Application of Volterra LMS Adaptive Filter Algorithm Based on Gaussian Distribution

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Abstract

This paper mainly studied the LMS adaptive filter algorithm to the Volterra system model. Through the construction of the second order Volterra system model, the application of respectively selecting the first order and second order variable step length de-correlation Volterra LMS algorithm in gaussian noise environment, when the input signals in different correlation coefficient, the iteration times are not more than 2000 times and all items can realize the convergence, which prove the accuracy of the algorithm paper presented. The Volterra LMS adaptive filter algorithm can be effectively applied into the mechanical vibration damping and noise elimination, which has a broad application prospect.

Keywords: *Volterra series, adaptive filter algorithm, LMS, system identification, gaussian distribution.*

1. Introduction

Adaptive filter research began in the 1950's. Widrow and Hoff, etc first puts forward the least mean square (LMS) algorithm. [1] LMS algorithm has the advantages of simple structure, small amount of calculation, and easy to realize real-time processing, so, in the field of the low noise elimination, spectrum enhancement, and system identification, etc, which has been widely used. [2] Among them, the traditional adaptive linear filtering theory based on the gaussian noise model has more mature, and has a wide range of applications in many engineering fields. However, with the expansion of signal processing field, nonlinear filter adaptive filter algorithm is gradually becoming the research hot spot in the world. Volterra adaptive filter algorithm have been successful applied in the military industry and other signal processing or modeling, which reflect its accuracy.

In this paper, we mainly research the Volterra LMS adaptive filtering algorithm [3]based on the gauss noise environment, and through the simulation of the fault vibration model of the nonlinear system identification, which proved the performance of the Volterra LMS adaptive filter algorithm.

2. Volterra series nonlinear system

A discrete causal nonlinear Volterra system relationship between the input signal $x(n)$ and its output $y(n)$ can be expressed as the Volterra series formula. [4]

$$\begin{aligned} y(n) &= h_0 + \sum_{m_1=0}^{\infty} h_1(m_1)x(n-m_1) \\ &+ \sum_{m_1=0}^{\infty}\sum_{m_2=0}^{\infty} h_2(m_1,m_2)x(n-m_1)x(n-m_2) + \ldots \\ &+ \sum_{m_1=0}^{\infty}\sum_{m_2=0}^{\infty}\ldots\sum_{m_p=0}^{\infty} h_p(m_1,m_2,\ldots,m_p)x(n-m_1)x(n-m_2)\ldots x(n-m_p) + \ldots \end{aligned} \tag{1}$$

Among the formula (1), $h_p(m_1,m_2,\ldots,m_p)$ is called as p orders Volterra kernel coefficient. It is called linear kernel when p=1. Volterra series can be seen as the Taylor series expansion with memory circumstance, which can approach to any continuous nonlinear system model. Formula (1) expresses there are infinity numbers Volterra kernel to the nonlinear system. In the fact application, we should carry out truncation process in the practical application.

Truncation process contains two aspects of the order number p and memory depth N. How to truncate is relevant to the specific nonlinear system type and the performance of the requirements. Usually, only considering the second order truncation model, that is p=2, and hypothesis h_0 = 0, memory depth is N. System can be simplified:

$$\begin{aligned} y(n) &= \sum_{m_1=0}^{\infty} h_1(m_1)x(n-m_1) \\ &+ \sum_{m_1=0}^{N-1}\sum_{m_2=m_1}^{\infty} h_2(m_1,m_2)x(n-m_1)x(n-m_2) \end{aligned} \tag{2}$$

Among the formula (2), we suggest the kernel of Volterra series is symmetrical. To any of $p!$ numbers $m_1, m_2, ..., m_p$ transposition, $h_p(m_1, m_2, ..., m_p)$ is equation.

Thus, formula (2) has $N(N+3)/2$ numbers Volterra kernel. Considering the symmetrical character to the Volterra series. We can define system kernel quantity in n times.

$$\begin{aligned}\boldsymbol{H}(n) = [&h_1(0;n), h_1(1;n), \cdots, h_1(N-1;n)\\ &, h_2(0,0;n), h_2(0,1;n), \cdots,\\ &h_2(0,N-1;n), h_2(1,1;n), .., h_2(N-1,N-1;n)]^T\end{aligned} \quad (3)$$

The same we can define system input vector in n time.

$$\begin{aligned}\boldsymbol{X}(n) = [&x(n), x(n-1), \cdots, x(n-N+1), x^2(n)\\ &, x(n)x(n-1), \cdots, x(n)x(n-N-1),\\ &x^2(n-1), .., x^2(n-N+1)]^T\end{aligned} \quad (4)$$

Thus the output can be expressed as formula (5) in n times.

$$y(n) = \boldsymbol{H}^T(n)\boldsymbol{X}(n) \quad (5)$$

Formula (2) and (5) state that a nonlinear system can be state extended to express as linear combination of the input vector $X(n)$ each component, which is the advantages of the nonlinear system Volterra series expressed.

3. Adaptive volterra filter

If the known system has the form style as the formula (5), but its kernel vector $\boldsymbol{H}(n)$ is unknown, so, we can use Figure 1 to identify the system kernel vector $\boldsymbol{H}(n)$ similar to the linear style. In the Figure 1, $\boldsymbol{W}(n)$ is Volterra filter coefficient vector with the length of $M=N(N+3)/2$.

If we define the Volterra filter coefficient vector $\boldsymbol{W}(n)$ is: $w(n) = [w_0(n), w_1(n), ..., w_{M-1}(n)]^T$, then the output of Volterra filter $c(n)$ is : $c(n) = \boldsymbol{W}^T(n)\boldsymbol{X}(n)$, among the formula, $X(n)$ is the input vector of the formula (3).

The purpose of the system identification is changing filter coefficient vector $\boldsymbol{W}(n)$ through a adaptive algorithm, which to make error information $e(n)$ into minimum in a sense. That is to say, it will make some a cost function $J(n)$ of $e(n)$ into minimum. When the cost function $J(n)$ approaching to minimum, we can think $H(n) \approx \boldsymbol{W}(n)$.

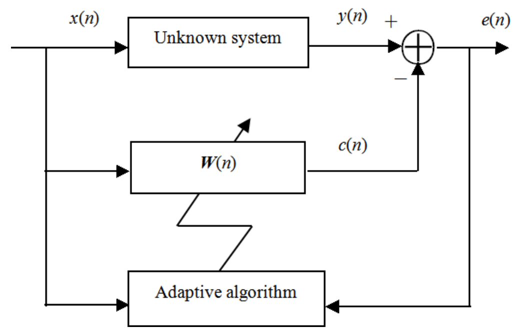


Fig. 1 Identification method based on adaptive filter.

If we define the cost function $J(n)$ as formula (6).

$$J(n) = e^2(n) = [y(n) - c(n)]^2 = [y(n) - \boldsymbol{W}^T(n)\boldsymbol{X}(n)]^2 \quad (6)$$

We can calculate the $\boldsymbol{W}(n)$ differential coefficient to $J(n)$. And make the changing direction is opposite to the differential coefficient direction of $\boldsymbol{W}(n)$. Then we can get the optimal $\boldsymbol{W}(n)$ recursion algorithm. That is LMS algorithm. [5] LMS algorithm process can be concluded as formula (7). [6]

$$\begin{aligned}e(n) &= y(n) - \boldsymbol{W}^T(n)\boldsymbol{X}(n)\\ W(n+1) &= W(n) + u\boldsymbol{X}(n)e(n)\end{aligned} \quad (7)$$

Among the formula (7), the initial value of $\boldsymbol{W}(n)$ can be determined according to the prior knowledge, or simply select $\boldsymbol{W}(n) = [0,0,...,0]^T$. Step length u decides convergence speed, tracing character, and the stability of the LMS algorithm. In order to assure the convergence speed, and stability system, we can adopt standardization method to determine the step length u.

$$u_n = u \| \boldsymbol{X}(n) \|^2 \quad (8)$$

Among the formula (7), $u_n(0 < u_n < 2)$, which called standardization step length. LMS algorithm has the advantage of little calculation amount, but because system has nonlinear character, it makes the correlation matrix eigenvalue of input signal extend to big, and lead the convergence speed to slow. In order to speed up the convergence speed, in this paper, we adopt different step length to the system linear part and the nonlinear part. [7] The first order and second order terms of Volterra series use different convergence factor, the weight vectors iterative formula of LMS algorithm [8] is shown as bellow.

$$\boldsymbol{W}(n+1)=\boldsymbol{W}(n)+\begin{bmatrix}\mu_1 & \cdots & 0 & 0 & \cdots & 0\\ 0 & \ddots & 0 & 0 & \ddots & 0\\ 0 & \cdots & \mu_1 & 0 & \cdots & 0\\ 0 & \cdots & 0 & \mu_2 & \cdots & 0\\ 0 & \ddots & 0 & 0 & \ddots & 0\\ 0 & \cdots & 0 & 0 & \cdots & \mu_2\end{bmatrix}\boldsymbol{X}(n)e(n) \quad (9)$$

If we adopt scalar style. It is shown as formula (10).

$$w(1,m_1;n+1)=w(1,m_1;n)+\mu_1 e(n)x(n-m_1)$$
$$w(2,m_1,m_2;n+1)=w(2,m_1,m_2;n)+\mu_2 e(n)x(n-m_1)x(n-m_2) \quad (10)$$

Among the formula (10), $m_1=0,1,\cdots,N-1$; $m_2=0,1,\cdots,N-1$. And in order to assure algorithm convergence in statistical sense, Volterra LMS algorithm convergence factor must choose in the following range shown as formula (11).

$$0<\mu<\frac{1}{tr[\boldsymbol{R}]}<\frac{1}{\lambda_{\max}} \quad (11)$$

Among the formula (11), $\lambda_{\max}$ is the eigenvalue of maximum of the input vector autocorrelation matrix $\boldsymbol{R}=E[\boldsymbol{X}(n)\boldsymbol{X}^T(n)]$. Form the style we can see, the condition of convergence of Volterra series is same as the traditional LMS algorithm, but because the definition of the input vector $\boldsymbol{X}(n)$ is different, which making Volterra LMS method input vector autocorrelation matrix $\boldsymbol{R}$ contains the input signal of high order statistics. So, even in white noise condition, it will lead matrix characteristic value expansion. When the correlation of the input signal become stronger, the convergence speed will become slow, even cannot realize convergence. [9]

4. Algorithm simulation and character analysis

This algorithm is used for a nonlinear system model [10] damping and de-noising, it was assumed that identify a nonlinear time-invariant system input and output relationship for:

$$c(n)=0.7512x(n)+0.3467x(n-1)+0.1231x(n-2)$$
$$+0.6892x^2(n)+0.2154x^2(n-1)$$
$$-1.4893x^2(n-2)+0.5625x(n)x(n-1)$$
$$-1.5903x(n-1)x(n-2)+2.3467x(n)x(n-2)$$
$$+noise(n)$$

The first order kernel coefficient respectively is 0.7512, 0.3467, and 0.1231; the second kernel coefficient respectively is 0.6892, 0.2154, -1.4893, 0.5625, -1.5903, and 2.3467. The input signal is $x(n)=ax(n-1)+noise(n)$, among it , $noise(n)$ is gaussian white noise with mean value is 0 and variance is 1. We set the input signal signal to noise ratio is 20dB. We adopt weight coefficient error to analyze algorithm performance. And take the second order Volterra filter with the memory length N of 3. The simulation result is obtained through 50 times independent simulation after taking average. When the input signal is for weak correlation signal, namely a=0.3, the kernel coefficient convergence curve of Volterra series by the Volterra LMS algorithm calculating is shown as Figure 2.

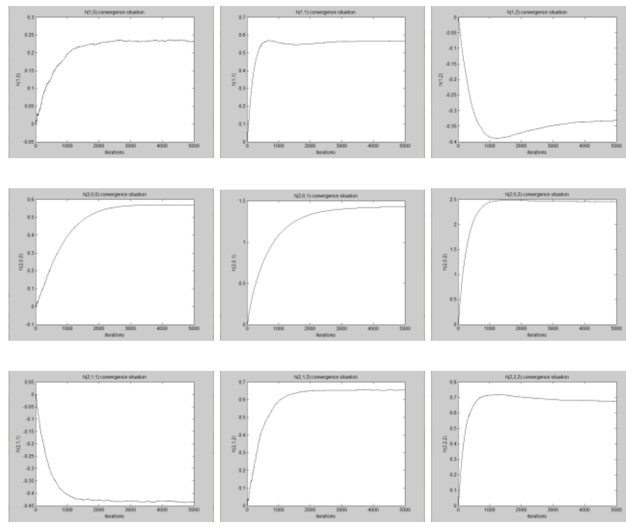
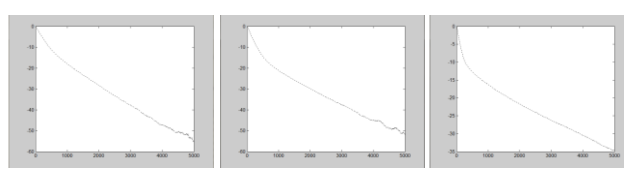


Fig. 2 Each weight value convergence situation comparison under the weak correlation.

From the Fig. 2 we can see, in the weak correlation cases, Volterra LMS algorithm can realize fast convergence, steady state disorder quantity is relatively low. However, in the medium correlation and strong correlation cases, the input signal weight value still can realize convergence, but the convergence speed is slow. Figure 3 is given in three related intensity input signal under the condition of weight coefficient error norm curve of the Volterra LMS algorithm.



a=0.3 a=0.6 a=0.9

Fig. 3 Weight coefficient error bound norm.

From the Fig. 3 we can see, with the increasing of the input signal correlation strength (From 0.3, 0.6 until 0.9), the weight coefficient error norm curve all decrease, that is error is smaller by smaller, but with the increasing of correlation strength, the mismatch error can not achieve minimum in a relatively short period of time.

5. Conclusion

This paper studies the model of Volterra LMS adaptive filter algorithm, through using the advantage of variable step length and decorrelation, respectively adopts different convergence factor to the first order and second order terms, which improving the performance of Volterra LMS algorithm. Simulation shows that this algorithm in different input signal correlation situation has better convergence performance and steady state performance. This Volterra LMS adaptive filter algorithm can better able to used in mechanical model damping and noise elimination, which has a broad application prospect.

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