A New Buy-back Contract Coordinating Dual-channel Supply Chain under Stochastic Demand

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Abstract

The conflict between the manufacturer and the retailer except the double marginalization is an important issue in order to coordinate the dual-channel supply chain. In the general case of the non-linear stochastic demand which also is affected by the sales effort of the retailer, this paper designs a new buy-back contract to coordinate the dual-channel supply chain. On the base of developing the Stackelberg game model between the manufacturer and the retailer, the value of every parameter for the new buy-back contract, which can coordinate the dual-channel supply chain, is achieved respectively. The numerical experiment shows that the effort of the retailer can promote the sales amount of the retailing channel and the direct selling, and increase the profits of the manufacturer and the retailer simultaneously, while the overmuch effort of the retailer isn’t good anymore.

Keywords: Supply Chain Coordination, Dual-Channel, New Buy-Back Contract, Stochastic Demand, Stackelberg Game.

1. Introduction

As the development of electronic pay technology, the manufacturers had the opportunities and abilities to sell to customer directly except the common retailing channel. More and more manufacturing firms attached importance to build their own online direct channel. Many famous brand manufactures, such as Hewlett-packard, Nike, Dell, IBM, Apple, have added their own online direct channel (Tsay and Agrawal, 2004). In China, the electric appliance manufacturers like Haier, Lenovo, also have their own direct marketing channel. In this context, the conventional single retail channel supply chain model changes to dual-channel supply chain model. Manufacturers might get benefit from their own direct channel. On the one hand, they can be closer to customers and understand their real demand information better. On the other hand, they can reduce the cost, expand the market shares, enhance his competitive power, increase the revenues, and avoid be dominated by the retailers. However, this will inevitably causes marketing competition between suppliers and retailers, and hence sharpen their conflict and contradiction. How to coordinate the dual-channel supply chain effectively in order to improve supply chain performance is an important theoretical and practical problem.

As a matter of fact, supply chain contract coordination has been research emphases. Cachon (2003) has reviewed and expanded a series of supply chain contracts, including wholesale price contract, buy-back contract, revenue sharing contract, quantity flexibility contract and quantity discount contract. The supply chain contract research progress also has been reviewed by Yang et al. (2006). There are many literatures on supply chain contract coordination with market demand influenced by the retailer’s sale effort, which includes improving the shelf space layout (Wang, 2001), increasing the advertising investment (Taylor, 2002). For example, Cachon (2003) showed that the quantity discount contract can coordinate such supply chain model, while the constrained buy-back contract by constraining the quantity of buying back products can also coordinate the supply chain (Xu et al., 2005). Suo et al. (2005) evaluated that the effect of the retailer’s loss aversion on supply chain coordination. He et al. (2009) examined how the dependence level of demand to price influences the coordination of such supply chain. However, they mainly focus on the contract coordination of the traditional single retail channel supply chain.

There are many literatures in the dual-channel supply chain conflict and coordination also. Tsay and Agrawal (2004) indicated that although there are conflicts, the direct channel may also increase the retailer’s profit. Dumrongsi et al. (2008) analyzed the equilibrium conditions of the direct to increase the retailer’s profit. Geng and Mallik (2007) discussed the equilibrium inventory strategies for increase the manufacturer and the retailer’s profit simultaneously under multi-channel inventory competition environment. Takahashi et al. (2011) developed a new control policy for the two-echelon dual-channel supply chain with setup costs of production and delivery. Huang et al. (2012) examined how to adjust the price and the production plan so that the potential maximal profit is obtained under a disruptive demand. They found that the optimal production quantity had some robustness under a demand disruption. Li et al. (2011) analyzed the effect of supply chain members’ risk preference on...
choosing of dual-channel supply chain operation mode. Chen et al. (2012) examined the coordination schemes for a dual-channel supply chain, found that a two-part tariff or a profit-sharing agreement, can coordinate the dual-channel supply chain and enable both the manufacturer and the retailer to realize win–win. Xu et al. (2010) designed a coordinating revenue sharing contract under linear demand, and Qu et al. (2010) designed a changed coordinating revenue sharing contract when the marker demand affected by the sales effort of the retailer. Yu and Liu (2012) found that the buy-back contract can improve the manufacturer and the retailer’s profit but can not coordinate the dual-channel supply chain with the linear demand and joint promotion. Chen et al. (2011) pointed out that the system of the innovation compensation also could realize the development of supply chain members’ profit, but cannot coordinate the dual-channel supply chain from the view of traditional distribution and online selling price competition. Wang et al. (2011) showed in the context of the switching fraction of non-linear demand that a revenue sharing contract of direct channel enable to decrease channel conflict, improve the channel system profit and coordinate the supply chain.

It is clear that the existing literatures about dual-channel supply chain coordination with linear demand and stochastic demand are focus on the study of revenue sharing contract, without considering the effect of retailer’s effort on demand, and they fail to design a buy-back contract to coordinate the dual-channel supply chain. In this paper, we consider the general case of the stochastic demand, which is affected by the sales effort of the retailer. On the base of developing the Stackelberg game model between the manufacturer and the retailer, we design a new buy-back contract coordinating the dual-channel supply chain. The value of every parameter for the new buy-back contract, which can coordinate the dual-channel supply chain, is achieved respectively. And by the numerical example, we analyze the effect of retailer’s effort level on the sale quantity of retail channel and direct channel, and on the manufacturer and the retailer’s profit.

2. Hypothesis and denotation

2.1 Hypothesis

As the literature practices (Cachon, 2003; Yang et al., 2006; Taylor, 2002; Qu et al., 2010), we makes the following hypothesis. The dual-channel supply chain includes only a manufacturer and a retailer, which both are the risk neutral. The manufacturer has the retailer and the direct channel. The retailer faces a newsvendor problem and only has one stochastic sales season. Before the sales season, the retailer has only one chance to order from the manufacturer. The market demand is non-linear stochastic, and influenced by the level of retailer’s effort. The retailer’s effort not only can increase the sales quantity of retail channel, but also can improve the direct channel’s sales quantity. However, the manufacturer selling products through direct channel will reduce the retailer’s sales quantity. So, the competition and conflict between dual-channel is inevitable. This is a special problem that makes the dual-channel supply chain difficult to coordinate beside the double marginalization. In addition, we assume that the retailer’s penalty cost, the manufacturer’s penalty cost, the direct channel’s penalty cost and the product’s net salvage value all are zero.

As two mutually independent decision makers, it is a typical Stackelberg game relationship between the manufacturer and the retailer. The manufacturer is the dominant one in the supply chain. The manufacturer first decides the direct price, wholesale price and buy-back price. Then, the retailer decides his effort level, his retail price. The common goal is to pursue own maximum expected profits.

2.2 Denotation

e : the retailer’s effort level, represents all the retailer’s effort activities;
G(e) : the retailer’s effort cost of exerting effort e, where
G(0) = 0, G(e) > 0, G’(e) > 0;
x : the total market demand, which is non-linear stochastic.
According to Taylor (2002), let x = ψ(e) · τ, where ψ(e) is non-negative, differentiable, ψ’(e) > 0, ψ(e) ≤ 0 .
τ is a random variable which independent to e, and whose distribution function is F(τ). density function is f(τ);
F_x (x|e) : the distribution function of total market demand, and it is differentiable, strictly increasing, denote
F_x (x|e) = 1 − F_x (x|e);
f_x (x|e) : the density function of total market demand;
y : the demand of traditional retail channel,
y = θx, where θ (0 < θ < 1) presentation of the market share of retailer, θ is the function about e with
θ(e) > 0, θ(e) < 0;
z : the demand of direct channel, z = (1 − θ)x;
F_r (y|e) : the demand distribution of retail channel;
f_r (y|e) : the demand density of retail channel;
F_d (z|e) : the demand distribution of direct channel;
f_d (z|e) : the demand density of direct channel;
\( w_m \): the wholesale price;
\( b_m \): the buy-back price, namely the price that the manufacturer pays the retailer for the remaining product, \( b_m < w_m \);
\( p_r \): the retail price;
\( p_k \): the direct price;
\( c_m \): the supplier’s production cost per unit;
\( c_r \): the retailer’s marginal cost per unit;
\( c_E \): the direct channel’s marginal cost per unit;
\( q_r \): the retailer’s quantity, \( q_r \) is increasing in \( e \);
\( q_E \): the direct channel’s quantity, \( q_E \) is decreasing in \( e \).

3. The Basic Dual-channel Supply Chain Model

The manufacturer’s profit includes the direct channel’s profit under the dual-channel supply chain. The direct channel leads to competition and conflict between the supply chain members, the retailer’s effort not only affects the retail channel, but also the direct channel. According to the above hypothesis, we can obtain that the distribution function of total market demand is \( F_X(x|e) = F\left(\frac{x}{\psi(e)}\right) \), the density function is \( f_X(x|e) = \frac{1}{\psi(e)} \frac{F\left(\frac{x}{\psi(e)}\right)}{\psi(e)} \); The demand distribution function of retail channel is \( F_T(y|e) = F\left(\frac{y}{\theta(e) \cdot \psi(e)}\right) \), the density function is \( f_T(y|e) = \frac{1}{\theta(e) \cdot \psi(e)} \frac{F\left(\frac{y}{\theta(e) \cdot \psi(e)}\right)}{\psi(e)} \).

Let \( S_r(q_r, e) \) be the expected sales of retailer, and \( S_E(q_E, e) \) be the expected sales of direct channel, \( Z(q_r) \) be the transfer payment function. Under the above conditions, through simple calculation, we can obtain:

The retailer’s expected sales is
\[
S_r(q_r, e) = \int_0^{q_r} y f_T(y|e) dy + \int_{q_r}^{\infty} q_r f_T(y|e) dy = q_r - \int_0^{q_r} F_T(y|e) dy = \frac{3}{2} q_r - \frac{1}{2 \theta(e) \cdot \psi(e)} q_r^2,
\]
the expected sales of direct channel is
\[
S_E(q_E, e) = \int_0^{q_E} y f_E(z|e) dz + \int_{q_E}^{\infty} q_E f_E(z|e) dz = q_E - \int_0^{q_E} F_E(z|e) dz = \frac{3}{2} q_E - \frac{1}{2 (1 - \theta(e)) \cdot \psi(e)} q_E^2.
\]

Therefore, the retailer’s expected profit function under dual-channel is
\[
\Pi_r(q_r, e) = p_r S_r(q_r, e) - c_r q_r - G(e) - Z(q_r) = \left(\frac{3}{2} p_r - c_r\right) q_r - \frac{p_r}{2 \theta(e) \cdot \psi(e)} q_r^2 - G(e) - Z(q_r) \tag{1}
\]
the manufacturer’s expected profit function is
\[
\Pi_m(q_r, q_E, e) = Z(q_r) - c_m q_r + p_k S_E(q_E, e) - c_E q_E = Z(q_r) - c_m q_r + \left(\frac{3}{2} p_k - c_E\right) q_E - \frac{p_k}{2 (1 - \theta(e)) \cdot \psi(e)} q_E^2 \tag{2}
\]
and the dual-channel supply chain’s profit function is
\[
\Pi_u(q_r, q_E, e) = p_r S_r(q_r, e) - \left(\begin{array}{c} c_r + c_m \end{array}\right) q_r + p_k S_E(q_E, e) - c_E q_E - G(e) = \left(\frac{3}{2} p_r - c_r - c_m\right) q_r - \frac{p_r}{2 \theta(e) \cdot \psi(e)} q_r^2 + \left(\frac{3}{2} p_k - c_E\right) q_E - \frac{p_k}{2 (1 - \theta(e)) \cdot \psi(e)} q_E^2 - G(e) \tag{3}
\]
Consider the problem of the centralized decision making. By Eq. (3), for a given effort level, it can be obtained that there is \( \frac{\partial^2 \Pi_u(q_r, q_E, e)}{\partial q_r^2} = -p_r / \theta(e) \cdot \psi(e) < 0 \) and \( \frac{\partial^2 \Pi_u(q_r, q_E, e)}{\partial q_E^2} = -p_k / (1 - \theta(e)) \cdot \psi(e) < 0 \), this means that the dual-channel supply chain system exists only one optimal solution, i.e., the retailer’s optimal order quantity \( q_r^* \) satisfies \( q_r^* = \arg\left[\frac{\partial \Pi_u(q_r, q_E, e)}{\partial q_r}\right] = 0 \). According to the first-order condition of Eq. (3), the specific retailer’s optimal order quantity is
\[
q_r^* = \frac{3}{2} p_r - c_r - c_m \frac{p_r}{p_k} - \theta(e) \cdot \psi(e) \tag{4}
\]
and the direct channel’s optimal order quantity \( q_E^* \) satisfies \( q_E^* = \arg\left[\frac{\partial \Pi_u(q_r, q_E, e)}{\partial q_E}\right] = 0 \), i.e.,
\[
q_E^* = \frac{3}{2} p_k - c_E \frac{p_k}{p_r} - (1 - \theta(e)) \cdot \psi(e) \tag{5}
\]
In a similar way, the retailer’s optimal effort level for a given \( q_r \) and \( q_E \) satisfies \( e^* = \arg\left[\frac{\partial \Pi_u(q_r, q_E, e)}{\partial e}\right] = 0 \). According to the first-order condition of Eq. (3), the specific retailer’s optimal effort level is
\[
e^* = \arg\left[p_r \frac{\partial S_r(q_r, e^*)}{\partial e} + p_k \frac{\partial S_E(q_E, e^*)}{\partial e} - G(e^*)\right] = 0 \tag{6}
\]
4. The New Buy-back Contract

With a buy-back contract the supplier charges the retailer \( w_m \) ( \( w_m < p_r \) ) per unit purchased, but pays the retailer \( b_m \) ( \( b_m < w_m \) ) per unit remaining at the end of the selling season. The transfer payment between the manufacturer and the retailer is

\[
Z(q_r, w_m, b_m) = w_m q_r - b_m (q_r - S(q_r, e)) = b_m S(q_r, e) + (w_m - b_m) q_r
\]

(7)

However, according to Cachon (2003), the buy-back contract cannot coordinate the supply chain with effort-dependent demand. In the way of the general processing method, we introduce a contract parameter \( \phi(0 < \phi < 1) \) (Qu et al., 2010), which presents the fraction of effort cost the retailer bears. So, \( (1 - \phi) \) is the fraction of the manufacturer. According to Eq. (1) and Eq. (7), the retailer’s expected profit function is

\[
\Pi_r(q_r, e, w_m, b_m) = (p_r - b_m) S_r(q_r, e) - (c_r + w_m - b_m) q_r - \phi G(e)
\]

(8)

By Eq. (2) and Eq. (7), the manufacturer’s expected profit is

\[
\Pi_m(q_r, q_E, e, w_m, b_m) = b_m S_r(q_r, e) - (c_m - w_m + b_m) q_r + p_E S_E(q_E, e) - c_E q_E - (1 - \phi) G(e)
\]

(9)

By Eq. (3) and Eq. (7), the dual-channel supply chain’s expected profit function is

\[
\Pi_d(q_r, q_E, e, w_m, b_m) = p_r S_r(q_r, e) - (c_r + c_m) q_r + p_E S_E(q_E, e) - c_E q_E - G(e)
\]

(10)

On the other hand, consider the problem of the decentralized decision making. Because it is clear that \( \partial^2 \Pi_d(q_r, q_E, e, w_m, b_m) / \partial q_r^2 = -(p_r - b_m) / (\theta(e) \cdot \psi(e)) < 0 \), the retailer has a unique optimal order quantity \( q_r^* \). Then, by Eq. (8), the retailer’s optimal order quantity for a given effort level satisfies

\[
q_r^* = \arg \frac{\partial \Pi_m(q_r, q_E, e, w_m, b_m)}{\partial q_r} = 0,
\]

i.e.,

\[
q_r^* = \frac{3}{2} \frac{p_r - 1}{2} \frac{b_m - c_r - w_m}{p_r - b_m} \theta(e) \cdot \psi(e)
\]

(11)

And the retailer’s optimal effort level for a given \( q_r \) satisfies \( e^* = \arg \frac{\partial \Pi_r(q_r, e, w_m, b_m)}{\partial e} = 0 \), i.e.,

\[
e^* = \arg \frac{3}{2} \frac{p_E - c_E}{p_r} (1 - \theta(e)) \cdot \psi(e)
\]

(12)

By \( \partial^2 \Pi_d(q_r, q_E, e, w_m, b_m) / \partial q_r^2 = -p_r / (1 - \theta(e)) \cdot \psi(e) < 0 \), the direct channel has unique optimal order quantity \( q_E^* \). By Eq. (9), the direct channel’s optimal order quantity for a given effort level satisfies

\[
q_E^* = \arg \frac{\partial \Pi_m(q_r, q_E, e, w_m, b_m)}{\partial q_E} = 0
\]

i.e.,

\[
q_E^* = \frac{3}{2} \frac{p_E - c_E}{p_r} (1 - \theta(e)) \cdot \psi(e)
\]

(13)

5. The Dual-channel Supply Chain Coordination with New Buy-back Contract

The competition and conflict between the supply chain members will occur when the manufacturer adds a direct channel. To make the dual-channel supply chain achieves coordination, the coordination of the retailer’s order quantity, effort level and the direct channel’s order quantity must be realized at the same time. We can prove that as long as the value of every parameter for the new buy-back contract are appropriate, the dual-channel supply chain can achieve coordination. We can obtain the following Propositions respectively.

**Proposition 1:** The retailer’s order quantity \( q_r \) can be coordinated, as long as the contract parameter \( w_m \) and \( b_m \) satisfies

\[
w_m = b_m + c_r - \frac{c_m + c_E}{p_r} b_m
\]

(14)

**Proof:** If the retailer’s order quantity achieved coordination, \( q_r = q_r^* \). By Eq. (4) and Eq. (11), Eq. (14) can be obtained through calculation. Q.E.D.

**Proposition 2:** The retailer’s effort level \( e \) can be coordinated, as long as the contract parameter \( b_m \) and \( \phi \) satisfies
1 - \phi = \frac{b_m \partial S_m(q_r,e) + p_E \partial S_E(q_E,e)}{G(e)} \tag{15}

**Proof:** If the retailer’s effort level achieved coordination, 
\(e^* = e^{**}\). By Eq. (6) and Eq. (12), Eq. (15) can be obtained through simple calculation. **Q.E.D.**

The Eq. (5) and Eq. (13) show that the direct channel optimal order quantity is relative to itself price, marginal cost and the retailer’s effort level. When the retailer’s effort level realizes coordination, the coordination of the direct channel order quantity will be achieved. So, we can receive the following conclusion:

**Conclusion:** The new buy-back contract can coordinate the dual-channel supply chain as long as the value of every contract parameter satisfies the certain condition.

### 6. Numerical experiment

In order to discuss the model and illustrate the conclusion more clearly, this section through numerical example analysis the optimal decision results of dual-channel supply chain members before and after channel coordination with the above new buy-back contract. Suppose a certain kind of product with the market characteristics: 
\(\psi(e) = 1000(1 - e^{-1})\), \(\theta(e) = e\sqrt{e}/(\sqrt{e} + 1)\), \(x = 1000(1 - e^{-1})\). \(\tau\), where the random variable \(\tau\) comply with uniform distribution at [0.5, 1.5]. \(G(e) = 50e^2\), \(p_r = 120\), \(c_r = 10\), \(c_m = 30\), \(p_E = 80\), \(c_E = 8\), \(w_m = 55\), \(b_m = 25\), \(\phi = 0.7\).

We put these parameters into the above model. And through Matlab software, we can obtain the optimal decision results of decision-makers under centralized decision and decentralized decision, and the value of every contract parameter when the supply chain achieve coordination, which is shown in the Table 1.

The results of Table 1 shows that the dual-channel supply chain system profit under centralized decision higher than buy-back contract model under decentralized decision before coordination. With the appropriate contract parameters value, the new buy-back contract make the system profit under decentralized decision equals to the system profit under centralized decision, which realize the dual-channel supply chain coordination. After the coordination, the retailer’s optimal order quantity and effort level are increased, but the direct channel’s optimal order quantity is reduced. According to Eq. (8), Eq. (9), Eq. (11) and Eq. (13), we can obtain the corresponding the retailer’s order quantity, the direct channel’s order quantity, the retailer’s profit and the manufacturer’s profit for a given effort level respectively. Thus, we can examine the effect of the retailer’s effort level on the decisions and profits of the supply chain members by the obtained data, which is shown in the following Figure 1 and Figure 2 respectively.

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<th>System index</th>
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<th>After coordination</th>
<th>Centralized decision</th>
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<td>Dual-channel supply chain</td>
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The results of Fig. 1 shows that with the increasing of the retailer’s effort level, the retailer’s order quantity increase gradually and tend to be stable. From the general view, the reason is that the greater effort level, the higher sales and order quantity. However, the total amount of the market is limited, and the retailer’s order quantity can not continue to increase with the effort level. So, the order quantity maintaining in a stable state, otherwise, the retailer will face the risk of big increase in the number of inventory and unmarketable. The order quantity of direct channel appears decreasing gradually. But, the reduced scope becomes more and more small. The reason is that the greater effort level, the higher sales and order quantity. However, the total amount of the market is limited, and the retailer’s order quantity can not continue to increase with the effort level. So, the order quantity maintaining in a stable state, otherwise, the retailer will face the risk of big increase in the number of inventory and unmarketable. The order quantity of direct channel appears decreasing gradually. But, the reduced scope becomes more and more small. The reason is that the greater effort level, the higher sales and order quantity.
a non-linear stochastic demand influenced by the retailer’s sales effort. Because the traditional buy-back contract cannot coordinate the supply chain with the effort-dependent demand, we introduce the effort cost sharing parameter into the buy-back contract, i.e., the supplier shares the fraction of the retailer’s effort cost. Our results reveal that the new buy-back contract can coordinate the dual-channel supply chain as long as the contract parameters satisfy certain conditions. In the numerical analysis, we verify the theoretical analysis by calculating the specific optimal decision results of supply chain members under centralized decision and decentralized decision, and further examine the effect of retailer’s effort level on the sale quantity of retail channel, direct channel, and on the manufacturer and the retailer’s profit. The results of numerical analysis show that the appropriate increase of the retailer’s effort level can promote the increase of the sale quantity of retail channel and direct channel simultaneously, and also the profit of the manufacturer and the retailer. However, as the continuous increasing of the retailer’s effort level, the direct channel’s order quantity and the manufacturer’s profit decrease slowly down, and the increasing of the retailer’s profit is also very limited. The research conclusions have certain realistic significance to supply chain members, while still have limitations. For example, it doesn’t consider the effort level of the manufacturer while the manufacturer may also increase the direct channel’s sales quantity through improve effort level, which is the next research direction.

**References**


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