right]$ is a concave function of w. Hence, the optimal strategy of the supplier can be obtained by solving $\frac{\mathrm{d}E\left[\tilde{\Pi}_S\right]}{\mathrm{d}w}=0$, which gives

$$w^*=\frac{E[\tilde{\alpha}]+E\left[\tilde{c}_s\tilde{\beta}\right]-E\left[\tilde{c}_r\tilde{\beta}\right]}{2E\left[\tilde{\beta}\right]}\,.$$

Theorem 3 The optimal strategy of the retailer in fuzzy decentralized decision-making model is

$$p^*=\frac{3E[\tilde{\alpha}]+E\left[\tilde{c}_s\tilde{\beta}\right]+E\left[\tilde{c}_r\tilde{\beta}\right]}{4E\left[\tilde{\beta}\right]}\,,$$

$$E\left[\tilde{Q}\right]^*=\frac{E[\tilde{\alpha}]-E\left[\tilde{c}_s\tilde{\beta}\right]-E\left[\tilde{c}_r\tilde{\beta}\right]}{4}\,.$$

Proof: From Theorems 1 and 2, we can easily obtain Theorem 3.

Therefore, the fuzzy expected profit of the retailer, of the supplier and of the two-stage supply chain are all given as below:

$$\begin{aligned}E\left[\tilde{\Pi}_R\right]^*&=\frac{\left(E[\tilde{\alpha}]-E\left[\tilde{c}_s\tilde{\beta}\right]-E\left[\tilde{c}_r\tilde{\beta}\right]\right)^2}{16E\left[\tilde{\beta}\right]}+\frac{E[\tilde{\alpha}]E\left[\tilde{c}_r\tilde{\beta}\right]}{E\left[\tilde{\beta}\right]}\\
&\quad-\tfrac{1}{2}\int_0^1\left((\tilde{c}_r)_\lambda^L\tilde{\alpha}_\lambda^R+(\tilde{c}_r)_\lambda^R\tilde{\alpha}_\lambda^L\right)\mathrm{d}\lambda\,,\end{aligned}\tag{14}$$

$$\begin{aligned}E\left[\tilde{\Pi}_S\right]^*&=\frac{\left(E[\tilde{\alpha}]-E\left[\tilde{c}_s\tilde{\beta}\right]-E\left[\tilde{c}_r\tilde{\beta}\right]\right)^2}{8E\left[\tilde{\beta}\right]}+\frac{E[\tilde{\alpha}]E\left[\tilde{c}_s\tilde{\beta}\right]}{E\left[\tilde{\beta}\right]}\\
&\quad-\tfrac{1}{2}\int_0^1\left((\tilde{c}_s)_\lambda^L\tilde{\alpha}_\lambda^R+(\tilde{c}_s)_\lambda^R\tilde{\alpha}_\lambda^L\right)\mathrm{d}\lambda\,,\end{aligned}\tag{15}$$

$$E\left[\tilde{\Pi}_{SC}\right]^{*}=\frac{3\left(E[\tilde{\alpha}]-E\left[\tilde{c}_s\tilde{\beta}\right]-E\left[\tilde{c}_r\tilde{\beta}\right]\right)^2}{16E\left[\tilde{\beta}\right]}+\frac{E[\tilde{\alpha}]\left(E\left[\tilde{c}_r\tilde{\beta}\right]+E\left[\tilde{c}_s\tilde{\beta}\right]\right)}{E\left[\tilde{\beta}\right]}$$
$$-\tfrac{1}{2}\int_0^1\left(\left(\tilde{c}_s\right)_\lambda^L\tilde{\alpha}_\lambda^R+\left(\tilde{c}_s\right)_\lambda^R\tilde{\alpha}_\lambda^L\right)\mathrm{d}\lambda-\tfrac{1}{2}\int_0^1\left(\left(\tilde{c}_r\right)_\lambda^L\tilde{\alpha}_\lambda^R+\left(\tilde{c}_r\right)_\lambda^R\tilde{\alpha}_\lambda^L\right)\mathrm{d}\lambda. \quad (16)$$

3.2 Fuzzy centralized decision-making model

Consider a supply chain occupied by an integrated-actor, which can also be regarded as the retailer and the supplier making cooperation. The integrated-actor tries to maximize its fuzzy expected value of profit $E\left[\tilde{\Pi}_{SC}\right]$ by choosing the optimal price of the product *p*, which solves the following model

$$\mathrm{Max}_p\, E\left[\tilde{\Pi}_{SC}\right]=E\left[\left(p-\tilde{c}_r-\tilde{c}_s\right)\left(\tilde{\alpha}-\tilde{\beta}p\right)\right]. \quad (17)$$

Theorem 4 The optimal strategy of the supply chain in fuzzy centralized decision-making model is

$$p^{**}=\frac{E[\tilde{\alpha}]+E\left[\tilde{c}_s\tilde{\beta}\right]+E\left[\tilde{c}_r\tilde{\beta}\right]}{2E\left[\tilde{\beta}\right]},$$
$$E\left[\tilde{Q}\right]^{**}=\frac{E[\tilde{\alpha}]-E\left[\tilde{c}_s\tilde{\beta}\right]-E\left[\tilde{c}_r\tilde{\beta}\right]}{2}.$$

Proof: The fuzzy expected profit of the two-stage supply chain is

$$E\left[\tilde{\Pi}_{SC}\right]=\tfrac{1}{2}\int_0^1\left(\left(\left(p-\tilde{c}_s-\tilde{c}_r\right)\left(\tilde{\alpha}-\tilde{\beta}p\right)\right)_\lambda^L+\left(\left(p-\tilde{c}_s-\tilde{c}_r\right)\left(\tilde{\alpha}-\tilde{\beta}p\right)\right)_\lambda^R\right)\mathrm{d}\lambda$$
$$=-E\left[\tilde{\beta}\right]p^2+\left(E[\tilde{\alpha}]+E\left[\tilde{c}_s\tilde{\beta}\right]+E\left[\tilde{c}_r\tilde{\beta}\right]\right)p$$
$$-\tfrac{1}{2}\int_0^1\left(\left(\tilde{c}_r\right)_\lambda^L\tilde{\alpha}_\lambda^R+\left(\tilde{c}_r\right)_\lambda^R\tilde{\alpha}_\lambda^L\right)\mathrm{d}\lambda$$
$$-\tfrac{1}{2}\int_0^1\left(\left(\tilde{c}_s\right)_\lambda^L\tilde{\alpha}_\lambda^R+\left(\tilde{c}_s\right)_\lambda^R\tilde{\alpha}_\lambda^L\right)\mathrm{d}\lambda. \quad (18)$$

Notice that the second-order derivative $\frac{\mathrm{d}^2E\left[\tilde{\Pi}_{SC}\right]}{\mathrm{d}p^2}=$ $-2E\left[\tilde{\beta}\right]<0$ since $\tilde{\beta}$ is a positive fuzzy variable.

Consequently, $E\left[\tilde{\Pi}_{SC}\right]$ is a concave function of p. Hence, the optimal retail price of the supply chain can be obtained by solving $\frac{\mathrm{d}E\left[\tilde{\Pi}_{SC}\right]}{\mathrm{d}p}=0$, which give

$$p^{**}=\frac{E[\tilde{\alpha}]+E\left[\tilde{c}_s\tilde{\beta}\right]+E\left[\tilde{c}_r\tilde{\beta}\right]}{2E\left[\tilde{\beta}\right]}.$$

The optimal order quantity of the retailer in centralized decision-making model can be obtained by

$$E\left[\tilde{Q}\right]^{**}=E\left[\tilde{\alpha}-\tilde{\beta}p^{**}\right]=E[\tilde{\alpha}]-E\left[\tilde{\beta}\right]p^{**}$$
$$=\frac{E[\tilde{\alpha}]-E\left[\tilde{c}_s\tilde{\beta}\right]-E\left[\tilde{c}_r\tilde{\beta}\right]}{2}.$$

Therefore, the fuzzy expected profit of the two-stage supply chain is given as

$$E\left[\tilde{\Pi}_{SC}\right]^{**}=\frac{4\left(E[\tilde{\alpha}]-E\left[\tilde{c}_s\tilde{\beta}\right]-E\left[\tilde{c}_r\tilde{\beta}\right]\right)^2}{4E\left[\tilde{\beta}\right]}+\frac{E[\tilde{\alpha}]\left(E\left[\tilde{c}_r\tilde{\beta}\right]+E\left[\tilde{c}_s\tilde{\beta}\right]\right)}{E\left[\tilde{\beta}\right]}$$
$$-\tfrac{1}{2}\int_0^1\left(\left(\tilde{c}_r\right)_\lambda^L\tilde{\alpha}_\lambda^R+\left(\tilde{c}_r\right)_\lambda^R\tilde{\alpha}_\lambda^L\right)\mathrm{d}\lambda$$
$$-\tfrac{1}{2}\int_0^1\left(\left(\tilde{c}_s\right)_\lambda^L\tilde{\alpha}_\lambda^R+\left(\tilde{c}_s\right)_\lambda^R\tilde{\alpha}_\lambda^L\right)\mathrm{d}\lambda. \quad (19)$$

Theorem 5 $E\left[\tilde{Q}\right]^{**}>E\left[\tilde{Q}\right]^{*}$, $E\left[\tilde{\pi}_{SC}\right]^{**}>E\left[\tilde{\pi}_{SC}\right]^{*}$.

Proof: Comparing Theorem 3 and Theorem 4, Eq.(16) and Eq.(19), we can easily verify that $E\left[\tilde{Q}\right]^{**}>E\left[\tilde{Q}\right]^{*}$ and $E\left[\tilde{\pi}_{SC}\right]^{**}>E\left[\tilde{\pi}_{SC}\right]^{*}$.

Theorem 5 shows that the retailer's optimal order quantity in fuzzy centralized decision-making is larger than that in fuzzy decentralized decision-making, the total fuzzy expected profit in fuzzy decentralized decision-making is less than the maximum fuzzy supply chain profit in fuzzy centralized decision-making. That is to say, there also exists double marginalization problem in fuzzy environment. Therefore, the supplier has to provide incentive mechanism such as a revenue sharing contract to encourage the retailer to order more to improve the performance of the actors and the channel.

3.3 Fuzzy revenue sharing contract

In a revenue sharing contract, the supplier shares with the retailer a percentage of his revenue. Let Φ be the fraction the retailer keeps, then $(1-\Phi)$ is the fraction the supplier earns. Thus, we can express the fuzzy profit of retailer and supplier as follows

$$\tilde{\Pi}_R=\left(\Phi p-w-\tilde{c}_r\right)\left(\tilde{\alpha}-\tilde{\beta}p\right), \quad (20)$$
$$\tilde{\Pi}_S=\left((1-\Phi)p+w-\tilde{c}_s\right)\left(\tilde{\alpha}-\tilde{\beta}p\right). \quad (21)$$

The retailer tries to maximize its fuzzy expected value of profit $E\left[\tilde{\Pi}_R\right]$ in fuzzy revenue sharing contract by choosing the optimal price of the product *p*, which solves the following model:

$$\mathrm{Max}_p\, E\left[\tilde{\Pi}_R\right]=E\left[\left(\Phi p-w-\tilde{c}_r\right)\left(\tilde{\alpha}-\tilde{\beta}p\right)\right]. \quad (22)$$

Theorem 6 For any $\Phi\in\left(\max\left(\frac{E\left[\tilde{c}_r\tilde{\beta}\right]}{E\left[\tilde{c}_s\tilde{\beta}\right]+E\left[\tilde{c}_r\tilde{\beta}\right]},0.25\right),0.5\right)$, the optimal strategy in fuzzy revenue sharing contract is

$$w^{***}=\frac{\Phi E\left[\tilde{c}_s\tilde{\beta}\right]-(1-\Phi)E\left[\tilde{c}_r\tilde{\beta}\right]}{E\left[\tilde{\beta}\right]}.$$

Proof: The fuzzy expected profit of the retailer $E\left[\tilde{\Pi}_R\right]$ in fuzzy revenue sharing contract is

$$E\left[\tilde{\Pi}_R\right]=-\Phi E\left[\tilde{\beta}\right]p^2+\left(\Phi E[\tilde{\alpha}]+E\left[\tilde{c}_r\tilde{\beta}\right]+E\left[\tilde{\beta}\right]w\right)p$$
$$-E[\tilde{\alpha}]w-\tfrac{1}{2}\int_0^1\left(\left(\tilde{c}_r\right)_\lambda^L\tilde{\alpha}_\lambda^R+\left(\tilde{c}_r\right)_\lambda^R\tilde{\alpha}_\lambda^L\right)\mathrm{d}\lambda. \quad (23)$$

Notice that the second-order derivative $\frac{\mathrm{d}^2E\left[\tilde{\Pi}_R\right]}{\mathrm{d}p^2}=-2\Phi E\left[\tilde{\beta}\right]<0$ since $\tilde{\beta}$ is a positive fuzzy variable and $\Phi>0$. Consequently, $E\left[\tilde{\Pi}_R\right]$ is a concave function of p. Hence, for any given w, the optimal reaction function of the retailer can be obtained by solving $\frac{\mathrm{d}E\left[\tilde{\Pi}_R\right]}{\mathrm{d}p}=0$, which give

$$p^{***}=\frac{E\left[\tilde{\beta}\right]w+\Phi E[\tilde{\alpha}]+E\left[\tilde{c}_r\tilde{\beta}\right]}{2\Phi E\left[\tilde{\beta}\right]}.$$

The optimal order quantity of the retailer in revenue sharing contract can be obtained by

$$E\left[\tilde{Q}\right]^{***}=E\left[\tilde{\alpha}-\tilde{\beta}p^{***}\right]=E[\tilde{\alpha}]-E\left[\tilde{\beta}\right]p^{***}$$
$$=\frac{\Phi E[\tilde{\alpha}]-E\left[\tilde{c}_r\tilde{\beta}\right]-E\left[\tilde{\beta}\right]w}{2\Phi}. \quad (24)$$

In order to fully coordinate of the supply chain, we make $E\left[\tilde{Q}\right]^{***}=E\left[\tilde{Q}\right]^{**}$. From Theorem 4 and Eq.(24), we can obtain

$$w^{***}=\frac{\Phi E\left[\tilde{c}_s\tilde{\beta}\right]-(1-\Phi)E\left[\tilde{c}_r\tilde{\beta}\right]}{E\left[\tilde{\beta}\right]}.$$

Since $w^{***}>0$, we have

$$\Phi\in\left(\frac{E\left[\tilde{c}_r\tilde{\beta}\right]}{E\left[\tilde{c}_s\tilde{\beta}\right]+E\left[\tilde{c}_r\tilde{\beta}\right]},1\right).$$

Therefore, the fuzzy expected profit of the retailer, the supplier can be obtained as

$$E\left[\tilde{\Pi}_R\right]^{***}=\frac{\Phi\left(E[\tilde{\alpha}]-E\left[\tilde{c}_s\tilde{\beta}\right]-E\left[\tilde{c}_r\tilde{\beta}\right]\right)^2}{4E\left[\tilde{\beta}\right]}+\frac{E[\tilde{\alpha}]E\left[\tilde{c}_r\tilde{\beta}\right]}{E\left[\tilde{\beta}\right]}$$
$$-\tfrac{1}{2}\int_0^1\left(\left(\tilde{c}_r\right)_\lambda^L\tilde{\alpha}_\lambda^R+\left(\tilde{c}_r\right)_\lambda^R\tilde{\alpha}_\lambda^L\right)\mathrm{d}\lambda, \quad (25)$$

$$E\left[\tilde{\Pi}_S\right]^{***}=\frac{(1-\Phi)\left(E[\tilde{\alpha}]-E\left[\tilde{c}_s\tilde{\beta}\right]-E\left[\tilde{c}_r\tilde{\beta}\right]\right)^2}{4E\left[\tilde{\beta}\right]}+\frac{E[\tilde{\alpha}]E\left[\tilde{c}_s\tilde{\beta}\right]}{E\left[\tilde{\beta}\right]}$$
$$-\tfrac{1}{2}\int_0^1\left(\left(\tilde{c}_s\right)_\lambda^L\tilde{\alpha}_\lambda^R+\left(\tilde{c}_s\right)_\lambda^R\tilde{\alpha}_\lambda^L\right)\mathrm{d}\lambda. \quad (26)$$

To gain the win-win condition in fuzzy revenue sharing contract, we should ensure that the fuzzy expected profit of each actor is larger than that in fuzzy decentralized decision-making model. Thus, we can have

$$\mathrm{E}\left[\tilde{\Pi}_R\right]^{***}>\mathrm{E}\left[\tilde{\Pi}_R\right]^{*} \text{ and } \mathrm{E}\left[\tilde{\Pi}_S\right]^{***}>\mathrm{E}\left[\tilde{\Pi}_S\right]^{*}.$$

From Eqs. (14) , (15), (25)and (26), we can obtain

$$\max\left(\frac{E\left[\tilde{c}_r\tilde{\beta}\right]}{E\left[\tilde{c}_s\tilde{\beta}\right]+E\left[\tilde{c}_r\tilde{\beta}\right]},0.25\right)<\Phi<0.5.$$

4. A numerical example

In this section, we illustrate the proposed fuzzy model by a numerical example. The parameters are given as follows: $\tilde{\alpha}=(580,600,650,670)$, $\tilde{\beta}=(9,10,11,12)$, $\tilde{c}_r=(1,2,3,4)$ and $\tilde{c}_s=(7,9,10,12)$.

From the Theorem 6, we can obtain the range of contract parameter Φ satisfies the condition $\Phi\in(0.25,0.5)$.

Based on the analysis showed in the section 3, we can get the retail price, the quantity of the retailer and the fuzzy expected profit of the two-stage supply chain in fuzzy decentralized decision-making model and in fuzzy revenue sharing contract as

$p^*=47.17$, $E\left[\tilde{Q}\right]^*=124.06$ and $E\left[\tilde{\Pi}_{SC}\right]^*=4650.43$.

$p^{***}=35.89$, $E\left[\tilde{Q}\right]^{***}=248.12$ and $E\left[\tilde{\Pi}_{SC}\right]^{***}=6116.29$.

The other equilibrium values in fuzzy decentralized decision-making model and fuzzy revenue sharing contract are reported in Table.1.

Table 1: The decisions in fuzzy models for different Φ

	Φ	w	$E\left[\tilde{\Pi}_R\right]$	$E\left[\tilde{\Pi}_S\right]$
Decentralized decision-making	—	33.29	1567.01	3083.42
Revenue sharing contract	0.26	0.58	1625.64	4490.65
	0.28	0.83	1742.91	4373.38
	0.30	1.08	1860.18	4256.11
	0.32	1.32	1977.45	4138.84
	0.34	1.57	2094.72	4021.57
	0.36	1.81	2211.99	3904.30
	0.38	2.06	2329.26	3787.03
	0.40	2.30	2446.52	3669.77
	0.42	2.55	2563.79	3552.50
	0.44	2.80	2681.06	3435.23
	0.46	3.04	2798.33	3317.96
	0.48	3.28	2915.60	3200.69

Table.1 shows that in fuzzy revenue sharing contract, the retailer's optimal order quantity is larger than that in fuzzy decentralized system, the wholesale price is lower than that in fuzzy decentralized decision-making, and each actor's expected profit in fuzzy revenue sharing contract improves a lot compared with that in decentralized decision-making. In addition, because of dominating, the fuzzy expected profit of the leader (the supplier) is greater than that of the follower (the retailer). It also shows that in fuzzy revenue sharing contract, the wholesale price and the retailer's fuzzy expected profit increase with Φ , and the supplier's fuzzy expected profit decrease with Φ . The contract parameter Φ improves the flexibility of supply chain coordination, reflects the demand risk sharing between the supplier and retailer.

5. Conclusions

This paper formulates fuzzy supply chain models in fuzzy linear demand environment, where the supplier and the retailer adopt more reasonable coordination strategy. In order to examine models performance in fuzzy demand, we use cut set theory to solve this problem. The supplier and the retailer can gain win-win condition in revenue sharing contract. The method proposed in this paper is easier to implement and requires less data. It is appropriate when the environment is complex, ambiguous, or there is lack of statistical data.

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