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Structural Fatigue Reliability Based on Extension of Random Loads into Interval Variables

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Abstract

According to the problem that for a structure under random loads, the structural fatigue life can't be directly calculated out by S-N curves and linear Miner cumulative damage rule. Owing to the uncertainty of loads, and the problem of the inaccuracy of calculated structural reliability index for the existence of deviation between measured data in projects and real data, the research method for structural fatigue reliability based on extension of random loads into interval variables is proposed. The innovation is that we can accurately calculate out the interval of the structural fatigue life and reliability index of a structure according to the probability density function of stress level of random loads and the coefficient of variation of measured loads. By practical calculation example, it is proved that this method is more suitable to practical engineering comparing to traditional methods. It will provide a perfect research approach for reliability analysis of the structure under random loads.

Keywords: *Random load, Structural reliability analysis, Fatigue life; Interval analysis, Rayleigh distribution*

1. Introduction

Study of structural fatigue lead by random loads, is an important aspect in the study of structural reliability. Fatigue failure is one of the main failure modes for a structure. According to the statistical data, 80% of structural failure belongs to fatigue fracture. For example, the shaft, bearings, springs and frames are mostly fractured under random loads. To accurately predict the fatigue life of a structure according to the random cyclic loads that the structure is bearing, is the foundation of limited fatigue lifetime design and reliability analysis for a structure. However, the current theoretical research mainly focuses on fatigue reliability of a structure under constant amplitude cyclic loads or multistage constant amplitude cyclic loads with ideal state, while fatigue reliability of a structure under random cyclic loads is relatively less. In fact, in practical engineering the structure under constant amplitude cyclic loads or multistage constant amplitude cyclic loads with ideal state is rare, and in the vast majority of cases, the structural fatigue failure is caused by random cyclic loads exerting repeatedly. In current theoretical analysis and practical engineering calculation, the conservative calculation method is commonly used, that is to choose a deterministic load with maximum peak value in all cyclic loads to substitute the actual loads, as well as to use the mathematical model of fatigue life of a structure under deterministic constant amplitude cyclic loads to predict the actual structural fatigue life. In fact, the results that calculated out are often far different from the practical condition of the structure.

2. Structural Fatigue Life Estimation Based on the Miner Rule

The calculation of the structural fatigue life is based on fatigue cumulative damage theory. When a structure is bearing constant amplitude cyclic loads with ideal state, according to S-N curves of the structural materials and structural characteristics, the fatigue life of the structure can be calculated out directly. When a structure is bearing multistage constant amplitude cyclic loads with ideal state, then the fatigue life of the structure can be calculated out according to linear Miner cumulative damage rule. Suppose that, a structure bears k stages stresses of fatigue loads in its fatigue life: $S_1, S_2, ..., S_k$, with the corresponding acting number for each stage of cyclic loads are respectively: $n_1, n_2, ..., n_k$. Then by Miner fatigue cumulative damage rule, and according to the S-N curves and structural characteristics of composing materials, we can get the fatigue cumulative damage of the structure produced by k stages of cyclic loads as follow:

$$\Delta D = \sum_{i=1}^{k} D(s_i) = \frac{n_1}{N_1} + \frac{n_2}{N_2} + ... + \frac{n_k}{N_k} = \sum_{i=1}^{k} \frac{n_i}{N_i} \quad (1)$$

During the above formula, n_i is the action number of cyclic loads of the i th stress level; N_i is the failure cycles number under the cyclic loads of the i th stress level. For a structure in service, when its fatigue cumulative damage produced by all levels of cyclic loads is up to the failure threshold (failure threshold is usually taken 1), it will fail.

The randomness of the cyclic loads will not only include the uncertainty of the stress level of loads, but also include the uncertainty of action times that the loads imposed to the structure. When taken the action times of cyclic loads as the lifetime measurement index, the uncertainty of loads mainly manifested as the uncertainty of stress level of loads, as shown in figure 1. Then, the uncertainty of stress level caused by the action of random cyclic loads can be described as the corresponding probability density function.

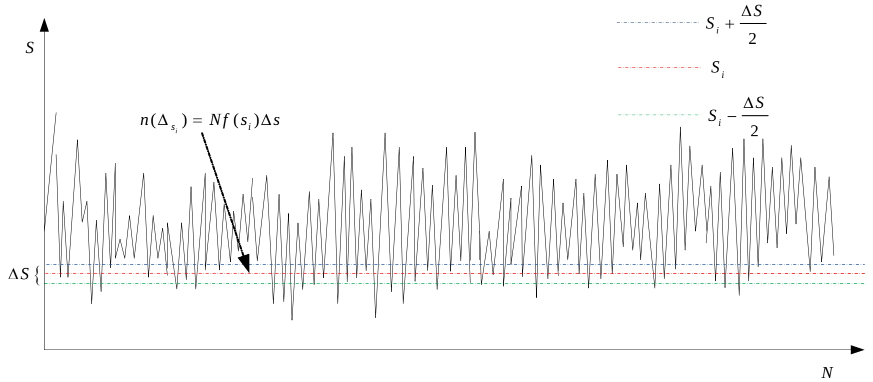

Fig. 1 Distribution of random cyclic loads

Suppose that, the stress levels of cyclic loads are random variables which obey probability density function $f(x)$. When the stochastic process is a narrow band process, the probability density of the peak values can be approximately considered as obeying Rayleigh distribution, that is:

$$f(S) = \frac{S}{\sigma^2}\exp\left(-\frac{S^2}{2\sigma^2}\right) \quad (2)$$

During the above formula, σ is the shape parameter of state curve of Rayleigh distribution, and is a constant.
As shown in figure 2, according to the characteristics of probability density function, the probability $P(S_i)$ of stress values corresponding to the effect of per random cyclic loads acting, located in interval $\left[S_i - \frac{\Delta S}{2}, S_i + \frac{\Delta S}{2}\right]$ nearby S_i is

$$P(S_i) = f(S_i)\Delta S = \frac{S_i}{\sigma^2}\exp\left(-\frac{S_i^2}{2\sigma^2}\right)\Delta S \quad (3)$$

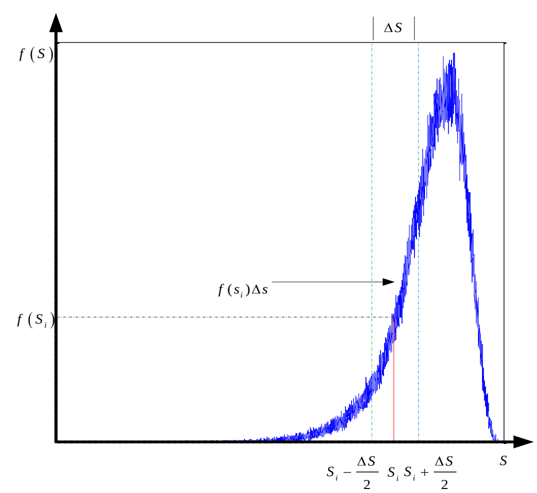

Fig. 2 Probability density distribution that the peaks of loads located in interval $\left[S_i - \frac{\Delta S}{2}, S_i + \frac{\Delta S}{2}\right]$ nearby S_i in Rayleigh distribution

If the action times of random cyclic loads is N, the times $n(\Delta_{S_i})$ that the stresses located in interval $\left[S_i - \frac{\Delta S}{2}, S_i + \frac{\Delta S}{2}\right]$ nearby S_i are

$$n(\Delta_{S_i}) = Nf(S_i)\Delta S = N\frac{S_i}{\sigma^2}\exp\left(-\frac{S_i^2}{2\sigma^2}\right)\Delta S \quad (4)$$

According to Miner fatigue cumulative damage rule, the structural fatigue cumulative damage $\Delta D_i(S_i)$ caused by the loads, whose stresses located in interval $\left[S_i - \frac{\Delta S}{2}, S_i + \frac{\Delta S}{2}\right]$, can be expressed as

$$\Delta D_i(S_i)=\frac{n(\Delta_{S_i})}{N_i}=\frac{N\frac{S_i}{\sigma^2}\exp\left(-\frac{S_i^2}{2\sigma^2}\right)\Delta S}{N_i} \quad (5)$$

For the reason of the structure are absorbing the acting of fatigue loads with k stress levels, let's divide the taking value of the whole interval of probability distribution of stress level into k subintervals correspondingly, and suppose that the midpoint of the i th subinterval is S_i. Form the above analysis, we can learn that, when the action times of the random cyclic loads is N, the stress acting times that located in the i th subinterval is $n(\Delta_{s_i})$, and the corresponding structural fatigue cumulative damage is $\Delta D_i(S_i)$.

According to formula (1), we can get the total structural fatigue cumulative damage under N times action of random cyclic loads is as follow

$$D=\Delta D_1(S_1)+\Delta D_2(S_2)+\ldots+\Delta D_k(S_k)=\sum_{k=1}^{k}\Delta D_i(S_i) \quad (6)$$

Substitute formula (5) into formula (6), we can get

$$D=\frac{Nf(S_1)\Delta S}{N_1}+\frac{Nf(S_2)\Delta S}{N_2}+\ldots+\frac{Nf(S_k)\Delta S}{N_k}=\sum_{k=1}^{k}\frac{N\frac{S_i}{\sigma^2}\exp\left(-\frac{S_i^2}{2\sigma^2}\right)\Delta S}{N_i} \quad (7)$$

Usually, the relationship between the fatigue life N of structural material and stress level S can be expressed as the form of power series, as follow:

$$S^m N=C \quad (8)$$

During the above formula, m and C are material constant.

According to the relationship between fatigue life and stress level shown in formula (8), the fatigue life N_i corresponding to stress level S_i can be expressed as:

$$N_i=\frac{C}{S_i^m} \quad (9)$$

Substitute formula (9) into formula (7), we can get

$$\begin{aligned}D&=\frac{Nf(S_1)S_1^m\Delta S}{C}+\frac{Nf(S_2)S_2^m\Delta S}{C}+\ldots+\frac{Nf(S_k)S_k^m\Delta S}{C}\\&=\sum_{k=1}^{k}\frac{Nf(S_i)S_i^m\Delta S}{C}=\sum_{k=1}^{k}\frac{N\frac{S_i}{\sigma_S^2}\exp\left(-\frac{S_i^2}{2\sigma_S^2}\right)S_i^m\Delta S}{C}\end{aligned} \quad (10)$$

When $k\to\infty$, formula (10) is equivalent to the following formula:

$$\begin{aligned}D&=\lim_{k\to\infty}\sum_{k=1}^{k}\frac{Nf(S_i)S_i^m\Delta S}{C}=\int_0^{+\infty}\frac{N\frac{S}{\sigma^2}\exp\left(-\frac{S^2}{2\sigma^2}\right)S^m}{C}ds\\&=N\int_0^{+\infty}\frac{\frac{S^{m+1}}{\sigma^2}\exp\left(-\frac{S^2}{2\sigma^2}\right)}{C}ds\end{aligned} \quad (11)$$

Furthermore, we can get the structural fatigue life N under the action of random cyclic loads as follow:

$$N=\frac{D}{\int_0^{+\infty}\frac{\frac{S^{m+1}}{\sigma^2}\exp\left(-\frac{S^2}{2\sigma^2}\right)}{C}ds} \quad (12)$$

When the fatigue failure of the structure happen, its fatigue cumulative damage is equivalent to the failure threshold, that is $D=\delta=1$. Then formula (12) can be written as:

$$N=\frac{1}{\int_0^{+\infty}\frac{\frac{S^{m+1}}{\sigma^2}\exp\left(-\frac{S^2}{2\sigma^2}\right)}{C}ds} \quad (13)$$

During formula (12) and formula (13), owing to δ, C, σ and m are all for constant. We can calculate out the structural fatigue life on the basis of the mathematical model established above, according to the known probability density function of the random cyclic loads stress levels and the material parameters of the structure.

3. Mathematical Modeling of Structural Fatigue Reliability Based on Extension of Random Loads into Interval Variables

Suppose that the designed life of the structure is n_D, then let's first establish the structural fatigue state equation g_n (safety margin), measured by loading action times, that is:

$$g_n = \frac{1}{\int_0^{+\infty} \frac{\frac{S^{m+1}}{\sigma^2}\exp\left(-\frac{S^2}{2\sigma^2}\right)}{C} ds} - n_D \qquad (14)$$

To taylor series expansion for the state equation g_n on the average point μ_g, and according to the linear feature of interval variables operation, the mean value and the deviation of the structural response are respectively:

$$\left.\begin{aligned} \mu_g &= g(x_1^c, x_2^c, ..., x_n^c) \\ \sigma_g &= \sum_{i=1}^{n}\left(\left|\frac{\partial g}{\partial x_i}\right|\right)_{\mu} \sigma_{x_i} \end{aligned}\right\} \qquad (15)$$

Substitute formula (14) into formula (15), we can get the mean value and deviation of the fatigue state equation of the structure under the action of random cyclic loads, which are respectively:

$$\left.\begin{aligned} \mu_g &= \frac{1}{\left(\int_0^{+\infty} \frac{\frac{S^{m+1}}{\sigma^2}\exp\left(-\frac{S^2}{2\sigma^2}\right)}{C} ds\right)_{\mu_S}} - n_D \\ \sigma_g &= \left(\left|\frac{\exp\left(-\frac{S}{2\sigma^2}\right)\left[\left(\frac{S^{m+2}}{2\sigma^4}\right) - \frac{(m+1)S^m}{\sigma^2}\right]}{\frac{1}{C}\left[\int_0^{+\infty}\frac{S^{m+1}}{\sigma^2}\exp\left(-\frac{S^2}{2\sigma^2}\right)ds\right]^2}\right|\right)_{\mu_S} \sigma_s \end{aligned}\right\} \qquad (16)$$

By the properties of Rayleigh distribution, we can get $\mu_S = \sigma\sqrt{\frac{\pi}{2}}$, $\sigma_S = \frac{4-\pi}{2}\sigma^2$.

Expand the random variables into interval variables, and the result is:

$$\left.\begin{aligned} \mu_g^I &= \left[\underline{\mu}_g, \overline{\mu}_g\right] = \left[\mu_g(1-\alpha_{\mu_g}), \mu_g(1+\alpha_{\mu_g})\right] \\ \sigma_g^I &= \left[\underline{\sigma}_g, \overline{\sigma}_g\right] = \left[\sigma_g(1-\alpha_{\sigma_g}), \sigma_g(1+\alpha_{\sigma_g})\right] \end{aligned}\right. \qquad (17)$$

$$\beta^I = [\underline{\beta}, \overline{\beta}] = \left[\frac{\underline{\mu}_g}{\overline{\sigma}_g}, \frac{\overline{\mu}_g}{\underline{\sigma}_g}\right] \qquad (18)$$

For the convenience of calculation, let be $\alpha_{\mu_g} = \alpha_{\sigma_g} = \alpha$, then substitute formula (17) into formulas (16) and (18), and the result is:

$$\underline{\beta} = \frac{\underline{\mu}_g}{\overline{\sigma}_g} = \frac{\frac{1}{\left(\int_0^{+\infty} \frac{\frac{S^{m+1}}{\sigma_S^2}\exp\left(-\frac{S^2}{2\sigma_S^2}\right)}{C} ds\right)_{\underline{\mu}_S = \mu_S(1+\alpha)}} - n_D}{\left(\left|\frac{\exp\left(-\frac{S}{2\sigma^2}\right)\left[\left(\frac{S^{m+2}}{2\sigma^4}\right) - \frac{(m+1)S^m}{\sigma^2}\right]}{\frac{1}{C}\left[\int_0^{+\infty}\frac{S^{m+1}}{\sigma^2}\exp\left(-\frac{S^2}{2\sigma^2}\right)ds\right]^2}\right|\right)_{\mu_S} \sigma_s(1+\alpha)} \qquad (19)$$

$$\overline{\beta}=\frac{\overline{\mu}_g}{\underline{\sigma}_g}=\frac{\dfrac{1}{\left(\displaystyle\int_0^{+\infty}\frac{\frac{S^{m+1}}{\sigma_S^2}\exp\left(-\frac{S^2}{2\sigma_S^2}\right)}{C}ds\right)_{\underline{\mu}_S=\mu_S(1-\alpha)}}-n_D}{\left(\left|\dfrac{\exp\left(-\frac{S}{2\sigma^2}\right)\left[\left(\frac{S^{m+2}}{2\sigma^4}\right)-\frac{(m+1)S^m}{\sigma^2}\right]}{\frac{1}{C}\left[\int_0^{+\infty}\frac{S^{m+1}}{\sigma^2}\exp\left(-\frac{S^2}{2\sigma^2}\right)ds\right]^2}\right|\right)_{\mu_S}\sigma_s(1-\alpha)} \tag{20}$$

4. Example

Suppose that the fatigue loads that a structure bears are random cyclic loads and the random process is a narrow band one, then the probability density of the peak value of stress levels for cyclic loads can be approximately deemed to obey Rayleigh distribution. Let be $\sigma=1$, and the mean stress level for cyclic loads is $\mu_S=\sigma\sqrt{\frac{\pi}{2}}$, while the deviation is $\sigma_S=\frac{4-\pi}{2}\sigma^2$. Given the influence of environment and other factors in the actual engineering process, there will be some errors between the actual situation and the measured mean stress levels of cyclic loads and deviation. Let the coefficient of variation be $\alpha_{\mu_S}=\alpha_{\sigma_S}=0.1$, and the value of the parameter curve m is 3, and C value is 1×10^{10}. The designed life is $n_D=10^6$. Try to analyze and calculate out the reliability of the structure system.

Solution: (1) When the coefficient of variation of stress levels for cyclic loads during data acquisition is not taken into account, the stress levels for cyclic loads will be random variables. Substitute the known conditions into formula (16), and the result is:

$$\mu_g=1.313742\times10^6$$

$$\sigma_g=0.495317\times10^6$$

And then

$$\beta=\frac{\mu_g}{\sigma_g}=2.652326$$

(2) When the coefficient of variation of stress levels for cyclic loads during data acquisition is taken into account, the stress levels for cyclic loads will be interval variables. Substitute the known conditions into formulas (17), (19), and (20), and the result is:

$$\overline{\mu}_g=1.445116\times10^6$$

$$\underline{\mu}_g=1.182368\times10^6$$

$$\overline{\sigma}_g=0.544849\times10^6$$

$$\underline{\sigma}_g=0.445785\times10^6$$

And then

$$\underline{\beta}=\frac{\underline{\mu}_g}{\overline{\sigma}_g}=2.170084$$

$$\overline{\beta}=\frac{\overline{\mu}_g}{\underline{\sigma}_g}=3.241733$$

According to the above analysis, conclusions can be drawn that due to the existing of coefficient of variation of mean value and deviation of stress levels for cyclic loads, there will be some deviations between the calculated index of the structure reliability and the true value. And if the true index of structure reliability is substituted by this calculated value, the calculated structure reliability may be too conservative or too optimistic, and thus not in line with the actual engineering process. However, the use of the theory of expanding random variables into interval variables can help identify approximately the interval range of indexes of structure reliability.

5. Conclusions

(1) Due to the indeterminacy of loads, S-N Curves and linear Miner cumulative fatigue damage rules are not suitable for the study on fatigue life of structures under the action of random loads. This article builds a model for calculation of fatigue life of structures under the actions of random loads, and can obtain the corresponding fatigue life of structures relatively more accurate , according to the probability density function of stress distribution for random cyclic loads. Then the corresponding indexes of structure reliability can be determined with the relationship between the calculated structure fatigue life and designed life.

(2) As there are inevitable deviations between

measured data in the actual engineering process and the authentic data, correspondingly, the calculated indexes of the structure reliability based on the measured data cannot be the exact values. If they are used for deduction of structure reliability, it will not be in line with the actual engineering process. This article expands the random variables of random cyclic loads into interval variables, builds a model for indexes of structure reliability with the method of interval analysis, and determines the fluctuation range for indexes of structure reliability, from which the lower limit can be taken in structure design.

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