The Evidential Reasoning Approach for Multiple Decision Analysis Using Normal Cloud Model

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Abstract

In this paper, normal cloud model and evidential reasoning (E-R) approach is used in multiple attribute decision analysis (MADA) problems. Different attributes Belief function are represented by cloud model interval. Using cloud model generating algorithm, belief degree interval is obtained without numerical computation. In addition, it is reasonable and it accords with human's mind. Evidential reasoning algorithm is also used to incorporate different attributes interval in different ranks. Maximum and minimum in belief degree interval is computed by software. Then aggregative index number of attribute value is computed. Thing's rank is decided by the index number. In the example, truck's integrated performances are analysed. Simulation results further illustrate the effectiveness of the design method.

Keywords: *Multiple attribute decision analysis (MADA), Evidential reasoning (E-R) approach, Cloud model, Cloud model belief degree.*

1. Introduction

Multiple attribute problem of quantitative and qualitative attribute are both exist in practice. Recently, it is a hot topic. There are different attributes in an object, which can be divided into two categories: data attribute and quality attribute. Data attribute is quantitative and quality attribute value is qualitative. Various factors should be taken into account in object analysis and evaluation. We use evidence reasoning method in multiple attribute decision analysis (MADA) problem[1]. Two attributes are in the same framework of MADA problem. We deal with the two attributes by unifying level estimating reliability structure and uncertainty of fuzzy linguistic variables [2].

MADA problem is mainly based on D-S theory, so it is lack of flexibility. In this paper, we use D-S 、E-R theory combined with normal cloud model to analyze multiple attribute problem. The advantage of this method is belief degree fuzzification. Fuzzy belief degree interval much more accords with human's mind than unfuzzed one. In belief degree calculating, X cloud model generating algorithm is used, which could adapt to MADA problem[3-7].

This work is organized as follow, the normal cloud model theory and evidential reasoning theory are reviewed in Section 2. Section 3 introduces effects of evidential reasoning using normal cloud model. Section 4 shows the simulation and results. Finally, Section 5 concludes the study of future work.

2. Review of Related Works

2.1 Normal Cloud Model

Definition 1:

Let U be the set, $U=\{x\}$,as the universe of discourse and T a linguistic term T , $C_T(x)$ is a random variable with a probability distribution $C_T(x)$ takes values in [0, 1]. A membership cloud is a mapping from the universe of discourse U to the unit interval [0,1], that is

$$C_T(x)\ :\ U \to [0,1]$$

$$\forall\ x\ \forall\ x \in U\ , x \to C_T(x)$$

In the society and science, the expected curve of membership cloud approach normal distribution, so we usually study the quality of normal membership cloud[1].Normal cloud curve can be describe using three important parameters $(Ex,\ En,\ He\ )$, Ex represents fixed quality conception or expected value, which is the center of normal cloud; En is entropy, which is the expected value and center value of He at the same time, is the scale in measuring the fuzzy degree and the only standard in measuring bandwidth[1]. He is super entropy (entropy's entropy), which represents the uncertain degree of En and shows the sparse degree of cloud. The three characteristics is the frame of cloud theory. Using the three characteristics, fixed conception could be represented by cloud model[3].

Normal half-ascended cloud model generated algorithm

(1) Give the expected value Enx, deviation Hex

(2) Generate a n dimensional normal random Enx, whose expected value is $E'nx$, deviation is Hex

(3) Generate a n dimensional normal random $x = x_i$, whose expected value is Ex, deviation is $E'nx$.

(4) Compute:

$$C_T = \exp[-\frac{1}{2}\frac{(x_i - Ex_i)^2}{E'nx_i^2}]$$

(5) Repeat (1) ~ (4) till the number of cloud model drops is enough, if $x = x_i$ is given, the algorithm is X cloud model algorithm.

2.2 Generation and Structure of Cloud Model Belief Degree

If M Objects are estimated, there are L attributes in every object, and there are N ranks in every attribute, which are independent respectively. The rank of object α in attribute e_i is H_n. Belief degree is $\beta_{n,i}(\alpha_l)$. The estimation is $S(e_i(\alpha_l))$ ={ $H_n, \beta_{n,i}(\alpha_l)$ $(n = 1\cdots N)$ }, $\beta_{n,i}(\alpha_l) \geq 0$

If there are N cloud model rank H_n $(n = 1\cdots N)$ in M objects , then they are independent. Cloud model belief of H_n in attribute e_i of object α_l $(l = 1\cdots M)$ is:

$[\inf \beta^-_{n,i}(\alpha_l), \sup \beta^-_{n,i}(\alpha_l)] \cup [\inf \beta^+_{n,i}(\alpha_l)$,where $\sup \beta^+_{n,i}(\alpha_l)]$, $\sup \beta^+_{n,i}(\alpha_l) \geq \sup \beta^-_{n,i}(\alpha_l)$,

$\inf \beta^+_{n,i}(\alpha_l) \geq \inf \beta^-_{n,i}(\alpha_l)$.

Cloud model belief ($\beta^-_{n,i}(\alpha_l)$ and $\beta^+_{n,i}(\alpha_l)$ are interval values, which are generated by X cloud model algorithm. It is based on attribute interval [x_i^-, x_i^+], as $S(e_i(\alpha_l))$ ={ H_n ,[$\inf \beta^-_{n,i}(\alpha_l)$, $\sup \beta^-_{n,i}(\alpha_l)$] $\cup$ $[\inf \beta^+_{n,i}(\alpha_l), \sup \beta^+_{n,i}(\alpha_l)]$, $n = 1\cdots N$ }, where $\beta^-_{n,i}(\alpha_l) \geq 0$. If $\inf \beta^-_{n,i}(\alpha_l) \equiv \inf \beta^+_{n,i}(\alpha_l)$, $\sup \beta^+_{n,i}(\alpha_l) \equiv \sup \beta^-_{n,i}(\alpha_l)$, then attribute value is precise. As follow, we give the definition:

Definition 2 :

$S(e_i(\alpha_l))$ ={ H_n,[$\inf \beta^-_{n,i}(\alpha_l), \sup \beta^-_{n,i}(\alpha_l)$] $\cup$ [$\inf \beta^+_{n,i}(\alpha_l)$, $\sup \beta^+_{n,i}(\alpha_l)$], is cloud model estimated vector of attribute value, if

$[\inf \beta^-_{n,i}(\alpha_l), \sup \beta^-_{n,i}(\alpha_l)] \cup$

$[\inf \beta^+_{n,i}(\alpha_l), \sup \beta^+_{n,i}(\alpha_l)]$ satisfies :

$\exists\ \beta_{n,i}(\alpha_l) \in [\inf \beta^-_{n,i}(\alpha_l), \sup \beta^-_{n,i}(\alpha_l)]$,

$\sum_{n=1}^{N}\beta_{n,i}(\alpha_l) \leq 1$, then $S(e_i(\alpha_l))$ is valid, otherwise it is invalid.

Definition 3:

$S(e_i(\alpha_l))$ ={ H_n,[$\inf \beta^-_{n,i}(\alpha_l), \sup \beta^-_{n,i}(\alpha_l)$] $\cup$ [$\inf \beta^+_{n,i}(\alpha_l)$, $\sup \beta^+_{n,i}(\alpha_l)$], $n = 1\cdots N$ } is attribute
cloud distribution estimation vector, if belief interval $[\inf \beta^-_{n,i}(\alpha_l), \sup \beta^-_{n,i}(\alpha_l)]$,

$\exists\ \beta^-_0(\alpha_l) \in [\inf \beta^+_{n,i}(\alpha_l), \sup \beta^+_{n,i}(\alpha_l)]$, $\beta^+_0(\alpha_l) \in [\inf \beta^+_{n,i}(\alpha_l), \sup \beta^+_{n,i}(\alpha_l)]$,

$\exists\ \forall\ \beta_{n,i}(\alpha_l) \in [\beta^-_0(\alpha_l), \beta^+_0(\alpha_l)]$,

where $\sum_{n=1}^{N}\beta_{n,i}(\alpha_l) = 1$,

$S(e_i(\alpha_l))$ is called complete estimated vector, or incomplete.

In complete cloud distribution estimation, there is only one rank estimation in α_l. The other belief distributes are in the whole set H, if cloud distribution is not incomplete.

Definition 4:

$S(e_i(\alpha_l))$ = { H_n,[$\inf \beta^-_{n,i}(\alpha_l), \sup \beta^-_{n,i}(\alpha_l)$] $\cup [\inf \beta^+_{n,i}(\alpha_l), \sup \beta^+_{n,i}(\alpha_l)]$, $n = 1\cdots N$ }

is not incomplete cloud model estimated vector. Belief degree
$\beta_{H,i}(\alpha_l)$ is assigned to H [4-5].

$$\inf \beta^-_{H,i}(\alpha_l) = \max(0, 1 - \max\sum_{n=1}^{N}\beta^-_{n,i}(\alpha_l)),$$

$$\sup \beta^-_{H,i}(\alpha_l) = \max(0, 1 - \inf\sum_{n=1}^{N}\beta^-_{n,i}(\alpha_l)) \quad (1)$$

$$\inf \beta^+_{H,i}(\alpha_l) = \max(0, 1 - \max\sum_{n=1}^{N}\beta^+_{n,i}(\alpha_l)),$$

$$\sup \beta^+_{H,i}(\alpha_l) = \max(0, 1 - \inf\sum_{n=1}^{N}\beta^+_{n,i}(\alpha_l)) \quad (2)$$

after L attributes in M objects are estimated[8-11], cloud model belief degree Decision-making matrix is given: $D=(S(e_i(\alpha_l)))_{L\times M}$

3. Attribute Dater Representation in Cloud Model Belief Degree

3.1 Attribute Data Representation in Cloud Model Belief Degree

Dater attribute usually could be divided into two parts

(1)Accurate data attribute representation
Attribute value is usually represented by accurate data. To deal with MADA problem in E-R method, we make all the cloud model rank Figure and get the belief interval of data. In order to describe the Evaluation rank of data attribute, we should know effect of every rank. H_1 is impossible rank and H_N the highest[12-13].

(2)Interval data attribute representation
Because interval data crosses over many ranks, representation of cloud model is much more complex. If $[x_i^-, x_i^+]$ crosses two ranks H_n, H_{n+1}, the other is the same. Belief degree $\beta_{n,i}$ is generated by X cloud model algorithm. Belief degree interval on H_n, H_{n+1} is

$\{H_n, [\inf \beta_{n,i}^-(\alpha_l), \sup \beta_{n,i}^-(\alpha_l)] \cup [\inf \beta_{n,i}^+(\alpha_l), \sup \beta_{n,i}^+(\alpha_l)], n=1\cdots N\}$; $\{H_{n+1}, [\inf \beta_{n+1,i}^-(\alpha_l), \sup \beta_{n+1,i}^-(\alpha_l)] \cup [\inf \beta_{n+1,i}^+(\alpha_l), \sup \beta_{n+1,i}^+(\alpha_l)], n=2\cdots N\}$

3.2 Data Integration of Attribute Cloud Model Distribution Belief Degree

E-R analysis algorithm can fully use and synthesize the evidence. Cloud model theory can strengthen capability of processing uncertain evidence data. Cloud model theory belief is transformed into mass function using formula(3)~(6)

$$m_{n,i}=m_i(H_n)=\omega_i\beta_{n,i}(\alpha_l), n=1\cdots N; i=1\cdots L \quad (3)$$

$$m_{H,i}=m_i(H)=1-\sum_{n=1}^{N} m_{n,i}=1-\omega_i\sum_{n=1}^{N}\beta_{n,i}(\alpha_l), \quad i=1\cdots L \quad (4)$$

$$\bar{m}_{H,i}=\bar{m}_i(H)=1-\omega_i, i=1\cdots L \quad (5)$$

$$\tilde{m}_{H,i}=\tilde{m}_i(H)=\omega_i(1-\sum_{n=1}^{N}\beta_{n,i}(\alpha_l)), \; i=1\cdots L \quad (6)$$

$$m_{H,i}=\bar{m}_{H,i}+\tilde{m}_i(H), \text{ and } \sum_{i=1}^{L}\omega_i=1$$

The possibility of set H is m_H and it is divided into two parts $\tilde{m}_H$, $\bar{m}_H$. Multiple attribute mass function are integrated in (7)~(12)

$$\{H_n\}: m(H_n)=k\{\prod_{i=1}^{L}[m_i(H_n)+m_i(H)]-\prod_{i=1}^{L} m_i(H)\}, n=1\cdots N \quad (7)$$

$$\{H\}: \tilde{m}_H=k\{\prod_{i=1}^{L} m_i(H)-\prod_{i=1}^{L}\bar{m}_i(H)\} \quad (8)$$

$$\{H\}: \bar{m}_H=k[\prod_{i=1}^{L}\bar{m}_i(H)] \quad (9)$$

$$k=\{\sum_{n=1}^{L}\prod_{i=1}^{L}[m_i(H_n)+m_i(H)]-(N-1)\prod_{i=1}^{L}\bar{m}_i(H)\}^{-1} \quad (10)$$

$$\{H_n\}: \beta_n=\frac{m(H_n)}{1-\bar{m}_H}, n=1\cdots N \quad (11)$$

$$\{H\}: \beta_H=\frac{\tilde{m}(H)}{1-\bar{m}_H} \quad (12)$$

Multiple attribute cloud model belief degree is based on (13~15)

$$m_{n,i}=m_i(H_n)\in[\inf m_{n,i}^-, \sup m_{n,i}^+]=[\omega_i \inf \beta_{n,i}^-, \omega_i \sup \beta_{n,i}^+], n=1\cdots N \;\; i=1\cdots L \quad (13)$$

$$\bar{m}_{H,i}=\bar{m}_i(H)=1-\omega_i, \; i=1\cdots L \quad (14)$$

$$\tilde{m}_{H,i}=\tilde{m}_i(H)\in[\inf \tilde{m}_{H,i}^-, \sup \tilde{m}_{H,i}^+]=[\omega_i \inf \beta_{H,i}^-, \omega_i \sup \beta_{H,i}^+] \quad (15)$$

$$\text{and} \sum_{n=1}^{N} m_{n,i}+\bar{m}_{H,i}+\tilde{m}_{H,i}=1, \sum_{i=1}^{L}\omega_i=1$$

3.3 Cloud Model Distribution Expectation Effect

$$u(S(e_i(\alpha_l)))=\sum_{n=1}^{N} u(H_n)\beta_n(\alpha_l), l=1\cdots M$$

$u(S(e_i(\alpha_l)))$ is cloud model distribution expectation effect and $u(H_n)$ is the effect of H_n. $\beta_{n,i}(\alpha_l)$ is belief degree of α_l on H_n. If distribution is complete,

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$\beta_H(\alpha_l)$ =0 and If distribution is complete, incomplete $u(S(e_i(\alpha_l)))$ has maximum and minimum: (16-17)

$$u_{\max}(\alpha_l)=\sum_{n=1}^{N-1}u(H_n)\beta_n(\alpha_l)$$
$$(\beta_N(\alpha_l)+\beta_H(\alpha_l))u(H_N),l=1\cdots M \qquad (16)$$
$$u_{\min}(\alpha_l)=\sum_{n=2}^{N}u(H_n)\beta_n(\alpha_l)+$$
$$(\beta_1(\alpha_l)+\beta_H(\alpha_l))u(H_1),l=1\cdots M \qquad (17)$$
$$u_{ave}(\alpha_l)=\frac{u_{\max}(\alpha_l)+u_{\min}(\alpha_l)}{2}$$

4. Experiment Result

Description of truck's attributes

There are many factors in truck's comprehensive estimation: acceleration time (s), braking (m), power (kw), gear-box property, weight ω_i =0.25 $(i=1\cdots 4)$.
rank of truck: top(T), excellent(E) , good(G), average(A) , poor(P) , worst(W).
H_j={ $H_j \mid j=1\cdots 7$ }={'top', 'excellent', 'good', 'average', ' poor', ' worst'}

Table 1: Attribute values

attribute	car1	Car 2
acceleration time	4.4	4.0
braking	[19.2,19.26]	[19.11,19.2]
power	288	223
gear-box property	5.4	[6,7]

Table 2: Belief degree of attributes in different ranks

attribute	truck 1	truck 2
acceleration time	P [0.03, 0.09] A [0.8, 0.86]	G [0.25, 0.42] E [0.25, 0.42]
braking	G [0, 0] E [1, 1] E [0.78,0.90] G [0,0.15]	T [0.02,0.16] E [0.81,0.88] T [0, 0] E [1, 1]
power	E [0.1, 0.17] T [0.54, 0.68]	P [0.31, 0.45] A [0.21, 0.34]
gear-box property	A [0.1, 0.3] P [0.5, 0.6]	G [1, 1] A [0, 0]

Data in Table 1 is transferred into belief degree interval in Table 2 using cloud model theory . Every attribute value corresponds to a belief interval.

Every different attribute has its rank (Fig.1~Fig.4)

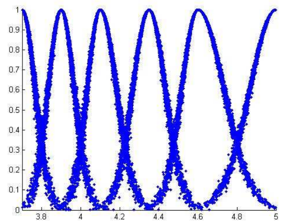

Fig.1 Cloud rank of acceleration time

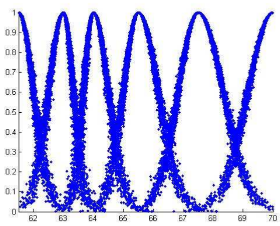

Fig.2 Cloud rank of braking

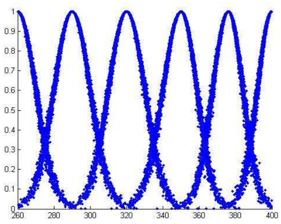

Fig.3 Cloud rank of power

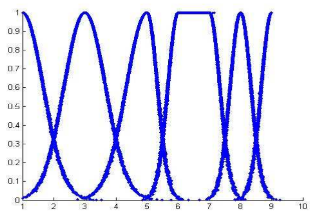

Fig.4 Cloud rank of gear-box property

Use (4)~(11) formula of dater integration
Truck 1, acceleration time:
$m_{1,1}$ =0, $m_{2,1}$ =0, $m_{3,1}$ =0, $m_{4,1}\in$ 0.25*[0.8,0.86], $m_{5,1}\in$ 0.25*[0.03,0.09], $m_{6,1}$ =0, $m_{1,H}\in 1-0.25$ *[0.83,0.95]=[0.76,0.79], $\bar{m}_1(H)\in$ [0.75, 0.75], $\tilde{m}_1(H)\in$ [0.01, 0.75]
braking:
$m_{1,2}$ =0, $m_{2,2}\in$ 0.25*[0.78,1], $m_{3,2}\in$ 0.25*[0, 0.15], $m_{4,2}$ =0, $m_{5,2}$ =0, $m_{6,2}$ =0, $m_{2,H}\in 1-0.25$*[0.78,1]=[0.75,0.8], $\bar{m}_2(H)\in$ [0.75,0.75], $\tilde{m}_2(H)\in$ [0, 0.06]
power:
$m_{1,3}\in$ 0.25*[0.54, 0.68], $m_{2,3}\in$ 0.25*[0.1, 0.17], $m_{3,3}$ =0, $m_{4,3}$ =0, $m_{5,3}$ =0, $m_{6,3}$ =0, $m_{3,H}\in$ $1-0.25$*[0.64, 0.85]=[0.79,0.84], $\bar{m}_3(H)\in$ [0.75, 0.75], $\tilde{m}_3(H)\in$ [0.04, 0.09]

gear-box property:
$m_{1,4} \in 0.25*[0.54, 0.68]$, $m_{2,4} \in 0.25*[0.1,0.17]$,
$m_{3,4}$ =0, $m_{4,4} \in 0.25*[0.1,0.3]$, $m_{5,4} \in 0.25*[0.5,0.6]$,
$m_{6,4}$ =0 $m_{4,H} \in 1-0.25*[0.6,0.9]$=
[0.77, 0.85], $\bar{m}_4(H) \in [0.75,0.75]$, $\tilde{m}_4(H) \in [0.02,0.1]$
Truck2:
$\bar{m}_H \in [0.32,0.5]$, $\tilde{m}_H \in [0.01,0.34]$, $\beta_1 \in [0,0.76]$,
$\beta_2 \in [0.18,1]$, $\beta_3 \in [0.2,1]$, $\beta_4 \in [0.03,0.84]$, $\beta_5 \in$
[0.04, 0.88], β_6 =0, $\beta_H \in [0.02,0.68]$

5. Conclusion

This paper has proposed a new method of evidence reasoning based on normal cloud model and introduced an method that belief degree is represented in interval value. To overcome the drawbacks of evidence reasoning, we adopted fuzzy method. Example in truck shows that the method could achieve better estimating effect than generic evidence reasoning, and own a good performance in truck quanlity estimation. We will further consider the selection of cloud model parameter in future work.

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