

# Modified PSO for Optimal Tuning of Fuzzy PID Controller

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## Abstract

Fuzzy PID controllers provide a promising approach for industrial applications with many desirable features. However, the large number of parameters and rule bases make self-tuning fuzzy PID controller optimization a complex task. In this paper, a novel tuning method based on the development of the standard particle swarm optimization (PSO) is proposed for optimum design of fuzzy PID controller for multivariable system. The parameters of membership functions and PID gains are optimized using modified PSO which is an efficient and simple tool for multi-dimensional problem. Based on the structure of the modified PSO, each particle in swarm population is divided into number of parts according to the number of inputs-outputs system, which means that each part of particle represents one input-output system controller. The new development in PSO for multi inputs-outputs system is based on tuning all the parts of each particle in swarm population in parallel. The system performance is enhanced by minimizing the error function between all inputs-outputs controller represented by the different parts of particle simultaneously instead of minimizing sum of error of whole inputs-outputs system controllers. The parameters of fuzzy controller and the PID gains are tuned simultaneously. Besides, a design methodology is introduced to combine the classical PID and fuzzy logic controller. The hybrid PID, FLC, and PSO is applied to an aerobic unit in wastewater treatment process for further improvements in steady state error and high system performance. The obtained results show that, the response of the biological system in both transient and steady state has improved significantly compared to both fuzzy PID and fuzzy PID tuned by the standard PSO.

**Keywords:** *Proportional Integral-Derivative (PID) control, Fuzzy logic control (FLC), Fuzzy PID controller and PSO.*

## 1. Introduction

PID controllers have been widely applied in industrial control process for about half century because of their simple structure and convenience of implementation [1]. However, it is hard to obtain optimal tuning for PID controller. Besides, a conventional PID controller may have poor control performance for nonlinear or complex systems for which there are no precise mathematical models. This motivates the interest in using fuzzy logic controller (FLC) which is based on fuzzy logic theory [2-3]. Fuzzy logic has

gradually adopted as one of major approaches for controller design. The conceptual framework of fuzzy logic is much closer to human thinking than the traditional logic systems. Fuzzy controllers are successfully applied to non-linear system because of their knowledge based nonlinear structural characteristics. It does not rely on the model of the system and can deal with nonlinear and stochastic problems. During the past years, control engineers apply fuzzy logic successfully in chemical process control systems, motor drives systems, robot systems, steam turbines systems, medicine diagnosis, and so on. Although fuzzy logic control method is flexible and adaptive, its stability is insufficient.

To exploit the beneficial sides of both above categories and overcome the above mentioned problems with the two methodology, the above two controllers is hybridized for providing a promising option for industrial applications with many desirable features. There have been numerous articles investigating different schemes of applying fuzzy logic to the design of PID controllers, which are generally termed as fuzzy PID controllers. Naturally various hybrid controller structures have been arisen in literatures [4-6]. The well-known pioneered and successful example in early stage was the design of a fuzzy proportional plus integral (PI) controller [7-8]. In some applications, these two control structures are combined using a switch [9]. In [10] a fuzzy switching method between fuzzy controller and conventional PID controllers are used. A multi stage fuzzy (PID) controller is proposed in [11], to solve the Load Frequency Control (LFC) problem in a restructured power system that operates under deregulation based on the bilateral policy scheme. The effort in [12], introduces the implementation of a FLC for the control of fluctuated AC line to a consumer home. The work in [13] proposes developed method to design a digital fuzzy logic controller with the aid of conventional PID controller using field programmable gate array (FPGA), in which Proportional - Derivative Fuzzy Logic Controller (PDFLC) and Proportional -Integral Fuzzy logic controller (PIFLC) connected in parallel through a summer. A hybrid fuzzy PID controller for

the Electro-Hydraulic Position Servo System (EHPSS) was proposed in [14].

Although, fuzzy PID controller is shown to be a versatile for controlling the nonlinear systems, the large number of parameters and rule bases make Self-tuning fuzzy PID controller optimization is a complex task. Some works in literature investigate the optimization of self tuning the parameters of fuzzy PID controller. The work in [15] introduced an optimization method of self tuning fuzzy PID controller for permanent magnet synchronous motor (PMSM) based on adaptive weighted PSO. In this work the performance of self tuning fuzzy PID is determined by FLC in which the optimal FLC makes PID gains attain appropriate values, robust, fast response and fine dynamic performance. The optimization of membership functions and rule bases of FLC in this work is presented by using adaptive weighted PSO. In [16] the parameters of both PID fuzzy controller and SVC-PI controllers are tuned by the adaptive particle swarm optimization (APSO) in order to regulate voltage variation caused by power loading and fault conditions in a power system stabilizer.

This paper proposes a modified PSO for optimum design of fuzzy PID controller to control a wastewater treatment process which is a multivariable nonlinear problem. In the proposed method, the parameters of membership functions of both inputs and outputs variables of fuzzy controllers and their scaling factors of PID gains are optimized by a new development in the standard PSO. In this manner, the PID gains are adaptive and fuzzy PID controller has more flexibility and capability than conventional versions with fixed gains. In the modified PSO, each particle in the population is divided into number of parts represented by the number of inputs-outputs controllers and all particles' parts are tuned in parallel. The development of PSO to control multi inputs-outputs system is based on minimizing all error functions between each input-output system controller simultaneously instead of minimizing sum of errors of whole inputs-outputs system controllers. By exchanging the bad parts represented inputs-outputs system controllers in each updated particle by the corresponding best ones of the particle in previous iteration, a new population is formed for the next iteration. Also, the best particle in this iteration is constituted by combining the best parts from all particles in the population. As, the steady state error is still out the range of the desired values and not equal to zero in our previous work [17], the modified PSO is proposed for solving the mentioned problems. The modified PSO is carried out for tuning fuzzy PID controller to wastewater treatment process, in which the centers and the widths of the triangle membership functions and the PID gains are all parameters to be determined simultaneously. The proposed controller is applied to the aerobic unit of wastewater treatment process for further improvements of the system response in both the transient and steady

state response compared to the system response with fuzzy PID controller without tuning.

The rest of paper is organized as follows: section 2 presents the Biomass dynamic model and activated sludge control design. Section 3 introduces the fuzzy PID controller and the main structure of the fuzzy PID controllers for aerobic unit. An overview of the standard PSO, the framework of the modified PSO and the error function used for evaluating the performance of fuzzy PID controller are described in details in section 4. Experimental results and discussions are presented in section 5. Finally, section 6 concludes the whole work.

## 2. The Biological Wastewater Treatment with an Activated-Sludge Process

Biological processes is the most common method for wastewater treatment in which the important part of the municipal wastewater treatment is the removal of organic matter which is dissolved in wastewater. The removal of organic matter by a biological process is an aerobic process which takes place in the aeration tank, in where the wastewater is aerated with oxygen using an activated sludge. The activated sludge process is probably the most versatile and effective of all waste treatment processes [18] and is usually constituted by a bioreactor (the aeration reactor) and a settler (secondary clarifier) as shown in figure 1.

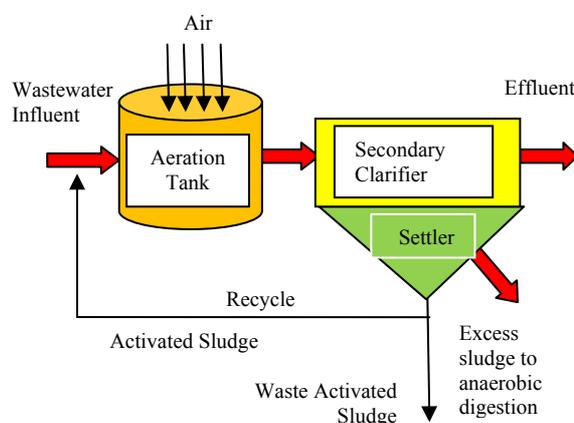


Fig. 1 Schematic diagram of Aerobic treatment unit

The oxygen is injected in the aerator by compressed air and the suspended micro-organisms are separated completely in the settler. In this process, microorganisms in the aeration tank convert dissolved organic material in wastewater to into their own biomass (microbial biomass) and carbon dioxide (CO<sub>2</sub>) [19]. Both of organic nitrogen and organic phosphorus is converted to ammonium ion or nitrate and orthophosphate. The microbial cell matter formed as part of the waste degradation processes is normally kept in the aeration tank until the microorganisms are past the log phase of growth, at which point the cells

flocculate relatively well to form settle-able solids (flocks). These solids collect in the bottom part of a settler and fraction of them is discarded. Part of the solids, the return sludge, is recycled to the head of the aeration tank and comes into contact with fresh sewage. The combination of a high concentration of "hungry" cells in the return sludge and a rich food source in the influent sewage provides optimum conditions for the rapid degradation of organic matter. The Dynamic Model of Activated Sludge process is represented by using the mass balance on the bioreactor and the settler which gives the following set of nonlinear differential equations:

$$X^*(t) = \mu(t)X(t) - D(t)(1+r)X(t) + rD(t)X_r(t) \quad (1)$$

$$S^*(t) = -\mu(t)X(t)/Y - D(t)(1+r)S(t) + D(t)S_{in} \quad (2)$$

$$C^*(t) = -K_o \mu(t)X(t)/Y - D(t)(1+r)C(t) + K_{La}(t)(C_s - C(t)) + D(t)C_{in} \quad (3)$$

$$X_r^*(t) = D(t)(1+r)X(t) - D(t)(\beta+r)X_r(t) \quad (4)$$

Where:

X(t): the state variable representing the biomass,

S(t): the state variable representing the substrate,

X<sub>r</sub>(t): the state variable representing the recycled biomass,

C(t): the state variable representing the dissolved oxygen,

D(t): the dilution rate (D(t) = q(t)/V) where q(t) and V are the influent flow rate and the inner aerator volume respectively,

S<sub>in</sub>: substrate concentrations in the feed stream

C<sub>in</sub>: dissolved oxygen concentrations in the feed stream

K<sub>La</sub>(t): Oxygen transfer rate coefficient, and

r and β : the ratio of recycled flow to influent flow and the ratio of waste flow to influent flow respectively.

The kinetics of the cell mass production are defined in terms of the specific growth rate μ and the yield of cell mass Y; the term K<sub>0</sub> is a constant, C<sub>s</sub> is the maximum dissolved oxygen concentration. In this study, it is assumed that the constants (C<sub>s</sub>, K<sub>0</sub>, Y) and the parameters (r, β) are known. The specific growth rate μ(t) is well defined and modeled by Olsson model, depending on substrate and dissolved oxygen concentrations as in Eq. (5)

$$\mu(t) = \mu_{max} S(t) / (K_s + S(t))(K_c + C(t)) \quad (5)$$

Where: μ<sub>max</sub> is the maximum specific growth rate, K<sub>s</sub> is the affinity constant and K<sub>c</sub> is the saturation constant [20].

## 2.1 Controller Design for Activated Sludge

The design of best controller for activated sludge wastewater treatment process based mathematical model is taking into consideration in this paper. Two main targets in treatment wastewater process must be achieved; the reduction of the organic matter concentration (pollutant substrate S(t)) and the dissolved oxygen concentration (air flow rate W(t))

must be kept above a critical level to maintain the microorganism activity. This quantity appears in equation (3) through the oxygen transfer rate coefficient K<sub>La</sub>(t) as follows:

$$K_{La}(t) = W(t)\alpha \quad \text{where } (\alpha : \text{const} > 0) \quad (6)$$

The objective of the control here is to regulate the substrate S(t) and the dissolved oxygen concentrations C(t) at desired set points S\* and C\* respectively by acting on the dilution rate D(t) and on the aeration rate W(t). The typical values of kinetic parameters and initial conditions are given in [17].

Two controllers are used to achieve the above goals, the first one will act on the air flow rate W(t) to maintain C(t) at the required set point, while the second controller will act on the dilution rate D(t) to maintain substrate concentration S(t) at the required set point. The output of each controller depends on both the error (e) which is defined as the difference between the set point and the controlled variable and error difference (derror) for efficient control. The main objective of our designed controller is to improve both the steady state response and the transient response by reducing the error, the settling time, the rise time, and eliminating or reducing the overshoots without causing sluggish response.

In the following section, the complete activated sludge control system using Fuzzy PID is introduced.

## 3. Fuzzy PID Controller structure

Fuzzy PID controller is often mentioned as an alternative to classical PID controllers for complex and high non linearity cases. It provides a promising option for industrial applications with many desirable features, as it has the ability to on-line adaptation to nonlinear, time varying, and uncertain systems. Fuzzy PID controllers in literature can be classified into three major categories as direct action type, fuzzy gain scheduling type, and hybrid type fuzzy PID controllers. The direct action type can also be classified into three categories according to number of inputs as single input, double input, and triple input direct action fuzzy PID controllers [5]. The most frequent versions of combining fuzzy with PID are: parallel combinations of Fuzzy PI+PD, PD+I, PI+D or P+I+D controllers. Fuzzy PI+PD controller settings can't be equivalent to classical PID controller settings due to double proportional gain included in fuzzy controller structure. The physical connection between integral and derivative time constants remains the same.

Fuzzy PID controller is carried out for wastewater treatment process in which the classical PID and fuzzy controller have been combined by a blending mechanism depending on a certain function of actuating error as follows:

**First**; digitize the conventional analog PI/PD controllers,

**Second**; Make PI/PD gains as two inputs to the fuzzy logic controllers. The structure of Fuzzy PID controller used as a controller for the activated sludge system is shown in figure 2.

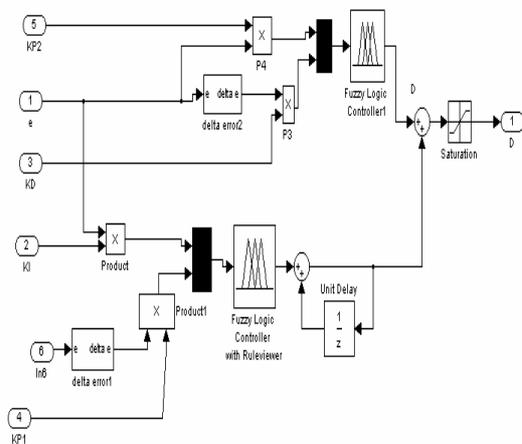


Fig. 2 The Structure of Fuzzy PID controller

Fuzzy controller in the above structure can be adjusted using both the same procedure as for classical controllers besides the procedure in tuning the fuzzy rules and the parameters of membership functions. Concerning the classical controllers, it means that the increase of proportional gain or decrease of integral time constant leads the system to higher oscillations. The oscillations can be compensated to some extent by increasing derivative time constant. In case of slow time response, we can increase the proportional gain or decrease integral time constant to make the response faster which is valid for both non symmetrical and non linear fuzzy set layout. Two Fuzzy PID controllers are implemented for the activated Sludge system; controller1 to control the dissolved oxygen concentration  $W(t)$  and controller2 to control mainly the substrate concentration  $S(t)$  as shown in figure 3.

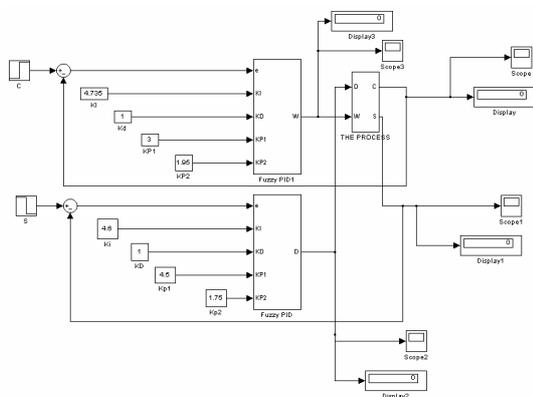


Fig.3 The complete activated sludge control system using two Fuzzy PID controllers.

In the following section, each controller will be introduced with more details.

### 3.1 Controller1

The first fuzzy PID controller consists of fuzzy PI and fuzzy PD and is constituted by both PI and PD parameters like  $K_{I1}$ ,  $K_{P1}$ ,  $K_{P2}$  and  $K_{D1}$  gains and both fuzzy controller parameters. These parameters are the parameters of the membership functions of two inputs (error and error difference) and the corresponding output, the corresponding rules and rule weigh of each one of them. **Fuzzy PD** has the membership functions of error and error difference (derror) of the first input (input1) and its corresponding output air flow rate  $W1(t)$ . Fuzzy PD inputs values are scaled to the interval of  $[-3 \ 3]$  and  $[-15 \ 15]$  and are composed of the five linguistic terms which are: Negative Big (--), Negative Medium (-), Zero (0), Positive Medium (+) and Positive Big (++)). The output of fuzzy PD which is  $W1(t)$  is partitioned into five fuzzy sets which are (VS, S, M, B, and VB). **Fuzzy PI** from fuzzy PID controller has membership functions of error and error difference of the first input (input1) and its corresponding output air flow rate  $W2(t)$ . Fuzzy PI inputs values are scaled to the interval of  $[-15:15]$  and  $[-10:10]$ . The two inputs are composed of the five linguistic terms for error which are Negative Big (nn), Negative Medium (n), Zero (z), Positive Medium (p) and Positive Big (pp) and three linguistic terms for derror which are respectively: negative (n), zero (z) and positive (p). The rules which manage the relation between the two inputs and the corresponding output of both Fuzzy PD and Fuzzy PI controllers of the first controller are given in table 1 and table 2 respectively.

Table 1: Fuzzy PD rules

1. If (error is --) and (derror is --) then (W1 is VS)	(1)
2. If (error is --) and (derror is -) then (W1 is VS)	(1)
3. If (error is --) and (derror is 0) then (W1 is VS)	(1)
4. If (error is --) and (derror is +) then (W1 is S)	(1)
5. If (error is --) and (derror is ++) then (W1 is M)	(1)
6. If (error is -) and (derror is --) then (W1 is VS)	(1)
7. If (error is -) and (derror is -) then (W1 is VS)	(1)
8. If (error is -) and (derror is 0) then (W1 is VS)	(1)
9. If (error is -) and (derror is +) then (W1 is M)	(1)
10. If (error is -) and (derror is ++) then (W1 is B)	(1)
11. If (error is 0) and (derror is --) then (W1 is VS)	(1)
12. If (error is 0) and (derror is -) then (W1 is S)	(1)
13. If (error is 0) and (derror is 0) then (W1 is VB)	(1)
14. If (error is 0) and (derror is +) then (W1 is B)	(1)
15. If (error is 0) and (derror is ++) then (W1 is VB)	(1)
16. If (error is +) and (derror is --) then (W1 is S)	(1)
17. If (error is +) and (derror is -) then (W1 is M)	(1)
18. If (error is +) and (derror is 0) then (W1 is VB)	(1)
19. If (error is +) and (derror is +) then (W1 is VB)	(1)
20. If (error is +) and (derror is ++) then (W1 is VB)	(1)
21. If (error is ++) and (derror is --) then (W1 is M)	(1)
22. If (error is ++) and (derror is -) then (W1 is B)	(1)
23. If (error is ++) and (derror is 0) then (W1 is VB)	(1)
24. If (error is ++) and (derror is +) then (W1 is VB)	(1)
25. If (error is ++) and (derror is ++) then (W1 is VB)	(1)

Table 2: Fuzzy PI rules

1. If (error is z) and (derror is z) then (W2 is z)	(1)
2. If (error is nn) and (derror is n) then (W2 is z)	(1)
3. If (error is nn) and (derror is z) then (W2 is n)	(1)
4. If (error is nn) and (derror is p) then (W2 is p)	(1)
5. If (error is pp) and (derror is n) then (W2 is z)	(1)
6. If (error is pp) and (derror is z) then (W2 is p)	(0.6)
7. If (error is pp) and (derror is p) then (W2 is p)	(1)
8. If (error is n) and (derror is n) then (W2 is n)	(1)
9. If (error is n) and (derror is z) then (W2 is n)	(0.8)
10. If (error is n) and (derror is p) then (W2 is p)	(1)
11. If (error is z) and (derror is n) then (W2 is z)	(1)
12. If (error is z) and (derror is p) then (W2 is z)	(1)
13. If (error is p) and (derror is n) then (W2 is z)	(1)
14. If (error is p) and (derror is z) then (W2 is p)	(1)
15. If (error is p) and (derror is p) then (W2 is p)	(1)

### 3.2 Controller2

Also, the second fuzzy PID controller consists of fuzzy PI and fuzzy PD. It is constituted by both PI and PD parameters like  $K_{I2}$ ,  $K_{P3}$ ,  $K_{P4}$  and  $K_{D2}$  gains and both fuzzy controller parameters. These parameters are the parameters of the membership functions of two inputs (error and error difference) and the corresponding output, the corresponding rules and rule weigh of each one of them. **Fuzzy PD** has the membership functions of error and error difference (derror) of the second input (input2) and its corresponding output air flow rate D1(t). Fuzzy PD inputs values are scaled to the interval of [-7.5 7.5] and [-15 15], both its inputs and output D1 are respectively composed of five linguistic terms which are the same membership functions of fuzzy PD to W1. **Fuzzy PI** of the second controller has the membership functions of error and error difference of the second input (input2) and its corresponding output air flow rate D2(t). Fuzzy PI inputs values are scaled to the interval of [-90: 90] and [-60:60] and composed of five linguistic terms for error and three linguistic terms for derror which are like the membership functions of fuzzy PI for W2. The rules which manage the relation between the two inputs and the corresponding output of both Fuzzy PI and Fuzzy PD of second fuzzy PID is described in table 3 and 4.

Table 3: Fuzzy PI rules

1. If (error is z) and (derror is z) then (D2 is z)	(1)
2. If (error is nn) and (derror is n) then (D2 is z)	(1)
3. If (error is nn) and (derror is z) then (D2 is n)	(1)
4. If (error is nn) and (derror is p) then (D2 is p)	(1)
5. If (error is pp) and (derror is n) then (D2 is z)	(1)
6. If (error is pp) and (derror is z) then (D2 is n)	(1)
7. If (error is pp) and (derror is p) then (D2 is p)	(1)
8. If (error is n) and (derror is n) then (D2 is n)	(1)
9. If (error is n) and (derror is z) then (D2 is z)	(1)
10. If (error is n) and (derror is p) then (D2 is z)	(1)
11. If (error is z) and (derror is n) then (D2 is z)	(1)
12. If (error is z) and (derror is p) then (D2 is z)	(1)
13. If (error is p) and (derror is n) then (D2 is n)	(1)
14. If (error is p) and (derror is z) then (D2 is p)	(1)
15. If (error is p) and (derror is p) then (D2 is p)	(1)

Table 4: Fuzzy PD rules

1. If (error is --) and (derror is --) then (D1 is B)	(0.1)
2. If (error is --) and (derror is -) then (D1 is VS)	(0.1)
3. If (error is --) and (derror is 0) then (D1 is VS)	(0.9)
4. If (error is --) and (derror is +) then (D1 is S)	(0.9)
5. If (error is --) and (derror is ++) then (D1 is M)	(0.1)
6. If (error is -) and (derror is --) then (D1 is VS)	(0.1)
7. If (error is -) and (derror is -) then (D1 is VS)	(0.1)
8. If (error is -) and (derror is 0) then (D1 is S)	(0.9)
9. If (error is -) and (derror is +) then (D1 is M)	(0.1)
10. If (error is -) and (derror is ++) then (D1 is B)	(0.1)
11. If (error is 0) and (derror is --) then (D1 is VS)	(0.1)
12. If (error is 0) and (derror is -) then (D1 is S)	(0.1)
13. If (error is 0) and (derror is 0) then (D1 is M)	(0.9)
14. If (error is 0) and (derror is +) then (D1 is B)	(0.1)
15. If (error is 0) and (derror is ++) then (D1 is VB)	(0.1)
16. If (error is +) and (derror is --) then (D1 is S)	(0.1)
17. If (error is +) and (derror is -) then (D1 is M)	(0.1)
18. If (error is +) and (derror is 0) then (D1 is B)	(0.9)
19. If (error is +) and (derror is +) then (D1 is VB)	(0.1)
20. If (error is +) and (derror is ++) then (D1 is VB)	(0.1)
21. If (error is ++) and (derror is --) then (D1 is M)	(0.1)
22. If (error is ++) and (derror is -) then (D1 is B)	(0.1)
23. If (error is ++) and (derror is 0) then (D1 is VB)	(0.9)
24. If (error is ++) and (derror is +) then (D1 is VB)	(0.1)
25. If (error is ++) and (derror is ++) then (D1 is VB)	(0.1)

The correct choice of membership functions of fuzzy sets and PID gains plays an essential role in the performance of Fuzzy PID. The following section introduces an overview of PSO as well as the optimum design of membership functions of FLC and the best PID gains using the modified PSO.

## 4. PSO Algorithm for Tuning Fuzzy PID

### 4.1 Overview of PSO

PSO introduced by Kennedy and Eberhart in 1995 is one of the most important swarm intelligence paradigms [21-23]. PSO uses a simple mechanism that mimics swarm behavior likes birds flocking to guide the particles to search for globally optimal solutions. As PSO is easy to implement, it has rapidly progressed in recent years and with many successful applications seen in solving real-world optimization problems [24-30]. Similar to other evolutionary computation algorithms, PSO is also a population-based iterative algorithm. It is initialized with a group of random particles (solutions) and then searches for optimum by updating generations. In every new generation, each particle is updated by two "best" values. The first one is the best solution (fitness) it has achieved so far and it is called *pbest*. The other best value is the global best in the whole swarm and it is called *gbest*. After finding the two best values, the particle updates its velocity and positions using the equations in [17]. This paper presents a development in the conventional PSO in which each particle in swarm population is first divided into number of parts according to the number of inputs-outputs system means each part of particle represents one input-output system controller. The modified PSO for multi inputs-outputs system is based

on tuning all the parts of each particle in swarm population in parallel. The framework of the modified PSO is described as follows:

**The framework of the modified PSO**

1. Generate the initial position and velocity randomly for each part represented one input-output system controller in particle according to its upper and lower values of the parameters of each fuzzy PID controller in system to form parents.
2. Evaluate the fitness ( $p_{best}$ ) for each part in each particle in the swarm.
3. Determine the global best fitness ( $g_{best}$ ) for all different parts from all particles in the swarm to find the best fuzzy PID controllers represented by the best parts in the swarm.
4. Evaluate the global best fitness of all particles in the population by summing all the best global fitness for all parts from each particle in swarm.
5. Update each particle to form offspring.
6. Compare the fitness  $p_{best}$  of each part of offspring with their corresponding parts in parents and select the best ones to form new children to the next generation.
7. Determine the global fitness ( $g_{best}$ ) of the different parts in the population according to the new children and summing to be the best ones for the next generation.
8. Stop if the stopping criterion is satisfied otherwise, go to step 5.

**4.2 The Proposed PSO Algorithm for Tuning Fuzzy PID**

The modified PSO is proposed for tuning the parameters of fuzzy PID controllers. As mentioned before, two fuzzy PID controllers are utilized for wastewater treatment process and each of them is implemented using fuzzy PI and fuzzy PD controller. Both of fuzzy PI and fuzzy PD have three variables; two inputs (error and derror) and one output. Fuzzy PD of each fuzzy PID controller has five fuzzy sets for each variable which corresponding to 15 MFs and 25 rules. Each fuzzy set is triangle shape and is represented by three parameters which are x-coordinates of the three vertices of the triangle. Consequently, there are 45 parameters (3x15) can be tuned for each fuzzy PD controller, means 90 parameters for the two fuzzy PD controllers. Also, each fuzzy PI in fuzzy PID controller has two inputs and one output, the first input with five fuzzy sets while the other input and its output have three fuzzy sets represented by three parameters for each one, which means 33 parameters (5x3+3x3x2) for each PI controller. This means 66 parameters for the two fuzzy PI controllers plus the above fuzzy PD parameters to give 156 parameters for the two fuzzy PID controller means. Besides, each fuzzy PID controller has four

gains which mean eight parameters for two controllers. Consequently, the total number of parameters for system controllers equals 164 elements means 82 parameters for each fuzzy PID controller to be tuned. PSO searches all of the antecedent and consequent parameters of fuzzy controllers (inputs and outputs) and their PID gains in 164 dimensional spaces. Therefore, each particle in the swarm population has 164 elements (genes) with the following order:

$$\begin{aligned}
 P_i = & \\
 & a_1^l \ b_1^l \ c_1^l \dots a_1^r \ b_1^r \ c_1^r \ a_2^l \ b_2^l \ c_2^l \dots a_2^r \ b_2^r \ c_2^r \ a_3^l \ b_3^l \ c_3^l \dots a_3^r \ b_3^r \ c_3^r \\
 & a_4^l \ b_4^l \ c_4^l \dots a_4^r \ b_4^r \ c_4^r \ a_5^l \ b_5^l \ c_5^l \dots a_5^r \ b_5^r \ c_5^r \ a_6^l \ b_6^l \ c_6^l \dots a_6^r \ b_6^r \ c_6^r \\
 & a_7^l \ b_7^l \ c_7^l \dots a_7^r \ b_7^r \ c_7^r \ a_8^l \ b_8^l \ c_8^l \dots a_8^r \ b_8^r \ c_8^r \ a_9^l \ b_9^l \ c_9^l \dots a_9^r \ b_9^r \ c_9^r \\
 & a_{10}^l \ b_{10}^l \ c_{10}^l \dots a_{10}^r \ b_{10}^r \ c_{10}^r \ a_{11}^l \ b_{11}^l \ c_{11}^l \dots a_{11}^r \ b_{11}^r \ c_{11}^r \ a_{12}^l \ b_{12}^l \ c_{12}^l \dots a_{12}^r \ b_{12}^r \ c_{12}^r \\
 & K_{P1}, K_{P2}, K_{D1}, K_{D2}, K_{P3}, K_{P4}, K_{D2}
 \end{aligned}
 \tag{7}$$

Where: b, a, and c represent the center and both the left and right deviations from the center of triangle membership (x-coordinate of the three vertices). In the above equation, the first and the second lines consist of the parameters of membership functions of two inputs and the corresponding output of fuzzy PD and fuzzy of the first fuzzy PID controller. Also, the third and fifth lines constitute the parameters of fuzzy PD and fuzzy PI of the second fuzzy PID controller. Finally, the last line represents the parameters of PID gains of both fuzzy PID controllers.

The initial values to the first particle from element one to element 156 representing fuzzy parameters are generated with the normal values by equally dividing the range of each input and output on the corresponding fuzzy sets. On the other hand, the corresponding elements of the remaining particles in the population are randomly generated in the first generation by associating an interval of performance for each element in those particles. Besides, the elements from 157 to 164 in all particles of the first generation are generated within its upper and lower values. The interval of performance from elements 1 to 156 will be the interval of adjustment for each correspondent variable and is described in details as follows.

The variables a, b and c from each fuzzy set has interval of performance that are:  $a \in (a^l, a^r)$ ,  $b \in (b^l, b^r)$  and  $c \in (c^l, c^r)$

These variables are described as follows:

$$a \in [a^l, a^r] = \left[ a - \frac{b-a}{2}, a + \frac{b-a}{2} \right] \tag{8}$$

$$b \in [b^l, b^r] = \left[ b - \frac{b-a}{2}, b + \frac{c-b}{2} \right] \tag{9}$$

$$c \in [c^l, c^r] = \left[ c - \frac{c-b}{2}, c + \frac{c-b}{2} \right] \tag{10}$$

The most crucial step in applying the modified PSO is to choose both the best membership parameters and PID gains of each controller by minimizing the error function between each input-output controller.

Consequently, the performance of each fuzzy PID controller is evaluated by the fitness of the corresponding part in the particle which is depending on the value of the used cost function. Two different cost functions are used to investigate the performance of each fuzzy PID controller tuned by the proposed PSO like Mean of Squared Error (MSE) and integral of Absolute Magnitude of the Error (IAE) which are described as follows:

**- Mean of the Square of the Error (MSE)**

$$I_{MSE1} = \frac{1}{n} \sum_{t=0}^n (e_1(t))^2 \tag{11}$$

$$I_{MSE2} = \frac{1}{n} \sum_{t=0}^n (e_2(t))^2$$

**-Integral of Absolute Magnitude of the Error (IAE)**

$$I_{IAE1} = \int_{t=0}^n |e_1(t)| dt \tag{12}$$

$$I_{IAE2} = \int_{t=0}^n |e_2(t)| dt$$

Where:  $e_1$  and  $e_2$  are the errors between system input1 and input2 and their corresponding outputs  $W(t)$  and  $D(t)$  calculated over a time interval ( $t$ ) respectively, and  $n$  is the number of samples. The performance of controlled system is enhanced by minimizing the above objective functions for each controller in parallel as follows:

$$I_{MSE \text{ system}} = \text{minimize}(I_{MSE1}) + \text{minimize}(I_{MSE2})$$

$$I_{IAE \text{ system}} = \text{minimize}(I_{IAE1}) + \text{minimize}(I_{IAE2})$$

The effectiveness of the proposed PSO for tuning fuzzy PID controller in comparison with the fuzzy PID tuned by the standard PSO and fuzzy PID is tested using the above two performance indices. The plant system with the tuned fuzzy PID parameters using the modified PSO is shown in figure 4. Membership functions of the two fuzzy PID controllers tuned by the modified PSO with IAE objective function are shown in the figure 5

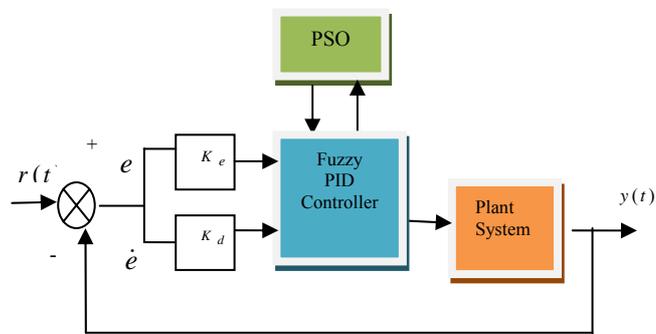


Fig. 4 The structure of the proposed algorithms in tuning Fuzzy PID controller for the plant system

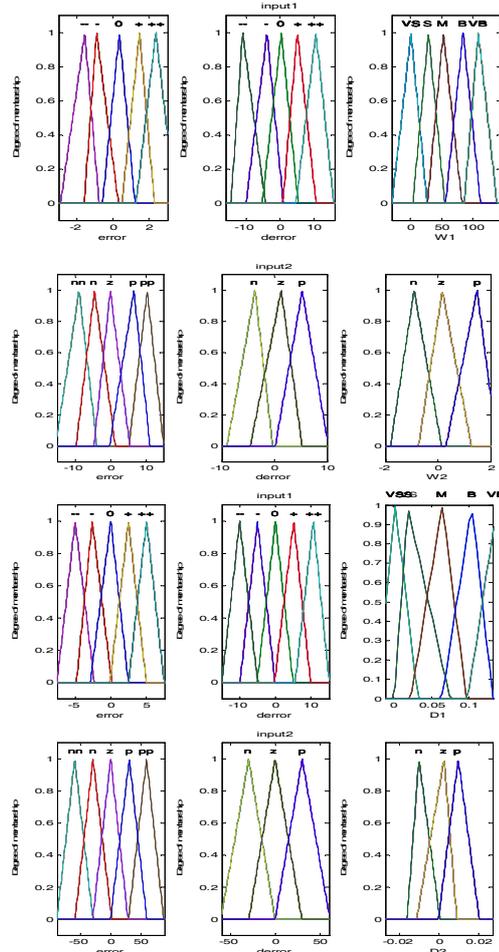


Fig. 5 The Membership function of fuzzy sets of inputs-outputs system with two fuzzy PID controller after tuning by the modified PSO and using IAE

**5. Simulation Results**

To test the performance of the system for sudden change situation and to check the robustness of the controllers, two set points for each controlled variable are applied in interval 100 hour (h). The set points represent the upper and lower bound of the controlled variable as depicted in table 5.

Table 5: The set points for both substrate and dissolved oxygen

Time interval	Dissolved oxygen set points C*	substrate set points S*
0 < t < 50 h	5mg/l	50mg/l
50 < t < 100h	6.5mg/l	30mg/l

The set point for each controller will take the shape of step representing the controlled variable bounds. To verify the efficiency of the proposed algorithm, the experiments have been carried for optimal tuning fuzzy PID controller to wastewater treatment process. The performance of fuzzy PID controller tuned by the modified PSO using IAE, and MSE are compared with

the fuzzy PID controller tuned by the standard PSO and fuzzy PID controller without tuning. The cost function is calculated as an average over 10 runs for 30 generations. The resulted time response of two outputs system using the two performance indices are shown in Figures 6-9 respectively. Also, the cost functions for the two performance indices are shown in Figures 10-11. Tables 6-9 give comparison of the transient response characteristics for the two outputs  $C(t)$  and  $S(t)$  with modified PSO-fuzzy PID controller and both the standard PSO- fuzzy PID and fuzzy PID controllers using IAE and MSE performance indices.

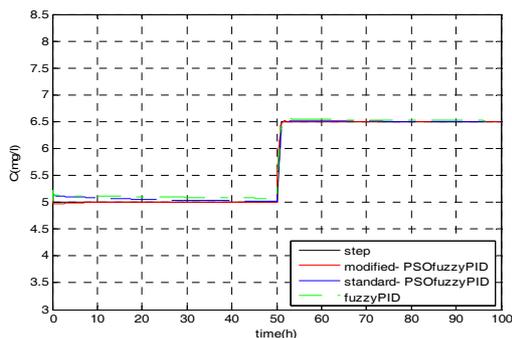


Fig.6 The output value of  $C(t)$  with modified PSO-Fuzzy PID and both the standard PSO- Fuzzy PID and fuzzy PID controller using IAE

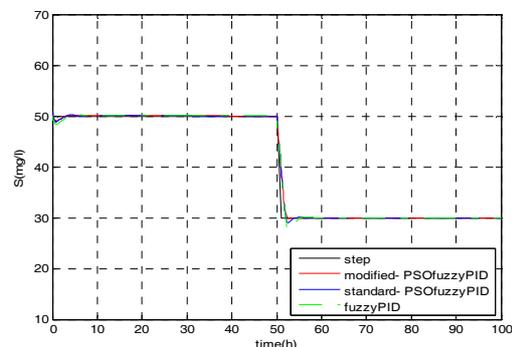


Fig.7 The output value of  $S(t)$  with modified PSO-Fuzzy PID and both the standard PSO- Fuzzy PID and fuzzy PID controller using IAE

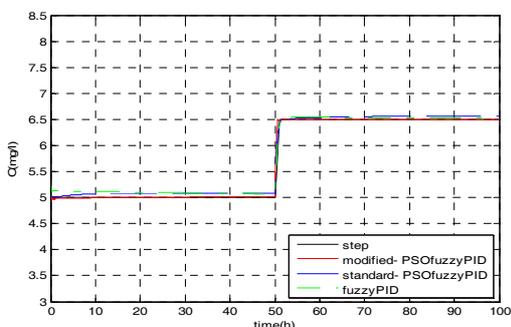


Fig.8 The output value of  $C(t)$  with modified PSO-Fuzzy PID and both the standard PSO- Fuzzy PID and fuzzy PID controller using MSE

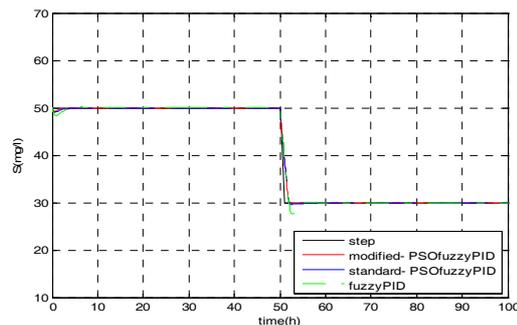


Fig.9 The output value of  $S(t)$  with modified PSO-Fuzzy PID and both the standard PSO- Fuzzy PID and fuzzy PID controller using MSE

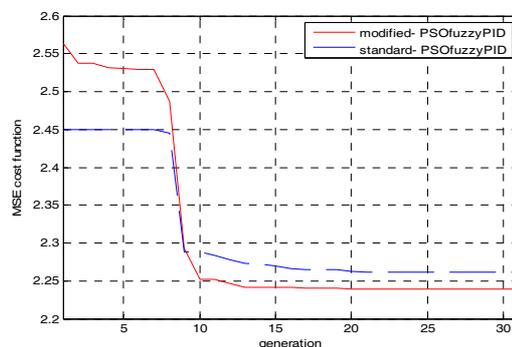


Fig. 10 Cost function using modified PSO-fuzzy PID and the standard PSO-fuzzy PID controller with MSE

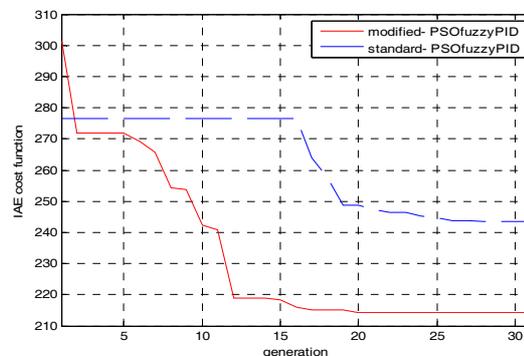


Fig. 11 Cost function using modified PSO-fuzzy PID and the standard PSO-fuzzy PID controller with IAE

Table 6: Transient response characteristics of dissolved oxygen concentration and steady state error of wastewater treatment process using IAE Criteria

IAE criterion	FLC controller		Standard PSO-Fuzzy PID controller		Modified PSO-Fuzzy PID controller	
	SP=5	SP=6.5	SP=5	SP=6.5	SP=5	SP=6.5
$t_r$ (h.)	25.1	.95	37.2	1.02	.07	.84
$M_p$	2.8%	1%	2.7%	0.5%	0%	0.1%
$t_s$ (h.)	48.3	4.5	45.7	2.1	21.9	1.1
Ess	1.54%	0.07%	.7%	0.4%	0%	0%

Table 7: Transient response characteristics of substrate concentration and Steady State Error of wastewater treatment process using IAE Criteria

IAE criterion	FLC controller		Standard PSO-Fuzzy PID controller		Modified PSO-Fuzzy PID controller	
	SP=50	SP=30	SP=50	SP=30	SP=50	SP=30
t <sub>r</sub> (h.)	.16	1.95	.15	1.7	.15	1.7
M <sub>p</sub>	0.1%	0.16%	0.02%	0.01%	0%	0%
t <sub>s</sub> (h.)	7.9	4.5	5.6	3.4	5.6	2.1
ess	0.1%	0.02%	.02%	.01%	0%	0%

Table 8: Transient response characteristics of dissolved oxygen concentration and steady state error of wastewater treatment process using MSE Criteria

MSE criterion	FLC controller		Standard PSO-Fuzzy PID controller		Modified PSO-Fuzzy PID controller	
	SP=5	SP=6.5	SP=5	SP=6.5	SP=5	SP=6.5
t <sub>r</sub> (h.)	.02	.95	19.49	1.38	.002	.43
M <sub>p</sub>	2.4%	1.2%	2.2%	0.9%	0.3%	0.1%
t <sub>s</sub> (h.)	46.3	4.5	24.28	12.7	24	1.17
ess	2.3%	1%	2%	0.8%	0%	0%

Table 9: Transient response characteristics of substrate concentration and Steady State Error of wastewater treatment process using MSE Criteria

MSE criterion	FLC controller		Standard PSO-Fuzzy PID controller		Modified PSO-Fuzzy PID controller	
	SP=50	SP=30	SP=50	SP=30	SP=50	SP=30
t <sub>r</sub> (h.)	.06	1.69	.05	1.69	.05	1.69
M <sub>p</sub>	1%	0.1%	.02%	0.01%	0%	0%
t <sub>s</sub> (h.)	7.5	4.6	3.6	2	3.4	2
ess	.02%	.01%	.01%	0%	0%	0%

Simulation results demonstrate the superiority of the modified PSO-Fuzzy PID controller comparing with both the standard PSO-Fuzzy PID and fuzzy PID controllers. As shown and comparing to the above mentioned controller, the modified PSO-Fuzzy PID controller has a lower overshoot also it has minimum settling time, and concerning the steady state error, the modified PSO-Fuzzy PID achieve lower error comparing with the other controllers.

## 6. Conclusion

In this paper a modified PSO for tuning both the parameters of membership functions of fuzzy logic controller and PID gains was proposed. The development of PSO for multi inputs-outputs system controllers is based on minimizing all the error

functions between each input-output system in parallel instead of minimizing sum of errors of the whole inputs-outputs controllers. The proposed algorithm was applied to control the aerobic unit of wastewater treatment process which considered as a multivariable nonlinear system. Two Fuzzy PID controllers have been implemented, one to control the dissolved oxygen concentration through acting on air flow rate, and the other to control the substrate concentration through acting on the dilution rate. The performance of the proposed algorithm has been analyzed based on two performance indices; IAE and MSE. The modified PSO for tuning fuzzy PID controllers was compared with fuzzy PID and the standard PSO for tuning fuzzy PID. Experimental results showed the superiority of the modified PSO-Fuzzy PID controller over the standard PSO-Fuzzy PID and fuzzy PID in metric of time response characteristic and steady state error value.

For IAE, the settling time taken by the modified PSO-Fuzzy PID decreased by (52% to 47%) compared with standard PSO Fuzzy PID, and by (54% to 75.5%) compared with fuzzy PID for the dissolved oxygen at set-points 5 and 6.5 respectively. Also for the substrate at set-points 50 and 30, the settling time of modified PSO-Fuzzy PID is (29% to 53%) less than standard PSO Fuzzy PID and (0% to 38%) less than Fuzzy PID. With MSE, the settling time taken by the modified PSO-Fuzzy PID decreased by (1.1% to 90%) compared with standard PSO Fuzzy PID, and by (48% to 74 %) compared with fuzzy PID for the dissolved oxygen at set-points 5 and 6.5 respectively. Also for the substrate at set-points 50 and 30, the settling time of the modified PSO-Fuzzy PID is (5.5% to 0%) less than the standard PSO Fuzzy PID and (54.6% to 56%) less than Fuzzy PID. The steady state error equals zero with the modified PSO-Fuzzy PID compared with the other controllers. Finally, the system with the modified PSO has lower overshoot which reaches to zero compared with the other controllers using either MSE or IAE.

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