Non-fragile H_∞ Filtering for a Class of Nonlinear Sampled-data System with Long Time-Delay

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Abstract

This paper studies the problem of non-fragile filtering for a class of nonlinear sampled-data system with long time-delay. The nonlinearity is assumed to satisfy global Lipschitz conditions, and the filter to be designed includes multiplicative gain variation, which results from filter implementations. Furthermore, to long time-delay, the direct distribution method is applied to design non-fragile filter that assures asymptotic stability and satisfy a prescribed performance index for the filtering error system. The proposed algorithm is given in terms of linear matrix inequality, whose feasibility and effectiveness has been shown by a numerical example.

Keywords: *Nonlinear Sampled-data System, Long Time-delay, Non-fragile H_∞ filtering, Uncertainties.*

1. Introduction

Sampled-data system extensively exists in lots of industrial processes, such as welding industry, aeronautics and astronautics (Tian 2012), chemical industry, etc. (Chen 1995), which is characterized by a continuous control plant and discrete controller. Due to uncertainties and time-delay (Xian 2012) frequently appearing in sampled-data system, which makes the system instable and its performance deteriorated. Therefore, robust control and robust filtering have gradually become hot topics of control field and signal processing (Wu 2001; Wu 2002; Theodor 1994; Xie 1991; Xie 1996; Li 1997). However, above-mentioned results are based on the accurate feedback controllers. In fact, because of the existence of the accuracy problem parameter drift, and other factors, it is shown that relatively small perturbation of the controller parameters might destabilize the closed-loop system, even lead to the performance degradation. Thus, it is necessary to design a controller which can tolerate some level of controller parameter variables. This is known as the non-fragile control problem. To data, this problem of non-fragile control and fitering has been widely investigated by many researchers, (Reinaldo 2001, Wang 2011; Wang 2011, Mahmoud 2004, Yang 2008, Che 2008, Al-Doori 2012).

On the other hand, H_∞ theory has been important effect on stability analysis and state estimation. In recent years, researches has made lots of works in H_∞ theory, such as (Mahmoud 1998, Zhang 2006).

This paper deals with non-fragile H_∞ filtering problem for nonlinear sampled-data systems whose time-delay is longer than a sampling period. The uncertain parameters are assumed to belong to a given norm-bounded type. A methodology for the design of a full-order stable linear filter that assures asymptotic stability and a prescribed H_∞ performance for the filtering error system, irrespective of the uncertainty and long time delay, is developed by solving a set of LMIs.

2. Problem Statement and Preliminaries

Consider the plant of sampled-data system

$$\begin{cases} x(t) = A_0 x(t) + A_1 x(t-\tau) + B_0 \omega(t) + f(x,u,t) \\ y(t) = C_0 x(t) + D_0 \omega(t) \\ z(t) = L_0 x(t) \\ x(t) = x_0 \,,\, t \in [-\tau, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, and $y(t) \in \mathbf{R}^r$ is the measured output, $z(t) \in \mathbf{R}^l$ is the signal to be estimated, $\omega(t) \in \mathbf{R}^p$ is the external disturbance input that belongs to $L_2[0, \infty]$, A_0, A_1, B_0, C_0, D_0 and L_0 are the constant matrices of appropriate dimensions. x_0 is the initial state vector. τ is time delay, which is uncertain and assumed to be evaluated between two adjacent sampling periods, namely, $(m-1)h \le \tau \le mh$, where $m \ge 1$ is a known constant.

Discretizing system (1) in one period, we can obtain the discrete state equation of the sampled-data system

$$\begin{cases} x(k+1) = G_0 x(k) + G_1 x(k-m+1) + \\ \qquad G_2 x(k-m) + H_0 \omega(k) + \bar{f}(x,u,t) \\ y(k) = C_0 x(k) + D_0 \omega(k) \\ z(k) = L_0 x(k) \\ x(k) = x_0, \quad k \le 0 \end{cases} \tag{2}$$

where

$$G_0 = e^{A_0 h}, G_1 = \int_0^{mh-\tau} e^{A_0 t} dt A_1$$
$$G_2 = \int_{mh-\tau}^{h} e^{A_0 t} dt A_1, H_0 = \int_0^h e^{A_0 t} dt B_0$$
$$\bar{f}(x,u,t) = \int_0^h e^{At} dt\, f(x,u,t)$$

Since time-delay τ is uncertain, G_1 and G_2 are uncertain matrices. Let

$$A_0 = L\,\mathrm{diag}\{\lambda_1, \cdots, \lambda_n\} L^{-1}$$

where L is a $n \times n$ nonsingular matrix, $\lambda_1, \cdots, \lambda_n$ are the eigenvalues of matrix A_0, here, assuming that $\lambda_1, \cdots, \lambda_n$ are unequal to 0, then

$$\begin{aligned} G_1(\tau) &= \bar{G}_1 + DF(\tau)E \\ G_2(\tau) &= \bar{G}_2 - DF(\tau)E \end{aligned} \tag{3}$$

where

$$\bar{G}_1 = -L\,\mathrm{diag}\{1/\lambda_1, \cdots, 1/\lambda_n\} E$$
$$\bar{G}_2 = L\,\mathrm{diag}\{e^{\lambda_1 h}/\lambda_1, \cdots, e^{\lambda_n h}/\lambda_n\} E$$
$$D = L\,\mathrm{diag}\{e^{\lambda_1 \beta_1}/\lambda_1, \cdots, e^{\lambda_n \beta_n}/\lambda_n\}, E = L^{-1} A_1$$
$$F(\tau) = \mathrm{diag}\{e^{\lambda_1 (mh-\tau-\beta_1)}, \cdots, e^{\lambda_n (mh-\tau-\beta_n)}\}$$

Selection of make $e^{\lambda_i (mh-\tau-\beta_i)} \le I$. Thus it is clear that

$$F^{\mathrm{T}}(\tau) F(\tau) \le I \tag{4}$$

Remark 1 If A_0 can't be transformed into a diagonal matrix or it has $j\ (0 \le j \le n)$ eigenvalues equal to 0, a similar result can be obtained, but $\bar{G}_1$, $\bar{G}_2$, D, $F(\tau)$, E, should be changed correspondingly.

The aim of this section is to design a full-order, linear, time-invariant asymptotically stable filter for system (2), The state-space realization of the filter has the form

$$\begin{cases} \hat{x}(k+1) = A_f \hat{x}(k) + B_f y(k) \\ \hat{z}(k) = C_f \hat{x}(k) \end{cases} \tag{5}$$

where $A_f \in \mathbf{R}^{n \times n}$, $B_f \in \mathbf{R}^{n \times r}$, $C_f \in \mathbf{R}^{l \times n}$ are filter parameters to be determined,

$$\begin{aligned} A_f &= A_{f1}(I + \Delta_1) \\ B_f &= B_{f1}(I + \Delta_2) \\ C_f &= C_{f1}(I + \Delta_3) \end{aligned} \tag{6}$$

and $\Delta_1, \Delta_2, \Delta_3$ represent filter parameter uncertainties to satisfy

$$\begin{bmatrix} \Delta_1 & \Delta_2 & \Delta_3 \end{bmatrix} = DF(\tau) \begin{bmatrix} E_2 & E_3 & E_3 \end{bmatrix} \tag{7}$$

In order to be convenient to solve the following linear matrix inequality, the corresponding linear transformation is used.

letting $\tilde{x}(k) = M\hat{x}(k)$

and then $\hat{x}(k) = M^{-1}\tilde{x}(k)$

filter is transformed to

$$\begin{cases} \tilde{x}(k+1) = MA_f M^{-1}\tilde{x}(k) + MB_f y(k) \\ \tilde{z}(k) = C_f M^{-1}\tilde{x}(k) \end{cases} \tag{8}$$

Denote

$$\xi(k) = \begin{bmatrix} x(k) \\ \tilde{x}(k) \end{bmatrix}, e(k) = z(k) - \tilde{z}(k)$$

Then, fitering error system

$$\begin{cases} \xi(k+1) = \tilde{G}_0 \xi(k) + \tilde{G}_{m-1}\xi(k-m+1) + \\ \qquad \tilde{G}_m \xi(k-m) + \tilde{H}_0 \omega(k) + \hat{f}(x,u,k) \\ e(k) = z(k) - \tilde{z}(k) = \tilde{L}_0 \xi(k) \end{cases} \tag{9}$$

where

$$\tilde{G}_0 = \begin{bmatrix} G_0 & 0 \\ MB_f C_0 & MA_f M^{-1} \end{bmatrix}, \tilde{G}_{m-1} = \begin{bmatrix} G_{m-1} & 0 \\ 0 & 0 \end{bmatrix}$$
$$\tilde{G}_m = \begin{bmatrix} G_m & 0 \\ 0 & 0 \end{bmatrix}, \tilde{H}_0 = \begin{bmatrix} H_0 \\ MB_f D_0 \end{bmatrix}$$
$$\hat{f}(x,u,k) = \begin{bmatrix} \bar{f}(x,u,k) \\ 0 \end{bmatrix}, \ \tilde{L}_0 = \begin{bmatrix} L_0 & -C_f M^{-1} \end{bmatrix}$$

The non-fragile H_∞ filtering problem in this section is stated as follows:

Given scalars $\gamma > 0$, find a full-order linear time-invariant, asymptotically stable filter with a state-space realization of the form (8) for system (2), such that,

(1) Filtering error system (9) with $\omega(k) = 0$ is asymptotically stable;

(2) H_∞ performance index satisfies

$$\|e(k)\|_2 < \gamma \|\omega(k)\|_2 \tag{10}$$

is guaranteed under zero-initial conditions for all non-zero $\omega(k) \in \boldsymbol{l}_2[0, +\infty]$。

Assumption 1 $\hat{f}(x,u,k)$ satisfies the quadratic inequality is the domain of continuity, namely

$$\begin{aligned} \hat{f}^T(x,u,k)\hat{f}(x,u,k) \le \xi^T(k) M_{11} \xi(k) + \\ \xi^T(k-m+1) M_{22} \xi(k-m+1) + \\ \xi^T(k-m+1) M_{33} \xi(k-m+1) \end{aligned} \tag{11}$$

Lemma 1[18] For given matrices $Q = Q^{\mathrm{T}}$, H and E, with appropriate dimension

$$Q + HF(k)E + E^T F^T(k) H^T < 0$$

holds for all $F(k)$ satisfying $F^T(k)F(k) \le I$ if and only if there exists $\varepsilon > 0$, such that

$$Q + \varepsilon HH^T + \varepsilon^{-1}E^T E < 0$$

3. Main Results

Theorem 1 Consider the filtering error system (9). For given scalar $\gamma > 0$, the system is asymptotically stable and (11) is satisfied under zero-initial conditions for all non-zero $\omega(k) \in \boldsymbol{l}_2[0,+\infty]$, if there exist matrices S, R_i $(i = 1,\cdots,5)$ such that the following matrix inequalities hold:

$$\begin{bmatrix} \Omega & * & * & * & * & * & * & * & * \\ \varepsilon_1\theta_1^{\mathrm{T}} & -\varepsilon_1 I & * & * & * & * & * & * & * \\ \theta_2 & 0 & -\varepsilon_1 I & * & * & * & * & * & * \\ \varepsilon_2\theta_3^{\mathrm{T}} & 0 & 0 & -\varepsilon_2 I & * & * & * & * & * \\ \theta_4 & 0 & 0 & 0 & -\varepsilon_2 I & * & * & * & * \\ \varepsilon_3\theta_5^{\mathrm{T}} & 0 & 0 & 0 & 0 & -\varepsilon_3 I & * & * & * \\ \theta_6 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_3 I & * & * \\ \varepsilon_4\theta_7^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_4 I & * \\ \theta_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_4 I \end{bmatrix} < 0 \quad (12)$$

where

$$\Omega = \begin{bmatrix} \Upsilon_1 & * & * & * & * & * & * & * & * & * \\ \Upsilon_1 & \Upsilon_2 & * & * & * & * & * & * & * & * \\ 0 & 0 & \Upsilon_3 & * & * & * & * & * & * & * \\ 0 & 0 & 0 & \Upsilon_4 & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & -\gamma^2 I & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & -I & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & -I & * & * & * \\ SG_0 & SG_0 & SG_{m-1} & SG_m & SH_0 & I & S & -S & * & * \\ \Upsilon_5 & \Upsilon_6 & R_0G_{m-1} & R_0G_m & \Upsilon_7 & I & R_0 & -I & -R_0 & * \\ \Upsilon_8 & L_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix}$$

$$\Upsilon_1 = -S + R_1 + R_2 + R_3$$

$$\Upsilon_2 = -R_0 + R_1 + R_2 + R_3$$

$$\Upsilon_3 = -R_1 + R_4$$

$$\Upsilon_4 = -R_2 + R_5$$

$$\Upsilon_5 = R_0G_0 + \hat{B}_f C_0 + \hat{A}_f$$

$$\Upsilon_6 = R_0G_0 + \hat{B}_f C_0$$

$$\Upsilon_7 = R_0H_0 + \hat{B}_f D_0$$

$$\Upsilon_8 = L_0 - \hat{C}_f$$

$$\theta_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & D^T S^T & D^T R_0^T & 0 \end{bmatrix}$$

$$\theta_2 = \begin{bmatrix} 0 & 0 & E_1 & -E_1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\theta_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & D^T\hat{B}_f^T & 0 \end{bmatrix}$$

$$\theta_4 = \begin{bmatrix} E_3C_0 & E_3C_0 & 0 & 0 & E_3D_0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\theta_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & D^T\hat{A}_f^T & 0 \end{bmatrix}$$

$$\theta_6 = \begin{bmatrix} E_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\theta_7 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & D^T\hat{C}_f^T \end{bmatrix}$$

$$\theta_8 = \begin{bmatrix} -E_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

then filter paramenter is solved by

$$A_{f1} = (S-R)^{-1}\hat{A}_f,\ B_{f1} = (S-R)^{-1}\hat{B}_f,\ C_{f1} = \hat{C}_f$$

Proof. Choose a Lyapunov functional candidate to be

$$V(\xi(k)) = \xi^{\mathrm{T}}(k)P_1\xi(k) + \sum_{j=k-m+1}^{k-1}\xi^{\mathrm{T}}(k)Q_1\xi(k) + \sum_{j=k-m}^{k-1}\xi^{\mathrm{T}}(k)Q_2\xi(k) \quad (13)$$

Denote

$$\tilde{\xi}(k) = \begin{bmatrix} \xi^{\mathrm{T}}(k) & \xi^{\mathrm{T}}(k-m+1) & \xi^{\mathrm{T}}(k-m) & \omega^{\mathrm{T}}(k) & \hat{f}^{\mathrm{T}}(x,u,k) \end{bmatrix}^{\mathrm{T}}$$

and then

$$\Delta V = V(k+1) - V(k)$$

$$= \tilde{\xi}^{\mathrm{T}}(k)\begin{bmatrix} -P_1 + Q_1 + Q_2 & * & * & * & * \\ 0 & -Q_1 & * & * & * \\ 0 & 0 & -Q_2 & * & * \\ 0 & 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}\tilde{\xi}(k) + \xi^{\mathrm{T}}(k)\begin{bmatrix} \tilde{G}_0^{\mathrm{T}} \\ \tilde{G}_{m-1}^{\mathrm{T}} \\ \tilde{G}_m^{\mathrm{T}} \\ \tilde{H}_0^{\mathrm{T}} \\ I \end{bmatrix} P_1 \begin{bmatrix} \tilde{G}_0 & \tilde{G}_{m-1} & \tilde{G}_m & \tilde{H}_0 & I \end{bmatrix}\xi(k)$$

By (11) and Schur complements lemma, we can attain

$$\begin{bmatrix} -P_1 + Q_1 + Q_2 + M_{11} & * & * & * & * & * & * \\ 0 & -Q_1 + M_{22} & * & * & * & * & * \\ 0 & 0 & -Q_2 + M_{33} & * & * & * & * \\ 0 & 0 & 0 & -\gamma^2 I & * & * & * \\ 0 & 0 & 0 & 0 & -I & * & * \\ P_1\tilde{G}_0 & P_1\tilde{G}_{m-1} & P_1\tilde{G}_m & P_1\tilde{H}_0 & P_1 & -P_1 & * \\ \tilde{L}_0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0 \quad (14)$$

In order to solve conveniently, let

$$P_1 = \begin{bmatrix} R & X_{12} \\ X_{12}^{\mathrm{T}} & X_{22} \end{bmatrix}, P_1^{-1} = \begin{bmatrix} S^{-1} & Y_{12} \\ Y_{12}^{\mathrm{T}} & Y_{22} \end{bmatrix}$$

where $R > 0, S > 0$, and assume matrices X_{12} and Y_{12} has full rank

Define

$$T_1 = \begin{bmatrix} S^{-1} & I \\ Y_{12}^{\mathrm{T}} & 0 \end{bmatrix}, T_2 = \begin{bmatrix} I & R \\ 0 & X_{12}^{\mathrm{T}} \end{bmatrix}$$

choose congruent transformation matrix

$$\mathrm{diag}\{T_1, \quad I, \quad I, \quad I, \quad I, \quad T_1, \quad I\}$$

pre- and post- multiplying both sides of inequality (14), one obtains the following inequality,

$$\begin{bmatrix} \phi_1 & * & * & * & * & * & * & * & * & * \\ \phi_2 & \phi_3 & * & * & * & * & * & * & * & * \\ 0 & 0 & \phi_4 & * & * & * & * & * & * & * \\ 0 & 0 & 0 & \phi_5 & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & -\gamma^2 I & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & -I & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & -I & * & * & * \\ G_0 S^{-1} & G_0 & G_{m-1} & G_m & H_0 & S^{-1} & I & -S^{-1} & * & * \\ \phi_6 & \phi_7 & R_0 G_{m-1} & R_0 G_m & \phi_8 & I & R_0 & -I & -R_0 & * \\ \phi_9 & L_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0 \quad (15)$$

where

$\phi_1 = -S^{-1} + S^{-1} R_1 S^{-1} + S^{-1} R_2 S^{-1} + S^{-1} R_3 S^{-1}$

$\phi_2 = -I + R_1 S^{-1} + R_2 S^{-1} + R_3 S^{-1}$

$\phi_3 = -R_0 + R_1 + R_2 + R_3$

$\phi_4 = -R_1 + R_4$

$\phi_5 = -R_2 + R_5$

$\phi_6 = R_0 G_0 S^{-1} + X_{12} M B_f C_0 S^{-1} + X_{12} M A_f M^{-1} Y_{12}^T$

$\phi_7 = R_0 G_0 + X_{12} M B_f C_0$

$\phi_8 = R_0 H_0 + X_{12} M B_f D_0$

$\phi_9 = L_0 S^{-1} - C_f M^{-1} Y_{12}^T$

choose congruent transformation matrix

$$\mathrm{diag}\{S, \quad I, \quad I, \quad I, \quad I, \quad I, \quad I, \quad S, \quad I, \quad I\}$$

pre- and post- multiplying both sides of inequality (15), the following inequality is obtain by,

$$\begin{bmatrix} \phi_{10} & * & * & * & * & * & * & * & * & * \\ \phi_{10} & \phi_{11} & * & * & * & * & * & * & * & * \\ 0 & 0 & \phi_{12} & * & * & * & * & * & * & * \\ 0 & 0 & 0 & \phi_{13} & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & -\gamma^2 I & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & -I & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & -I & * & * & * \\ SG_0 & SG_0 & SG_{m-1} & SG_m & SH_0 & I & S & -S & * & * \\ \phi_{14} & \phi_{15} & R_0 G_{m-1} & R_0 G_m & \phi_{16} & I & R_0 & -I & -R_0 & * \\ \phi_{17} & L_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0 \quad (16)$$

where

$\phi_{10} = -S + R_1 + R_2 + R_3$

$\phi_{11} = -R_0 + R_1 + R_2 + R_3$

$\phi_{12} = -R_1 + R_4$

$\phi_{13} = -R_2 + R_5$

$\phi_{14} = R_0 G_0 + (S - R) B_f C_0 + (S - R) A_f$

$\phi_{15} = R_0 G_0 + (S - R) B_f C_0$

$\phi_{16} = R_0 H_0 + (S - R) B_f D_0$

$\phi_{17} = L_0 - C_f$

Separate uncertainties from (15), we can get

$$\Omega + \theta_1 F(\tau)\theta_2 + \theta_2^{\mathrm{T}} F(\tau)\theta_1^{\mathrm{T}} + \theta_3 F(\tau)\theta_4 + \theta_4^{\mathrm{T}} F(\tau)\theta_3^{\mathrm{T}} + \theta_5 F(\tau)\theta_6 + \theta_6^{\mathrm{T}} F(\tau)\theta_5^{\mathrm{T}} + \theta_7 F(\tau)\theta_8 + \theta_8^{\mathrm{T}} F(\tau)\theta_7^{\mathrm{T}} < 0$$

and lemma 1 is applied to get

$$\Omega + \varepsilon_1 \theta_1 \theta_1^{\mathrm{T}} + \varepsilon_1^{-1} \theta_2^{\mathrm{T}} \theta_2 + \varepsilon_2 \theta_3 \theta_3^{\mathrm{T}} + \varepsilon_2^{-1} \theta_4^{\mathrm{T}} \theta_4 + \varepsilon_3 \theta_5 \theta_5^{\mathrm{T}} + \varepsilon_3^{-1} \theta_6^{\mathrm{T}} \theta_6 + \varepsilon_3 \theta_7 \theta_7^{\mathrm{T}} + \varepsilon_3^{-1} \theta_8^{\mathrm{T}} \theta_8 < 0$$

Finally, (11) is attained.

4. Numerical Simulation

We consider the system (1) with

$$A_0 = \begin{bmatrix} 0.5 & -3.5 \\ -1.2 & 0.6 \end{bmatrix}, A_1 = \begin{bmatrix} -1 & 0.5 \\ 0 & 0.1 \end{bmatrix}, B_0 = \begin{bmatrix} -5.5 \\ 1 \end{bmatrix}$$

$$C_0 = [-3 \quad 0.2], D_0 = 0.5, L_0 = [-2 \quad 1]$$

Letting $h = 0.1, m = 2$, and discretizing system (1), a new state equation is attained with corresponding parameter

$$G_0 = \begin{bmatrix} 1.0735 & -0.3724 \\ -0.1277 & 1.0841 \end{bmatrix}, H_0 = \begin{bmatrix} -0.5862 \\ 0.1381 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} -0.1033 & 0.0498 \\ 0.0062 & 0.0073 \end{bmatrix}, G_2 = \begin{bmatrix} 0.1011 & -0.2337 \\ 0.6279 & -0.3188 \end{bmatrix}$$

$$D = \begin{bmatrix} 0.4982 & 0.4277 \\ 0.2847 & -0.2566 \end{bmatrix}, E = \begin{bmatrix} 0.5899 & -0.3933 \\ -0.5689 & 0.1849 \end{bmatrix}$$

Apply MATLAB LMI Toolbox to solve, and then, filter parameter is given, as follows.

$$A_f = \begin{bmatrix} 1.8721 & -1.136 \\ -1.1152 & -0.0325 \end{bmatrix}$$

$$B_f = \begin{bmatrix} 2.2305 & -1.4230 \end{bmatrix}^T$$

$$C_f = \begin{bmatrix} -1.2175 & 0.6215 \end{bmatrix}$$

By numerical experiment, fitering effect of non-fragile H_∞ filter is better than regular filter obviously.

5. Conclusions

The problem of the non-fragile H_∞ filtering for a class of nonlinear sampled-data system with long time-delay has been investigated. Using direct distribution approach, dimension of state variables is deceased, a novel filter is established, and sufficient condition for the H_∞ performance index of the combined system is derived via linear matrix inequality (LMI). Finally, a simulation example is presented to show the validity and advantages of the proposed method.

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