Explicit travelling wave solutions in a magneto-electro-elastic circular rod

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Abstract

The abstract A modified (G'/G)-expansion method is proposed for constructing exact travelling wave solutions of nonlinear wave equations, and this method finds travelling wave solutions in a straightforward manner and in a neat and helpful form than (G'/G)-expansion method. The abundant exact travelling wave solutions of nonlinear longitudinal wave equation(NLWE) with dispersion caused by the transverse Poisson's effect in a long magneto-electro-elastic(MEE) circular rod are successfully obtained by the modified (G'/G)-expansion method. The relation between solitary wave velocity with wave number are derived strictly. Numerical examples are further presented for the wave in a rod made of five different materials. The obtained results show that the solitary wave not only exists in such rods but also shows different features in different materials, which could have potential applications in non-destructive evaluation of structures made of the advanced MEE material.

Keywords: *magneto-electro-elastic(MEE), modified (G'/G)-expansion method, Riccati like equation, travelling wave solution, exact solution.*

1. Introduction

The In the last two decades, nonlinear elastic effects on solitary waves have received considerable attention in solid mechanics[1-6]. With increasing usage of magneto-electro-elastic (MEE) structures in various engineering fields (such as sensors, actuators, etc), wave propagation in MEE media has also attracted many researchers[7-10]. Very recently, Xue et. al.[11] had derived the longitudinal wave equation with dispersion caused by the transverse Poisson's effect in a MEE circular rod, and the solitary waves had been successfully derived by Jacobi elliptic function method, the NLWE reads:

$$\frac{\partial^2 u}{\partial t^2}-c_0^2\frac{\partial^2 u}{\partial z^2}=\frac{\partial^2}{\partial z^2}\left(\frac{c_0^2}{2}u^2+N\frac{\partial^2 u}{\partial t^2}\right), \qquad (1)$$

where c_0 is the linear longitudinal wave velocity for a MEE circular rod and N is the dispersion parameter, both depending on the material properties as well as the geometry of the rod.

Here, assume the infinite homogeneous MEE circular rod is mad of composite $\mathrm{BaTiO_3}$-$\mathrm{CoFe_2O_4}$ with different volume fractions (v_f) of $\mathrm{BaTiO_3}$, The rod has a radius R =0.05m. The material properties of the composite are estimated using the simple rule of mixture according to the volume fraction[11]. Denoting for the composite the volume fraction of $\mathrm{BaTiO_3}$ as v_f , and that of $\mathrm{CoFe_2O_4}$ as $1-v_\mathrm{f}$, we then have $M_C=M_\mathrm{E}v_\mathrm{f}$ $+M_\mathrm{M}(1-v_\mathrm{f})$,where M represents an arbitrary material constant, and the subscripts C, E, and M indicate the composite, piezoelectric phase and piezomagnetic phase, respectively. In the following, we consider three different cases of material combinations, by taking the volume fraction of $\mathrm{BaTiO_3}$ as 0% (PM), 50% (MEE) and 100% (PE), respectively. Obviously, when v_f =0, the composite is piezomagnetic (PM), whilst v_f =100% corresponds to a piezoelectric (PE) material[14]. Another two purely elastic materials are also considered. One is the transversely isotropic elastic material (TI) taking from 50% (MEE) only the elastic coefficients. The other one is the effective elastic isotropy (EI) obtained from the TI by making it isotropic (i.e., letting $c_{11}=c_{33}$ and $c_{12}=c_{13}$). Xue et al.[11] have calculated the linear wave velocity c_0 , dispersion parameter N , as listed in table 1.

On the other hand, searching for explicit solutions of nonlinear wave equations by using various methods has being a main goal for many authors, and many powerful methods to construct explicit solutions of nonlinear wave equations have been established and developed, such as the tanh-function expansion method, the extended tanh-function method, the F-expansion method, the sub-ODE method, the Jacobi elliptic function expansion method, the homogeneous balance method, the Exp-function method, the (G'/G)-expansion method, the sine-cosine method [12,15-24], and so on. The above methods derived many exact solutions from most nonlinear wave equations.

Table 1: Linear wave velocity and dispersion parameter for different material

v_f	$c_0(\mathrm{ms}^{-1})$	$N(\times 10^{-4}\mathrm{m}^2)$
0%(PM)	5.2131	1.7350
50%(MEE)	5.1446	1.4890
100%(PE)	5.0498	1.0560
TI	4.8003	1.5700
EI	4.8398	1.6200

Based on the main idea of the (G'/G)-expansion method and the extended tanh-function method, we introduced a new method called modified (G'/G)-expansion method. This new method contains more parameters than the (G'/G)-expansion method, and all the solutions obtained by the (G'/G)-expansion method can be obtained by the modified (G'/G)-expansion method. Moreover, the modified (G'/G)-expansion method can obtain some new exact solutions in a neat and helpful form, and some of them can not be obtained by (G'/G)-expansion method.

So far, however, there has been no report on exact travelling wave solution of nonlinear wave equations of a MEE circular rod, which motivates this study, we will apply the modified (G'/G)-expansion method to construct the exact travelling wave solutions of nonlinear wave equations of a MEE circular rod. Therefore, this paper is organized as follows, in section 2, we describe the basic idea of the modified (G'/G)-expansion method. In Section 3, we apply the modified (G'/G)-expansion method to solve nonlinear wave equations of a MEE circular rod for exact traveling wave solution. Numerical examples are given in section 4 and conclusions are drawn in section 5.

2. Basic idea of the modified (G'/G)-expansion method

In this section, according to Wang's work [22], we describe basic idea of the (G'/G)-expansion method for finding travelling wave solutions of nonlinear partial differential equations. Suppose that a nonlinear equation, say in two independent variables x and t, is given by

$$F(u,u_t,u_x,u_{tt},u_{xt},u_{xx},\cdots)=0, \tag{2}$$

where $u=u(x,t)$ is an unknown function, F is a polynomial in $u=u(x,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following steps, we give the main steps of the modified (G'/G)-expansion method.

Step 1. Use the travelling wave transformation:

$$u=u(x,t)=u(\xi),\quad \xi=k(x-Vt+\xi_0), \tag{3}$$

where k and V is a constant to be determined later, ξ_0 is an arbitrary constant. The travelling wave variable (3) permits us to reduce (2) to an ODE for $u=u(\xi)$

$$F(u,-kVu',ku',k^2V^2u'',-k^2Vu'',k^2u'',\cdots)=0. \tag{4}$$

Step 2. Suppose that the solution of ODE (4) can be expressed by a polynomial in $(\frac{G'}{G}+\frac{\lambda}{2})$ as follows:

$$u(\xi)=\sum_{i=-m}^{m}\alpha_i(\frac{G'}{G}+\frac{\lambda}{2})^i, \tag{5}$$

where $|\alpha_{-m}|+|\alpha_m|\neq 0$, and $G=G(\xi)$ satisfies the second order LODE in the form

$$G''+\lambda G'+\mu G=0, \tag{6}$$

where prime denotes derivative with respect to ξ, $\alpha_i(i=\pm1,\pm2,\cdots,\pm m)$, λ and μ are constants to be determined later. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in ODE (4).

From the second order LODE (6), after some manipulation we find that

$$(\frac{G'}{G}+\frac{\lambda}{2})'=h-(\frac{G'}{G}+\frac{\lambda}{2})^2, \tag{7}$$

where $h=(\lambda^2-4\mu)/4$, and the h is determined by λ and μ.

So, $(\frac{G'}{G}+\frac{\lambda}{2})$ now satisfies the Riccati like equation (7). It is found that the Riccati like equation (7) admits several types of solutions[16]

$$\frac{G'}{G}+\frac{\lambda}{2}=\begin{cases}\sqrt{h}\tanh(\sqrt{h}\xi), & h>0;\\ \sqrt{h}\coth(\sqrt{h}\xi), & h>0;\\ 1/\xi, & h=0;\\ -\sqrt{-h}\tan(\sqrt{-h}\xi), & h<0;\\ \sqrt{-h}\cot(\sqrt{-h}\xi), & h<0.\end{cases}$$

(8)

Step 3. By substituting (5) into (4) and using first order ODE (7), collecting all terms with the same order of

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$(\frac{G'}{G}+\frac{\lambda}{2})$ together, the left-hand side of Eq. (4) is converted into another polynomial in $(\frac{G'}{G}+\frac{\lambda}{2})$. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $\alpha_i(i=\pm1,\pm2,\cdots,\pm m)$, V, λ and μ.

Step 4. Assume that the constants $\alpha_i(i=\pm1,\pm2,\cdots,\pm m)$, V, λ and μ can be obtained by solving the algebraic equations in Step 3. And the general solutions of the Riccati like equation (7) has been well known for us, as (8). And then substituting $\alpha_i(i=\pm1,\pm2,\cdots,\pm m)$, V and the general solutions (8) into (5) we have more travelling wave solutions of (2).

3. Exact travelling wave solution of NLWE in a MEE circular rod

To construct exact travelling wave solution of the nonlinear longitudinal wave equation in a magneto-electro-elastic circular rod by the modified (G'/G)-expansion method. By using the transformation

$$u=u(z,t)=u(\xi),\quad \xi=k(z-Vt+\xi_0), \tag{9}$$

where k and V are the wave number and wave velocity, respectively, ξ_0 is an arbitrary real constant. Then Eq. (1) can be converted into an ordinary differential equation (ODE) for $u(\xi)$, we have

$$k^2u'''' + \frac{c_0^2-V^2}{NV^2}u'' + \frac{c_0^2}{2NV^2}(u^2)''=0, \tag{10}$$

where prime denotes derivative with respect to ξ.

Integrating Eq.(10) twice with respect to ξ, and letting the integral constants be zero, we then have

$$k^2u'' + \frac{c_0^2-V^2}{NV^2}u + \frac{c_0^2}{2NV^2}u^2=0. \tag{11}$$

Balancing u'' with u^2 in Eq. (11) gives $m=2$. This means that we can write (5) as

$$\begin{aligned} u(\xi)=\alpha_2(\frac{G'}{G}+\frac{\lambda}{2})^2+\alpha_1(\frac{G'}{G}+\frac{\lambda}{2})+\alpha_0+ \\ \alpha_{-1}(\frac{G'}{G}+\frac{\lambda}{2})^{-1}+\alpha_{-2}(\frac{G'}{G}+\frac{\lambda}{2})^{-2}, \end{aligned} \tag{12}$$

where $|\alpha_2|+|\alpha_{-2}|\neq 0$.

Substituting (12) into (11), collecting the coefficients of $(\frac{G'}{G}+\frac{\lambda}{2})^i(i=\pm1,\pm2,\cdots,\pm4)$, and solving the resulting system with the aid of MATHEMATICA, we have the following sets of solutions:

The 1*th* solutions set:

$$\alpha_0=\frac{12hk^2N}{1-4hk^2N},\alpha_2=\frac{-12k^2N}{1-4hk^2N},$$
$$V^2=\frac{c_0^2}{1-4hk^2N},\alpha_{-2}=\alpha_{-1}=\alpha_1=0; \tag{13}$$

The 2 *th* solutions set:

$$\alpha_0=\frac{4hk^2N}{1+4hk^2N},\alpha_2=\frac{-12k^2N}{1+4hk^2N},$$
$$V^2=\frac{c_0^2}{1+4hk^2N},\alpha_{-2}=\alpha_{-1}=\alpha_1=0; \tag{14}$$

The 3*th* solutions set:

$$\alpha_0=\frac{12hk^2N}{1-4hk^2N},\alpha_{-2}=\frac{-12h^2k^2N}{1-4hk^2N},$$
$$V^2=\frac{c_0^2}{1-4hk^2N},\alpha_2=\alpha_{-1}=\alpha_1=0; \tag{15}$$

The 4 *th* solutions set:

$$\alpha_0=\frac{4hk^2N}{1+4hk^2N},\alpha_{-2}=\frac{-12h^2k^2N}{1+4hk^2N},$$
$$V^2=\frac{c_0^2}{1+4hk^2N},\alpha_2=\alpha_{-1}=\alpha_1=0; \tag{16}$$

The 5 *th* solutions set:

$$\alpha_0=\frac{24hk^2N}{1-16hk^2N},\alpha_{-2}=\frac{-12h^2k^2N}{1-16hk^2N},$$
$$\alpha_2=\frac{-12k^2N}{1-16hk^2N},V^2=\frac{c_0^2}{1-16hk^2N},$$
$$\alpha_{-1}=\alpha_1=0; \tag{17}$$

The 6 *th* solutions set:

$$\alpha_0=\frac{-8hk^2N}{1+16hk^2N},\alpha_{-2}=\frac{-12h^2k^2N}{1+16hk^2N},$$
$$\alpha_2=\frac{-12k^2N}{1+16hk^2N},V^2=\frac{c_0^2}{1+16hk^2N},$$
$$\alpha_{-1}=\alpha_1=0; \tag{18}$$

where $h=(\lambda^2-4\mu)/4\neq 0$, λ and μ are arbitrary real constants.

Substituting (13)-(18) into (12) and recall the general solutions (8), we have the solutions of (11) as follows:

When $\lambda^2-4\mu>0$, we have the hyperbolic function travelling wave solutions

$$u_1(\xi)=\frac{12hk^2N}{1-4hk^2N}\mathrm{sech}^2(\sqrt{h}\xi),$$
$$\xi=k\left[z\pm(\frac{c_0^2}{1-4hk^2N})^{1/2}t+\xi_0\right]; \qquad (19)$$
$$u_2(\xi)=\frac{4hk^2N}{1+4hk^2N}[1-3\tanh^2(\sqrt{h}\xi)],$$
$$\xi=k\left[z\pm(\frac{c_0^2}{1+4hk^2N})^{1/2}t+\xi_0\right]; \qquad (20)$$
$$u_3(\xi)=\frac{12hk^2N}{-1+4hk^2N}\mathrm{csch}^2(\sqrt{h}\xi),$$
$$\xi=k\left[z\pm(\frac{c_0^2}{1-4hk^2N})^{1/2}t+\xi_0\right]; \qquad (21)$$
$$u_4(\xi)=\frac{4hk^2N}{1+4hk^2N}[1-3\coth^2(\sqrt{h}\xi)],$$
$$\xi=k\left[z\pm(\frac{c_0^2}{1+4hk^2N})^{1/2}t+\xi_0\right]; \qquad (22)$$
$$u_5(\xi)=\frac{48hk^2N}{-1+16hk^2N}\mathrm{csch}^2(2\sqrt{h}\xi),$$
$$\xi=k\left[z\pm(\frac{c_0^2}{1-16hk^2N})^{1/2}t+\xi_0\right]; \qquad (23)$$
$$u_6(\xi)=-\left\{8hk^2N+12k^2hN[\tanh^2(\sqrt{h}\xi)+\coth^2(\sqrt{h}\xi)]\right\}(1+16hk^2N)^{-1},$$
$$\xi=k\left[z\pm(\frac{c_0^2}{1+16hk^2N})^{1/2}t+\xi_0\right]; \qquad (24)$$

and when $\lambda^2-4\mu<0$, then we have the trigonometric solutions

$$u_7(\xi)=\frac{12hk^2N}{1-4hk^2N}\sec^2(\sqrt{-h}\xi),$$
$$\xi=k\left[z\pm(\frac{c_0^2}{1-4hk^2N})^{1/2}t+\xi_0\right]; \qquad (25)$$
$$u_8(\xi)=\frac{4hk^2N}{1+4hk^2N}[1+3\tan^2(\sqrt{-h}\xi)],$$
$$\xi=k\left[z\pm(\frac{c_0^2}{1+4hk^2N})^{1/2}t+\xi_0\right]; \qquad (26)$$
$$u_9(\xi)=\frac{12hk^2N}{1-4hk^2N}\csc^2(\sqrt{-h}\xi),$$
$$\xi=k\left[z\pm(\frac{c_0^2}{1-4hk^2N})^{1/2}t+\xi_0\right]; \qquad (27)$$
$$u_{10}(\xi)=\frac{4hk^2N}{1+4hk^2N}[1+3\cot^2(\sqrt{-h}\xi)],$$
$$\xi=k\left[z\pm(\frac{c_0^2}{1+4hk^2N})^{1/2}t+\xi_0\right]; \qquad (28)$$
$$u_{11}(\xi)=\frac{48hk^2N}{1-16hk^2N}\csc^2(2\sqrt{-h}\xi),$$
$$\xi=k\left[z\pm(\frac{c_0^2}{1-16hk^2N})^{1/2}t+\xi_0\right]; \qquad (29)$$
$$u_{12}(\xi)=\left\{-8hk^2N+12k^2hN[\tan^2(\sqrt{-h}\xi)+\cot^2(\sqrt{-h}\xi)]\right\}(1+16hk^2N)^{-1},$$
$$\xi=k\left[z\pm(\frac{c_0^2}{1+16hk^2N})^{1/2}t+\xi_0\right]. \qquad (30)$$

where $h=(\lambda^2-4\mu)/4$, λ, μ and ξ_0 are arbitrary real constants, wave number k is a arbitrary positive real constant.

The obtained solutions existence must meet the following condition, Eqs. (19), (21), (25) and (27) must satisfied $hk^2N<0.25$, Eqs. (20), (22), (26) and (28) must satisfied $hk^2N>-0.25$, Eqs. (23) and (29) must satisfied $hk^2N<0.0625$, Eqs. (24) and (30) must satisfied $hk^2N>-0.0625$.

Here, comparing our obtained results with Xue's exact solitary wave solution in Ref.[11]. Xue et al. only obtained one solitary wave solution in Ref.[11]. We obtained twelve exact traveling wave solutions of Eq. (1). Not only the solitary wave solutions have been given, but also many other period exact travelling wave solutions of the NLWE in a MEE circular rod are successfully obtained by the modified (G'/G)-expansion method in this work. And the obtained solutions Eqs. (19)-(24) are soliton solutions, and Eqs.(25)-(30) are periodic solutions. All the exact travelling wave solutions reported in this paper have been checked with MATHEMATICA.

4. Numerical results and discussion

In section 3, the exact travelling wave solution have been successfully constructed. In the obtained solutions, the parameters c_0 and N are both depending on the material properties as well as the geometry of the rod, c_0

and N have been listed in table 1, wave number k, $h=(\lambda^2-4\mu)/4$ and ξ_0 are free real constants.

For example, the first exact travelling wave solution (19) is a solitary wave solution of Eq.(1). The solitary wave amplitude A and wave velocity V of Eq. (19) can be expressed as:

$$A=\frac{12hk^2N}{1-4hk^2N}, V=c_0(1-4hk^2N)^{-1/2}. \quad (31)$$

Furthermore, from Eq.(31) we can obtain that the maximum wave number $k_{\max}$ must satisfy

$$k_{\max}=(2\sqrt{Nh})^{-1}. \quad (32)$$

Hence, the maximum wave number $k_{\max}$ for five different materials in table 1 sequence are 37.959, 40.975, 48.656, 39.904 and 39.284, respectively. For a solitary wave solution, the wave number k must satisfy $k<k_{\max}$.

If parameters k, h and ξ_0 are given special value. To facilitate our study, we set $\xi_0=0$, h =1 or 2, k =5 or 6. According to the data of table 1 and Eq. (31), we can obtain the solitary wave amplitude and wave velocity, the obtained solitary wave amplitude and wave velocity are list in table 2.

The relations between the solitary wave velocity V and wave number k for the five different materials when h =1 are plotted in Fig.1. If wave number $k>k_{\max}$, the solitary wave velocity V will break.

It is observed that when the wave number k is small, the wave velocity in the coupled class (PM, MEE, and PE) is higher than that in the purely elastic class (EI and TI). However, with increasing wave number k, these five materials form three new classes: PM is the first with the highest velocity; in the middle, we have MEE, EI and TI; and finally PE has the lowest velocity.Eq. (19) is a bell-shaped sech^2 solitary wave solution, and it is a soliton solution.

Solitons are special kinds of solitary wave. The soliton solution is spatially localized solution, hence $u'(\xi)$, $u''(\xi)$ and $u'''(\xi)\to\pm\infty$, $\xi=k(z-Vt)$. Soliton have a remarkable soliton property in that it keeps its identity upon interacting with other soliton. And soliton's graph is a bell-shaped sech^2 soliton solution characterized by infinite wings or infinite tails. Fig.2 shows the solitary wave u in the 50% MEE rod versus the variable time t and z of Eq. (19) with $h=1$ and $k=5$. It is clear that the maximum of u is reached at the center $t=0$ and $z=0$. Obviously, the solitary wave amplitude is symmetrical about the center.

In the same way, Fig.3 shows the solitary wave u in the 50% MEE rod versus the variable time t and z of Eq. (24) with $h=1$ and $k=5$. It is clear that the minimum of u is reached at the center $t=0$ and $z=0$. And it shape like a cone, obviously, the solitary wave amplitude is symmetrical about the center. To the best of our knowledge, the obtained exact solitary solution (24) is a new solution, it have not been reported.

Similarly, we can give the numerical result and figure of other four materials and other exact travelling wave solution of Eq. (1), which are omitted for convenience.

Table 2: The solitary wave amplitude and wave velocity for different k and h

v_f	h	k	0%(PM)	50%(MEE)	100%(PE)	TI	EI
A(m)	1	5	0.05297	0.04535	0.03202	0.04785	0.04940
V(m/s)	1	5	5258.92	5183.33	5076.68	4838.43	4879.49
A(m)	2	5	0.10784	0.09208	0.06473	0.09725	0.10045
V(m/s)	2	5	5305.97	5222.96	5103.99	4877.49	4920.16
A(m)	2	6	0.15779	0.13441	0.09410	0.14207	0.14682
V(m/s)	2	6	5348.44	5258.59	5128.39	4912.65	4956.81
A(m)	1	6	0.07687	0.06573	0.04632	0.06939	0.07166
V(m/s)	1	6	5279.47	5200.66	5088.64	4855.50	4897.26

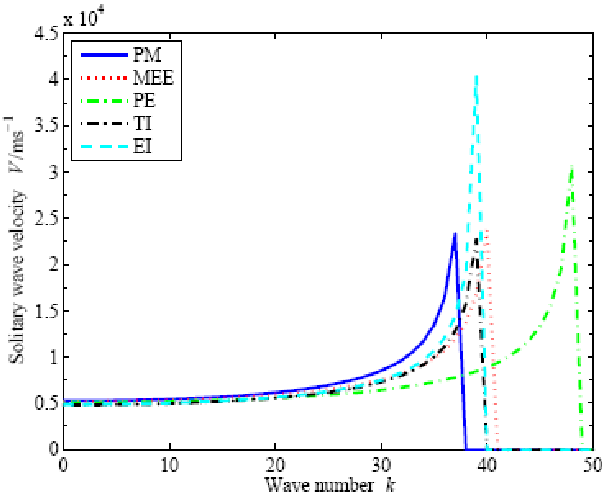

Fig. 1 Wave velocity V governing by (31) versus wave number k in a rod with $h=1$.

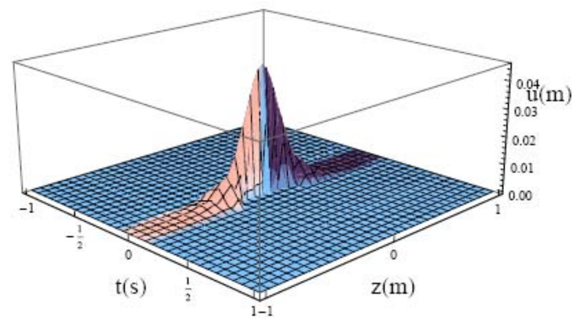

Fig. 2 Soliton versus t and z in a 50% MEE rod of Eq.(19) with $h=1$ and $k=5$.

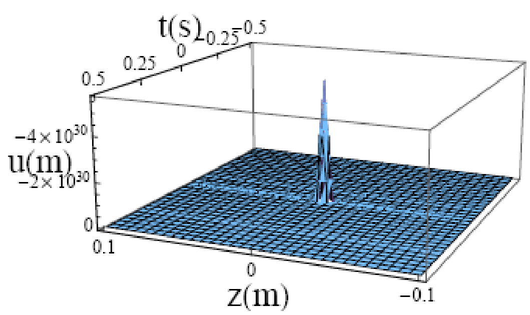

Fig. 3 Solitary waves versus t and z in a 50% MEE rod of Eq.(24) with $h=1$ and $k=5$.

5. Conclusion

In this paper, we present a modified (G'/G)-expansion method based on (G'/G)-expansion method. We have applied the new method to find the exact travelling wave solutions of nonlinear solitary wave equation in a long MEE circular rod. And the obtained exact travelling wave solutions are expressed by the hyperbolic functions, the rational functions and the trigonometric functions. When the parameters are taken as special values, the solitary wave solutions are derived from the hyperbolic functions. The obtained results show the modified (G'/G)-expansion method is direct, concise and effective with the help of MATHEMATICA, and this method can be applied to many other nonlinear partial differential equations in mathematical physics. Some numerical examples are further presented for the wave in a rod made of five different materials: the three-phase fully coupled MEE, coupled piezoelectric PE, coupled piezomagnetic PM, purely elastic but transverse isotropy TI and purely elastic isotropy EI. It is demonstrated that the solitary wave not only exists in such rods but also shows different features in different materials, which could have potential applications in non-destructive evaluation of structures made of the advanced MEE material.

Acknowledgments

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References

[1] Porubov A V, Velarde M G, "Dispersivedissipative solitons in nonlinear solids", Wave Motion, Vol. 31, No. 1, 2000, pp. 197–207.
[2] Samsonov A M, "Strain Solitons in Solids and How to Construct Them", New York: Chapman and Hall/CRC, 2001
[3] Guo J G, Zhou L J, Zhang S Y, "The geometrical nonlinear waves in finite deformation elastic rods", Appl. Math. Mech., Vol. 26, 2005, pp. 667–674.
[4] Zhang S Y, Liu Z F, "Three kinds of nonlinear dispersive waves in elastic rods with finite deformation", Appl. Math. Mech., Vol. 29, 2008, pp. 909–917.
[5] Liu Z F, Zhang S Y, "Nonlinear waves and periodic solution in finite deformation elastic rod", Acta Mech. Solida Sin., Vol. 9, No. 1, 2006, pp. 1–8.
[6] Christov C I, Marinov T T, Marinova R S, "Identification of solitary wave solutions as an inverse problem: application to shapes with oscillatory tails", Math. Comput. Simul., Vol. 80, No. 1, 2009, pp. 56–65.
[7] Chen J Y, Pan E, Chen H L, "Wave propagation in magneto-electro-elastic multilayered plates", Int. J. Solids Struct., Vol. 44, 2007, pp. 1073–1085.
[8] Chen P, Shen Y, "Propagation of axial shear magneto-electro-elastic waves in piezoelectricpiezomagnetic composites with randomly distributed cylindrical inhomogeneities", Int. J. Solids Struct., Vol. 44, 2007, pp. 1511–1532.
[9] Wu B, Yu J G, He C F, "Wave propagation in non-homogeneous magneto-electro-elastic plates", J. Sound Vib., Vol. 317, No. 1, 2008, pp. 250–264.

[10] Maity Niladri Pratap, Maity Reshmi, "Surface plasmon waves on noble metals at optical wavelengths", Int. J. Comput. Sci. Issues, Vol. 8, No. 3 3-2, 2011, pp. 485–490.

[11] C X Xue, E Pan, S Y Zhang, "Solitary waves in a magneto-electro-elastic circular rod", Smart Mater. Struct., Vol. 20, 2011, pp. 105010-17.

[12] W. Malfliet, "Solitary wave solutions of nonlinear wave equations", Am. J. Phys., Vol. 60, No. 7, 1996, pp. 650–654.

[13] W. Malfliet, "The tanh method: a tool for solving certain classes of nonlinear evolution and wave equation", J. Comput. Appl. Math. Vol. 164, 2004, pp. 529–541.

[14] A.M. Wazwaz, "The tanh method for traveling wave solutions of nonlinear equations", Appl. Math. Comput., Vol. 154, No. 3, 2004, pp. 713–723.

[15] E.J. Parkes, B.R. Duffy, "An automated tanhfunction method for finding solitary wave solutions to nonlinear evolution equations", Comput. Phys. Commun., Vol. 98, 1996, pp. 288–300.

[16] Engui Fan, "Extended tanh-function method and its applications to nonlinear equations", Phys. Lett. A, Vol. 277, 2000, pp. 212–218.

[17] M.L. Wang, X.Z. Li, "Extended F-expansion method and periodic wave solutions for the generalized Zakharov equations", Phys. Lett. A, Vol. 343, 2005, pp. 48–54.

[18] X.Z. Li, M.L. Wang, "A sub-ODE method for finding exact solutions of a generalized KdV-mKdV equation with higher order nonlinear terms", Phys. Lett. A, Vol. 361, 2007, pp. 115–118.

[19] S.K. Liu, Z.T. Fu, S.D. Liu, Q. Zhao, "Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations", Phys. Lett. A, Vol. 289, 2001, pp. 69–74.

[20] M.L.Wang, Y.B. Zhou, Z.B. Li, "Application of a homogeneous balance method to exact solutions of nonlinear evolution equations in mathematical physics", Phys. Lett. A, Vol. 216, 1996, pp. 67–75.

[21] J.H. He, X.H. Wu, "Exp-function method for nonlinear wave equations", Chaos Solitons Fractal, Vol. 30, 2006, pp.700–708.

[22] M.L. Wang, X.Z. Li, J.L. Zhang, "The (G'/G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics", Phys. Lett. A, Vol. 372, 2008, pp. 417–423.

[23] X.M. Ma, Y.T. Pan, L.Z. Chang. "The Modified (G'/G)-expansion Method and Its Applications to KDV Equation", Int. J. Nonlinear Sci., Vol. 12, No. 4, 2011, pp. 400–405.

[24] A.M.Wazwaz, "A study on nonlinear dispersive partial differential equations of compact and noncompact solutions", Appl. Math. Comput., Vol. 135, 2003, pp. 399–409.

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