

A Novel Robust Backstepping Control for Nonaffine Nonlinear Processes and Application to An Active Magnetic Bearing System

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Abstract

In this paper, a novel continuous-time robust nonlinear control scheme, is proposed for nonaffine nonlinear systems with unknown uncertainties/disturbance, which is based on backstepping methodology and sliding mode control technique. Firstly, a novel approximation technique is developed to approximate the nonaffine nonlinear dynamic systems. Then, the robust backstepping control for nonaffine nonlinear systems is proposed via the novel approximation technique. In the controller design procedure, the sliding model control method is introduced to avoid the possibility of the over parameterization problem and deal with the unknown uncertainties/disturbance. And, a second-order sliding mode integral filter is employed to facilitate the development of the derivation of the virtual control input with uncertainty terms included. Finally, the designed robust control strategy is applied to three-pole active magnetic bearing system, and simulation results are provided to demonstrate the effectiveness of the theoretic results obtained.

Keywords: Nonaffine Nonlinear Systems, Robust Backstepping Control, Active Magnetic Bearing System, Simulation.

1. Introduction

In the past decade, there has been significant progress in the area of control design for nonlinear plants. Isidori [1] developed important results related to the geometric approach for analysis and control design of nonlinear plants. An overview of available nonlinear control techniques is given by [2]. Many of these results have been extended to the case of nonlinear plants with uncertainty [1-2]. Up to now, few research articles related to the nonaffine nonlinear systems [3-4], in addition to the intelligent control algorithms. However, intelligent control algorithms require a lot of expertise or modeling data. For some plants, expertise or modeling data is not easy to obtain.

The problem of controlling the plants characterized by models that are nonaffine in the control input vector is a difficult one [5]. An approach widely used in practice is

that based on linearization of the nonlinear plant model around an operating point. In some controls, the nonlinear model of dynamics is generally nonaffine in input u and is commonly linearized around a trim point, that is, an operating point dependent on the current states.

Based on the aforementioned works, this paper develops a novel robust backstepping control (NRBC) methodology for nonaffine nonlinear dynamic systems. A novel approximation technique is firstly employed to approximate the nonaffine nonlinear dynamic system. Then, based on backstepping control, NRBC is proposed. However, in the NRBC controller design procedure, the sliding model control technique is introduced in the backstepping procedure so as to develop an easy-implemented controller, as well as to avoid the possibility of the overparameterization problem and deal with the unknown uncertainties/disturbance. And, a second order sliding mode integral filter is introduced to facilitate the development of the derivation of the virtual control input with uncertainty terms included. Finally, the proposed strategy is also applied to three-pole AMB system suffering from unknown uncertainties/disturbance. The tracking performances of three-pole AMB system could also be well guaranteed.

2. Problem Formulation

Consider the nonaffine nonlinear MIMO system which is represented by the following set of differential equations:

$$\dot{x}_1 = x_2 \quad (1)$$

$$\dot{x}_2 = f[\bar{x}(t), u(t)] + d \quad (2)$$

where $x_1 \in \mathfrak{R}^{n_1}$ and $x_2 \in \mathfrak{R}^{n_2}$ are the state vectors, and

$\bar{x}(t) = (x_1, x_2)^T \in \mathfrak{R}^{n_1+n_2}$, $u(t) = (u_1, u_2, \dots, u_m)^T \in \mathfrak{R}^m$

is the input vector of the system, respectively.

$f = (f_1, f_2, \dots, f_n)^T$, $f_i : \mathfrak{R}^n \times \mathfrak{R}^m \rightarrow \mathfrak{R}^1$ are known smooth nonlinear functions whose first derivatives with

respect to $\bar{x}(t)$ and $u(t)$ exist. $d \in \mathfrak{R}^{n_2}$ denotes the function uncertainty with $\|d\| \leq \psi$, which is due to the modeling errors and external disturbances.

Most of the nonlinear control methods developed in this context are applicable to nonlinear plant models that are linear in unknown parameters and affine in the control input vector u , that is, characterized by appearing linearly in the equation. However, for nonaffine nonlinear MIMO system, the problem of controlling the plants characterized by models that are nonaffine in the control input vector is difficult one. Without any effective methods to solve this problem. One nonlinear approach to this problem is that based on directly inverting the nonlinear function of on domain. Although the existence of an inverse function can be guaranteed by the implicit function theorem [2], it is generally difficult to prescribe technique to actually obtain such an inverse. However, in the proposed NRBC, such time consuming algorithms are totally avoided and thus the controller design is greatly simplified. Further speaking, for the continuous time nonaffine nonlinear systems, robust control research has not been studied.

In order to convenient unfold the following work, short assumption is given as following

Assumption 1: The input vector u of the system must be measurable or available.

3. Novel NRBC Algorithm Nonaffine Nonlinear Dynamic Systems

A novel NRBC algorithm is proposed here using newly developed nonaffine nonlinear approximation for continue-time systems, which not only avoids complex control development and intensive computation, but also overcomes the shortcomings of other existing methods. unique and rigorous stability proof will be given and its superior performance will be demonstrated in later simulations.

3.1 Novel Nonaffine Nonlinear Approximation

For the nonaffine nonlinear model (1), the Taylor expansion of the nonlinear function $f[\bar{x}(t), u(t)]$ with respect to $u(t)$ around $u(t - \tau)$ can result in

$$\dot{x}_1 = x_2 \tag{3}$$

$$\dot{x}_2 = f(\bar{x}(t), u(t - \tau)) + f_d(\bar{x}(t), u(t - \tau))\Delta u(t) + R_p + d \tag{4}$$

where

$$\Delta u(t) = \begin{bmatrix} u_1(t) - u_1(t - \tau) \\ u_2(t) - u_2(t - \tau) \\ \dots \\ u_m(t) - u_m(t - \tau) \end{bmatrix}$$

$$f_d(\bar{x}(t), u(t - \tau)) = \left. \frac{\partial f(\bar{x}(t), u(t))}{\partial u(t)} \right|_{u(t)=u(t-\tau)}$$

$$f_{dd} = \left. \frac{\partial^2 f(\bar{x}(t), u(t))}{\partial^2 u(t)} \right|_{u(t)=\zeta}$$

$$R_p = \frac{[\Delta u(t)]^T G_{dd} \Delta u(t)}{2}$$

$\zeta = [\zeta_1, \zeta_2, \dots, \zeta_m]^T$ with ζ_j being a point between $u_j(t)$ and $u_j(t - \tau)$. Let $0 \leq \|f_{dd}\| \leq r_p, r_p$ is finite

positive number, thus $\|R_p\| \leq \frac{r_p \|\Delta u(t)\|^2}{2}$. The parameter

$\tau > 0$ is the updating input, It may be chosen as the sampling-time in sampled-data control system, or as an integer multiple of the sampling-time. A better choice of the parameter τ is the sampling because a larger τ may lead to an inaccurate approximation when the system function $f[\bar{x}(t), u(t)]$ varies quickly.

It is easy that (3)-(4) can be representation as the following form.

$$\dot{x}_1 = x_2 \tag{5}$$

$$\dot{x}_2 = f_n(\bar{x}(t), u(t - \tau)) + f_d(\bar{x}(t), u(t - \tau))u(t) + d_\xi \tag{6}$$

Where $d_\xi = R_p + d$, and

$$f_n(\bar{x}(t), u(t - \tau)) = f(\bar{x}(t), u(t - \tau)) - f_d(\bar{x}(t), u(t - \tau))u(t)$$

To approximation accuracy, control input must satisfy the following assumption.

Assumption 2: $|\Delta u(t)| \in [0, \delta]$ and $0 < \left| \frac{\partial f}{\partial u(t)} \right| \leq \beta$, δ and β

are two finite positive vectors.

In Assumption 2 : $0 < \left| \frac{\partial f}{\partial u(t)} \right| \leq \beta$ means that the system

(1) has a well defined relative degree [4]. $|\Delta u(t)|$ should not be too large in order to limit the approximation error of the model (5)-(6) for a computed $u(t)$. In many actual process control systems and flight control systems, $|\Delta u(t)| \in [0, \delta]$ is a physical restriction of many practical systems because their states and outputs (actuators) cannot change too fast because of system 'inertia'.

Remak 1: If there is control input saturation constraints, $u(t - \tau)$ must be the actual control input of τ times before,

rather than control input command of τ times before.

3.2 Nonlinear Controller and Stability Analysis

In this subsection, based on above proposed novel non-affine nonlinear approximation algorithm, the robust backstepping procedure and the sliding model control techniques are introduced so as to develop a NRBC, whose function is to track the reference signal with an acceptable accuracy. The following assumptions are used in the design and analysis procedure.

Assumption 3: The reference signal x_{1d} , virtual input x_{2d} and their first order derivatives are piecewise continuous and bounded, they are

$$\begin{aligned} \|x_{1d}\| &\leq \|x_{1d}\|_{\max} = \Delta_1 \\ \|x_{2d}\| &\leq \|x_{2d}\|_{\max} = \Delta_2 \\ \|\dot{x}_{1d}\| &\leq \|\dot{x}_{1d}\|_{\max} = \dot{\Delta}_1 \\ \|\dot{x}_{2d}\| &\leq \|\dot{x}_{2d}\|_{\max} = \dot{\Delta}_2 \end{aligned}$$

Moreover, the function uncertainty is assumed to be unknown. However, the upper boundary of its magnitude is known as

$$\|d_\xi\| \leq \Psi_\xi$$

Before we start, respectively, the state tracking error variables e_1 and e_2 as follows

$$e_1 = x_1 - x_{1d} \quad (7)$$

$$e_2 = x_2 - x_{2d} \quad (8)$$

where x_{1d} and x_{2d} are the desired trajectories of x_1 and x_2 , respectively. Note that x_{1d} is given by command signals and x_{2d} will be defined later. From (5) and (7), we have

$$\dot{e}_1 = \dot{x}_1 - \dot{x}_{1d} = x_2 - \dot{x}_{1d} \quad (9)$$

We further assume that x_{2d} is the virtual input to (9), and the desired virtual control is

$$x_{2d} = -E^{-1}(C_1 e_1 - \dot{x}_{1d}) \quad (10)$$

where C_1 is a designed positive diagonal matrix, and E is a unit matrix.

As stated previously, with the inclusion of the uncertainty or disturbance in the virtual input (10), it is difficult in finding, its derivatives because the signal may not be practically differentiable due to noises and/or disturbances, and the problem of overparameterization will occur with the increase of steps as well.

In view of this, a second-order sliding model integral filter is presented in this paper so as to eliminate the analytic computation of \dot{x}_{2d} , which will be used as reference in the backstepping procedure. It is worth stressing that the proposed filter works also for the high-order backstepping procedures, just using the output of the $(i-1)$ th filter as the input to the i th filter, $i=1,2,\dots,n$. The proposed integral filters are presented as follows

$$\dot{\hat{\lambda}}_1 = -\frac{\hat{\lambda}_1 - x_{2d}}{\varepsilon_1} - \frac{Q_1(\hat{\lambda}_1 - x_{2d})}{\|\hat{\lambda}_1 - x_{2d}\| + \varsigma_1} \quad (11)$$

$$\dot{\hat{\lambda}}_2 = -\frac{\hat{\lambda}_2 - \hat{\lambda}_1}{\varepsilon_2} - \frac{Q_2(\hat{\lambda}_2 - \hat{\lambda}_1)}{\|\hat{\lambda}_2 - \hat{\lambda}_1\| + \varsigma_2} \quad (12)$$

where ε_i is the time constant of the filter, Q_i and ς_i are the designed constants, $i=1,2$.

Obviously, with Q_i assumed to be zero, the proposed filters are reduced to a classical integral filters. It should be pointed out that, with the inclusion of the sliding model control component, the fast convergence of the estimation error produced by the proposed integral filters is guaranteed, which will be analytically studied during the stability analysis. Similar integral filters associated with different control schemes can also be found in various applications [6-7], and the performance demonstrates their feasibility within the backstepping procedure.

Let us take x_2 as the virtual control variable of x_1 -subsystem, and select $x_{2d} \approx x_2$ as the ideal control input.

It is noted that, in this step, the task is to stabilize (7) with respect to the Lyapunov function.

$$V_1 = \frac{1}{2} e_1^T e_1 + \frac{1}{2} (\hat{\lambda}_1 - x_{2d})^T (\hat{\lambda}_1 - x_{2d}) \quad (13)$$

Obviously, the third term in (13) is used to stabilize the estimation error of the proposed filters. Consequently, evaluating its time derivative along the solutions of the system(9), results

$$\dot{V}_1 = e_1^T \dot{e}_1 + (\hat{\lambda}_1 - x_{2d})^T (\dot{\hat{\lambda}}_1 - \dot{x}_{2d}) \quad (14)$$

Substituting (9), (10) and (11) into (14) yields

$$\dot{V}_1 \leq -C_1 \|e_1\|^2 - \|(\hat{\lambda}_1 - x_{2d})^T\| \left(\frac{Q_1(\hat{\lambda}_1 - x_{2d})}{\|\hat{\lambda}_1 - x_{2d}\| + \varsigma_1} - \|\dot{x}_{2d}\| \right) \quad (15)$$

According to Assumption 3, the parameter Q_1 can be designed as $Q_1 = \mathcal{G}_1 \dot{\Delta}_2$, where $\mathcal{G}_1 > 1$. Hence, we have

$$\dot{V}_1 \leq -C_1 \|e_1\|^2 - \|(\hat{\lambda}_1 - x_{2d})^T\| \Delta_2 \left(\frac{\mathcal{G}_1(\hat{\lambda}_1 - x_{2d})}{\|(\hat{\lambda}_1 - x_{2d})\| + \varsigma_1} - 1 \right) \quad (16)$$

Apparently, if $\hat{\lambda}_1 - x_{2d} \neq 0$ and the following relation is satisfied, the time derivative of the Lyapunov function will be rendered to negative

$$\|\hat{\lambda}_1 - x_{2d}\| > \frac{\varsigma_1}{\mathcal{G}_1 - 1}$$

With the preceding condition, the system will be bounded stable at the origin (i.e., $e_1 = 0, e_2 = 0$), and also, with such condition, the actual estimation error of the proposed filter can be guaranteed within a compact set determined in the form:

$$\|\hat{\lambda}_1 - x_{2d}\| > \frac{\varsigma_1}{\mathcal{G}_1 - 1} \quad (17)$$

Obviously, the estimation error of the filter can be adjusted sufficiently small by choosing ς_1 appropriately, and with the inclusion of the sliding mode control component, (17) can be arrived in finite time.

Next, from (8), we have

$$\dot{e}_2 = f_n(\bar{x}(t), u(t - \tau)) + f_d(\bar{x}(t)) - u(t - \tau)u(t) + d_\xi - \dot{x}_{2d} \quad (18)$$

The candidate Lyapunov function in this case is defined as

$$V_2 = V_1 + \frac{1}{2} e_2^T e_2 + \frac{1}{2} (\hat{\lambda}_2 - \hat{\lambda}_1)^T (\hat{\lambda}_2 - \hat{\lambda}_1) \quad (19)$$

Then the time derivative of V_2 is given by

$$\dot{V}_2 = \frac{\partial V_2}{\partial e_1} \dot{e}_1 + \frac{\partial V_2}{\partial e_2} \dot{e}_2 + (\hat{\lambda}_2 - \hat{\lambda}_1)^T (\dot{\hat{\lambda}}_2 - \dot{\hat{\lambda}}_1) \quad (20)$$

Consequently, the parameter \mathcal{Q}_2 can be designed as $\mathcal{Q}_2 = \mathcal{G}_2 \Delta_3$, where $\mathcal{G}_2 > 1$. We have

$$\begin{aligned} \dot{V}_2 \leq & -C_1 \|e_1\|^2 + e_1^T e_2 + e_2^T (f_n(\bar{x}, u_{-\tau}) + f_d(\bar{x}, u_{-\tau})u \\ & + d_\xi - \dot{x}_{2d}) - \|(\hat{\lambda}_2 - \hat{\lambda}_1)^T\| \Delta_3 \left(\frac{\mathcal{G}_2(\hat{\lambda}_2 - \hat{\lambda}_1)}{\|(\hat{\lambda}_2 - \hat{\lambda}_1)\| + \varsigma_1} - 1 \right) \end{aligned} \quad (21)$$

where $\|\hat{\lambda}_1\|_{\max} = \Delta_3$. Then, then we can design the global NRBC law as

$$u(t) = -f_d^{-1}(\bar{x}, u_{-\tau})(C_2 e_2 + E e_1 + f_n(\bar{x}, u_{-\tau}) + r - \hat{\lambda}_2) \quad (22)$$

where C_2 is a designed positive diagonal matrix. r is a robust term designed to cancel the function uncertainty, and

$$r = \begin{cases} \frac{e_2 \mathcal{W}_\xi}{\|e_2\|^2} & e_2 \neq 0 \\ e_2 & e_2 = 0 \end{cases} \quad (23)$$

Hence, in this case

$$\begin{aligned} V_2 \leq & -C_1 \|e_1\|^2 - C_2 \|e_2\|^2 \\ & - \|(\hat{\lambda}_2 - \hat{\lambda}_1)^T\| \Delta_3 \left(\frac{\mathcal{G}_2(\hat{\lambda}_2 - \hat{\lambda}_1)}{\|\hat{\lambda}_2 - \hat{\lambda}_1\| + \varsigma_2} - 1 \right) \end{aligned} \quad (24)$$

In the same way as (16), in order to render $V_2 < 0$, we must have

$$\|\hat{\lambda}_2 - \hat{\lambda}_1\| > \frac{\varsigma_2}{\mathcal{G}_2 - 1}$$

Consequently, with such controller, the estimation error of the filter can be guaranteed within the set determined in the form

$$\|\hat{\lambda}_2 - \hat{\lambda}_1\| > \frac{\varsigma_2}{\mathcal{G}_2 - 1}$$

Therefore, the proposed control system is overall asymptotically stable in its origin ($e_1 = e_2 = 0$), and the estimated errors of the filters are all bounded and converge exponentially to a predetermined set. Also, since the included designed parameters do not depend on each other, the size of the set can be made sufficiently small by adjusting the corresponding parameters $\varsigma_i (i=1,2)$ appropriately.

In summary, we have the following results.

Theorem 1: Under Assumptions (1-3), using the NRBC controller (10) and (22) with the robust term (23) for nonaffine nonlinear dynamic systems (1). The solutions of error system (7-8) are UUB (Uniformly Ultimately Bounded) for $t \rightarrow \infty$.

Remark 2: Strictly speaking, when the dimension of control inputs is not equal to that of state variables, the inverse matrix of f_d is in the nonexistence. Thus, in this study, the generalized matrix inverse of f_d can be also obtained as $f_d^T (f_d f_d^T)^{-1}$. If $f_d f_d^T$ is well-conditioned, the inverse of $f_d f_d^T$ exists. However, $f_d f_d^T$ may be ill-conditioned. A diagonal matrix is defined as $\alpha = \text{diag}(a_1, a_2, \dots, a_m)$ with α_j being a given small positive number and thus matrix $f_d f_d^T + \alpha$ is invertible. Based on the approximation model (6), a global NRBC law (an approximation solution of (6) can then be determined as follows.

$$u(t) = -f_d^T (f_d f_d^T + \alpha)^{-1} (C_2 e_2 + E e_1 + f_n + r - \hat{\lambda}_2)$$

$$(25)$$

Remark 3: In order to obtain the smooth signal r , the unknown d_ξ can be approximately estimated by

$$r = \frac{e_2 \mathcal{W}_\xi}{\|e_2\|^2 + \beta} \quad (26)$$

β is a given small positive number.

Remark 4: If the nonlinear dynamic characteristics of the process plant can be accurately described by the mathematical model (1), the robust compensation term (23) is not needed. Then, the NRBC can be degraded into a novel backstepping control (NBC) for certain nonaffine nonlinear dynamic systems.

4. Illustrative Example

In this section the objective to evaluate the performance of the NRBC. The evaluation is carried out on the three-pole AMB system [8-9]. First, the three-pole AMB mathematical model is described. It will be shown that the three-pole AMB is a nonaffine nonlinear system.

With the configuration of Fig.1, a magnetic circuit is given in Fig.2, assuming that the reluctance exist only on air gaps, the differential equations the three-pole AMB system is given by

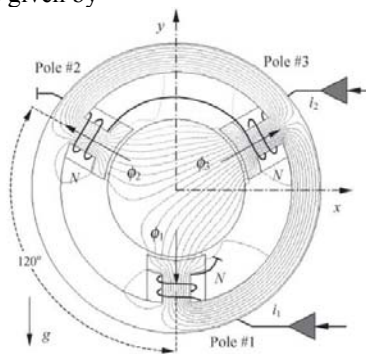


Fig. 1 Nonlinear control of a 3-pole AMB system.

$$\begin{aligned} \ddot{x}_r &= \frac{4}{3} \gamma m \Phi_1 \Phi_2 \\ \ddot{y}_r &= \frac{2}{3} \gamma m (\Phi_1^2 - \Phi_2^2) - g \end{aligned} \quad (27)$$

where (x_r, y_r) is the position of the rotor center, m is the rotor mass and g is the gravitational acceleration. $\gamma = \mu AN^2$, μ is the magnetic permeability of the air, A is the pole face area and N is the coil turns. The relationship

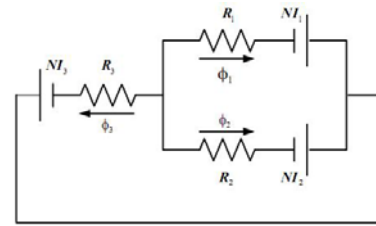


Fig. 2. Magnetic circuit for the 3-pole AMB system.

between (Φ_1, Φ_2) and (i_1, i_2) can be expressed in a matrix form by

$$\begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} = -\frac{1}{L} \begin{bmatrix} x_r & \sqrt{3}(2l_0 + y_r) \\ 2l_0 - y_r & \sqrt{3}x_r \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \quad (28)$$

where $L = 4l_0^2 - (x_r^2 + y_r^2)$ is always positive due to physical constrain $x_r^2 + y_r^2 \leq l_0^2$. The determinant of the matrix in (28) is $-\sqrt{3}L$, which is always non-zero. Therefore, there is a one-to-one correspondence between (i_1, i_2) and (Φ_1, Φ_2) .

Define the states of three-pole AMB system (27) are $\bar{x} = (x_r, y_r, \dot{x}_r, \dot{y}_r)$, and control input are $u = (i_1, i_2)$. The nominal values of the three-pole AMB system parameters are defined in Table 1.

Table 1 Nominal values of 3-pole AMB model parameters

Description	Value
rotor mass, \hat{m}	0.635 / kg
nominal air gap, \hat{l}_0	9.45×10^{-4} / m
Magnetic permeability of the air, μ	$4\pi \times 10^{-7}$ H / m
pole face area, A	4×10^{-4} / m ²
coil turns, N	300

Suppose that there is uncertainty caused by two parameters: the nominal air gap l_0 and the lumped parameter c_0 . Let v_l and v_c denote the percentage of the variations in l_0 and c_0 respectively, i.e., $l_0 = \hat{l}_0(1 + v_l)$ and $c_0 = \hat{c}_0(1 + v_c)$. Two levels of parameter variations are considered: $(v_l = 0; v_c = 0)$ and $(v_l = 1.2\%; v_c = -1.2\%)$. The uncertainty case use the same initial states $x = (2 \times 10^{-4}, 2 \times 10^{-4}, 0.015, 0.02)$. In order to verify the proposed control algorithm robustness. NRBC and NBC are designed and implemented for nonaffine nonlinear systems.

(1) The desired tracking commands are $x_{1d} = [0.0]^T$, / m. The other parameters are selected as $C_1 = C_2 = 40, \alpha = 10^{-5}$. According to Assumption 2, $|\Delta u(t)|$ should not be too large in order to limit the

approximation error of the model (5)-(6) for a computed $u(t)$, the parameters of robust compensation term (23) are selected as $\psi_\xi = 0.001$ and $\beta = 10^{-7}$.

(2) Compared to NRBC law, NBC has a similar design process NRBC apart from robust term (23). So the controller parameters are also selected as $C_1 = C_2 = 40, \alpha = 10^{-5}$. With the above controller parameters, the control currents i_1 and i_2 of the above controller are all within the range $(-2A, 2A)$.

Fig.3-5 show the rotor trajectories in the case of uncertainty. As can be seen from Fig.3-5, NRBC and NBC all can stabilize the system. Although both NBC and NRBC can stabilize the system in this uncertain case, the latter achieves better performance. Fig. 6 shows the control currents using NRBC law in the case of uncertainty, and Fig.7 shows the control currents using NBC law.

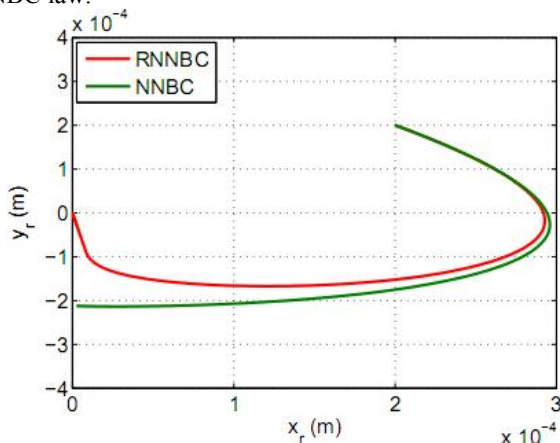


Fig. 3. Rotor trajectory with NRBC and NBC controller.

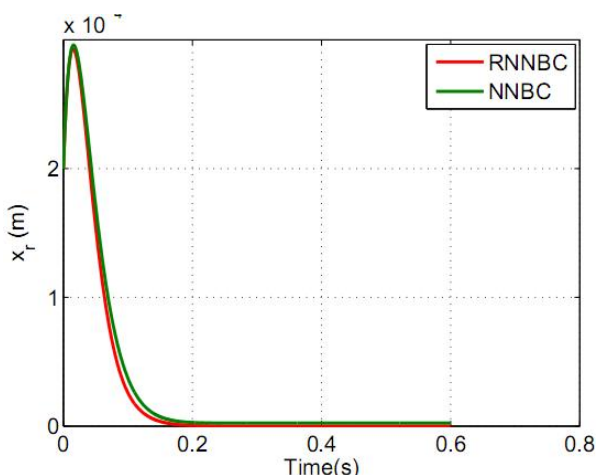


Fig.4.Rotor x_r displacement with NRBC and NBC controller.

5. Conclusion

A continuous-time nonaffine nonlinear controller design scheme for a class of nonlinear systems is presented in this paper. The strategy combines sliding mode and backstepping technique based on a novel approximation technique. The NRBC controller is designed to track the state commands against unknown uncertainties/disturbance. It is shown that, if the controller is applied, the tracking errors exponentially converge to a compact set and the size of the set can be made arbitrarily small by tuning the design parameters, and its stability is analyzed using Lyapunov theory. The proposed approach is then applied to three-pole AMB system, and simulation results demonstrate and illustrate the effectiveness and capabilities of our scheme.

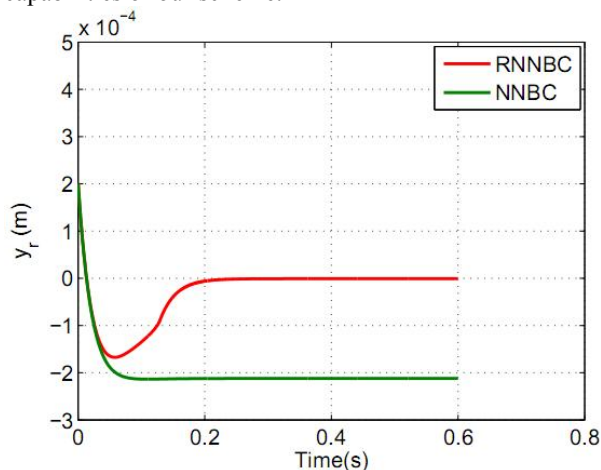


Fig.5. Rotor Y_r displacement with NRBC and NBC controller.

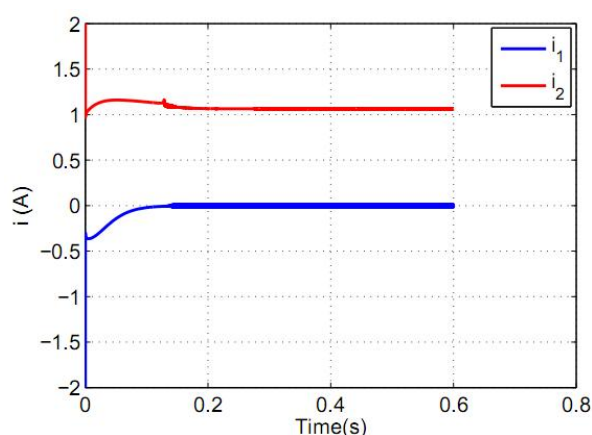


Fig. 6. Input coil current with NRBC controller.

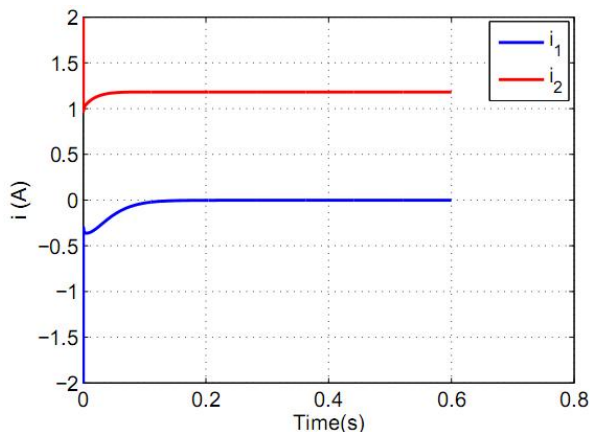


Fig. 7. Input coil current with NRBC controller.

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